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RESEARCH ARTICLE

Hybrid entrepreneurship and risk

Ignacia Benitez · Claudio A. Bonilla · Marcos Vergara

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Abstract In this paper, we study the impact of risk on time allocation decisions between occupations by modeling a hybrid entrepreneur who must decide how to allocate time between paid employment (labor) and working on a venture (entrepreneurship). We argue that hybrid entrepreneurs self-insure in response to income risk by managing the time they allocate between the two occupations. We provide the conditions under which an uninsurable risk (in paid employment or the entrepreneurial sector) has an unambiguous precautionary effect on the optimal time allocated to each occupation, and these conditions are based on the strengths of risk aversion and downside risk aversion. We focus on three cases: when risk affects only the entrepreneurial sector, which is the classical case studied in the occupational choice literature; when risk affects only the paid employment sector; and finally, when risk affects both sectors, as we experienced during the recent pandemic.

Plain English Summary In this paper, we study the impact of risk on time allocation decisions between occupations by modeling a hybrid entrepreneur who must decide how to allocate time between paid employment (labor) and working on a venture (entrepreneurship). We argue that hybrid entrepreneurs self-insure in response to income risk by managing the time they allocate between the two occupations. We provide the conditions under which an uninsurable risk (in paid employment or the entrepreneurial sector) has an unambiguous precautionary effect on the optimal time allocated to each occupation, and these conditions are based on the strengths of risk aversion and downside risk aversion. We focus on three cases: when risk affects only the entrepreneurial sector, which is the classical case studied in the occupational choice literature; when risk affects only the paid employment sector; and finally, when risk affects both sectors, as we experienced during the recent pandemic.

Keywords Hybrid entrepreneurship · Risk · Risk apportionment · Precautionary effect

JEL Classification D81 · L26 · M13

I. Benitez · M. Vergara
School of Economics and Business, Universidad del Desarrollo, Concepción, Chile
e-mail: ibenitezl@udd.cl
marcosvergara@udd.cl

C. A. Bonilla (✉)
School of Economics and Business, University of Chile, Santiago, Chile
e-mail: cbonilla@fen.uchile.cl

1 Introduction

The COVID-19 disruption has permanently altered how we understand work. In many activities and professions, individuals now enjoy more flexible employment relationships that provide more freedom in the use of their time. In this new context, the idea that some individuals have the possibility to choose how

to divide their working time between paid employment and developing a venture (becoming an entrepreneur) is a reality.[1] Therefore, the study of hybrid entrepreneurs regains relevance and realism. Following the time allocation theory initially developed by Becker (1965), this paper studies the role of risk in precautionary time allocation by hybrid entrepreneurs.

The nature of entrepreneurship and wage employment is distinct. As asserted by Burmeister-Lamp et al. (2012), a key determinant of the time allocation decision between working on a new venture and a wage job is the sense of certainty associated with wages coming from the paid employment sector versus the risk surrounding the potential returns from entrepreneurship. Traditionally, entrepreneurship has been described as a risky activity, while paid employment is an activity that enjoys a certain degree of stability over time. However, assuming that risk is concentrated only in the entrepreneurial sector is unrealistic. Risk is a transversal phenomenon that affects a wide range of productive activities: evidence of this is the COVID-19 pandemic and its consequent economic impact, which is an event of greater social and economic magnitude than the typical event we were used to experiencing. The different policy decisions adopted worldwide to fight COVID-19, such as lockdowns, business operating restrictions, hygiene protocols, and social distancing, among other prevention practices, had an adverse effect on labor markets, causing a decline in working hours, an increase in job loss, inactivity, and lost income. According to ILO, in 2020, the consequences of pandemic disruption for labor markets worldwide were on an historically unprecedented scale, so an obvious response to such economic hardships, derived from a weakened and riskier labor market, was to develop small new ventures as hybrid entrepreneurs.

Hybrid entrepreneurs represent a significant share of all entrepreneurial activity (Folta et al., 2010). Boosting entrepreneurial activity after the COVID-19 pandemic is pivotal because of the role entrepreneurs play in economic recovery through technological innovations and job creation (e.g., Faria et al., 2009; Faria, 2015; Glaeser et al., 2015). Therefore, it is important to understand how risk impacts the way hybrid entrepreneurs allocate their time to different activities by a process of optimization that considers the risk involved in each activity.

However, what is time management, and why is it so important? A response to this question is easy to find in the management literature. For example, Lévesque and Stephan (2020) defined it as the systematic process of accounting for time and then allocating it more efficiently among a series of prioritized tasks and activities. Aeon and Aguinis (2017) defined it as a form of decision-making used by individuals to structure, protect, and adapt their time to changing conditions. As asserted by Lévesque and MacCrimmon (1998), time is one of the most important inputs contributed by the entrepreneur and the most valuable and scarcest resource in business (Zachary et al., 2015). The relevance of efficiently allocating time between occupations is that it constitutes a control variable that individuals—as hybrid entrepreneurs—have to buffer the adverse effects of negative shocks in the economy.

The buffer is known as precautionary behavior in the economic literature. For example, an exogenous shock such as the pandemic creates risk about future labor income, which risk-averse consumers see as "bad" and therefore risk-averse individuals want less of it. On the other hand, precautionary behavior leads into more working hours and, as a result, a higher expected payment, i.e., a "good" that allows consumers to better face risk in the new scenario. This indicates that consumers can utilize work-hour reallocation as a control variable to maximize expected utility by mitigating the pain associated with future income risk. Essentially, they blend a bad with a good to improve their well-being.

In essence, the hybrid entrepreneur divides his or her time between two potentially risky activities: labor and entrepreneurship. On the one hand, risk aversion causes the hybrid entrepreneur to reduce the time allocated to a risky activity, whereas prudence causes the entrepreneur to increase the time allocated to the risky activity to address increased risk. The proportional importance of these forces will determine the final outcome.

As we will see in this paper, risk aversion is not a concept that exhaustively describes decision-makers risk preferences. It is just one piece of the puzzle and needs to be complemented with higher-order risk preferences such as prudence that produces the precautionary effect. A risk-averse decision-maker dislikes mean-

[1] When we discuss allocating time between paid employment and developing a venture (allocating time to an entrepreneurial endeavor), our analytic context is the time allocation decision of a hybrid entrepreneur.

preserving increases in risk (second-order risk aversion), while a prudent decision-maker dislikes transferring risk from a good to a bad state of nature. In this case, it is said that he is averse to downside risk (third-order risk aversion). Both risk preference orders will be present in our model and act as forces operating in opposite directions.

In this paper, we study how risk affects time allocation and under what conditions a hybrid entrepreneur will exhibit precautionary behavior in entrepreneurial time allocation. Our analysis yields several results. First, we show that precautionary behavior is affected by the relationship between entrepreneurship and labor as substitutes or complements. Second, the sufficient condition for precautionary behavior is related to the relative strength of downside risk aversion of entrepreneurship, labor, and leisure.

In our analysis, the precautionary behavior of hybrid entrepreneurs is associated with the idea of harm disaggregation recently developed in the risk literature. Therefore, we show a direct connection between the study of the economics of entrepreneurship and the classical study of decision-making under risk. One of the primary benefits of this approach is that it enables us to develop an intuitive method for working with bivariate risks that can be tested experimentally in the future. The works of Deck and Schlesinger (2014); Ebert and Wiesen (2014), and Krieger and Mayrhofer (2017) are examples of laboratory experiments based on this behavioral approach of harm disaggregation[2] that emphasize risk preferences. In addition, the behavioral approach may provide new insights that the traditional expected utility approach cannot capture (Bonilla & Vergara, 2021), implying that policy implications regarding entrepreneurship based solely on expected utility must be evaluated with caution, particularly when the behavioral approach accurately reflects the individual's decision-making process. This connection opens up multiple possibilities for exploration in the study of entrepreneurship using the modern decision theory tools. We believe that our paper takes advantage of this unexploited connection and provides new insights into the entrepreneurship literature.

The study of hybrid entrepreneurship is the study of future labor interactions. Advances in communication technology and process automation create greater opportunities for traditional workers to transition from traditional 9–5 jobs to hybrid entrepreneurship. Decoding hybrid entrepreneurship is comparable to decoding labor's future. It is a matter of adaptability, overcoming uncertainty, and making the most of a variety of opportunities that will be available to most people.

The remainder of this paper is organized as follows. Section 2 discusses some related literature on risk and entrepreneurship. Section 3 develops the optimal decision problem faced by a hybrid entrepreneur and provides a graphical interpretation of the equilibrium. Section 4 defines precautionary behavior in our context and harm disaggregation following the behavioral approach. Section 5 presents the optimal time allocation for hybrid entrepreneurs and the main results of the paper. In Sect. 5, we analyze optimal behavior under three specific sources of risks: when risk affects only the returns of the entrepreneurial sector, when risk affects only the paid employment sector, and when risk affects both occupations. In the last section, we provide the main conclusions of the paper.

2 Related literature

Our paper relates first to the entrepreneurship and occupational choice literature since the question of who becomes an entrepreneur and the role that risk plays is a longstanding question in the economics of entrepreneurship literature. Much of the literature has treated this decision as a discrete choice between entrepreneurship and paid employment. Traditionally, authors have argued that this decision depends on the individual's risk preferences (Kihlstrom & Laffont, 1979), credit constraints (Evans & Jovanovic, 1989), and differences in wealth under DARA preferences (Cressy, 2000). They all assumed that risk is only present in the entrepreneurial sector, which is the traditional perspective found in the literature of occupational choice.

However, other authors have also addressed the role that wage risk plays in the decision to engage in self-employment or entrepreneurship. For instance, Parker et al. (2005) focused on the impact that wage risk has on

[2] In the economics of risk literature, the behavioral approach of harm disaggregation refers to the study of decision-making under risk that can be analyzed through lotteries and thus can be experimentally tested. Therefore, the behavioral characteristic is affixed to decision-making with lotteries and does not necessarily imply a departure from traditional expected utility theory and rationality as in the case of behavioral economics.

the labor supply of self-employed workers. When wage risk is specified as an additive shock, they found that prudence ($u''' > 0$) is the condition explaining a greater labor supply of the self-employed in response to greater risk since that the self-employed self-insure against this risk by working harder to make the deterministic part of their income relatively higher. However, they could not extend this argument to a multiplicative shock. They found inconclusive results and made an unclear prediction about how self-employed labor supply responds to greater multiplicative wage risk. Given these theoretical results, they conducted an empirical analysis of self-employed American workers and found that wage risk is a key influence in explaining self-employed labor supply. Berkhout et al. (2016) showed that the paid employment wage distribution affects the probability of becoming an entrepreneur. Individuals with lower mean wages, a higher wage variance, and a lower-wage skew are more likely to leave paid employment and choose entrepreneurship. Our paper extends the contribution of this literature by considering two types of risk sources. We argue that labor income risk about wages and risky returns from self-employment plays a key role in a hybrid entrepreneur's decision to allocate more time to entrepreneurship.

At present, in the more flexible labor market with more flexible working arrangements since the COVID-19 pandemic, the decision to become an entrepreneur undeniably transcends the employee-versus-entrepreneur dichotomy and is instead a continuous choice, reviving interest in the phenomenon of hybrid entrepreneurship . Folta et al. (2010) defined hybrid entrepreneurs as individuals who engage in self-employment while also holding a primary job in wage work, which turns out to be a more widespread practice than previously thought (Thorgren et al., 2014), with a large and growing portion of individuals starting their businesses while holding a primary job in wage work (Burke et al., 2008; Folta et al., 2010). Like other authors, we assume that hybrid entrepreneurship is a permanent condition for many individuals. Therefore, we are only interested in understanding the entrepreneurial behavior of these individuals that persist in both simultaneous activities, and we do not seek to identify a transition to full-time entrepreneurship.

According to related research, these individuals pursue hybrid entrepreneurship as an alternative and safer approach to entrepreneurship than full-time entry. Since hybrid entrepreneurs remain salaried, this entry mode allows them to experience entrepreneurship and test the entrepreneurial waters, on the one hand, (Folta et al., 2010). On the other hand, they do so to reduce downside risk (Folta et al., 2010; Raffiee & Feng, 2014). Prior research understands downside risk with respect to real options theory, where the potential for downside loss has a limit because hybrid entry provides individuals the flexibility to start a venture at a slower pace with fewer sunk costs. However, to the best of our knowledge, downside risk has not yet been incorporated into hybrid entrepreneurship research through the lens of decision-making theory under risk. Therefore, acknowledging that an increase in downside risk corresponds to a dispersion transfer from higher to lower income levels and implies a reduction in skewness (Menezes et al., 1980) adds to the growing body of literature by providing insights into how the heterogeneity in individual behavioral responses to the risk exists and depends on their heterogeneous risk preferences.

In accordance with the current idea of risk aversion, individuals who are less risk averse are more likely to start their own full-time ventures than those who are not. Raffiee and Feng (2014) confirmed this and also suggested for the first time that risk preferences may affect how rather than whether someone enters the entrepreneurial world. They discovered that risk aversion influences the process of how an individual decides to start a business (i.e., full-time versus hybrid).

The literature on occupational choice and entrepreneurship is also relevant to our paper. In recent years, the literature has examined the antecedents of becoming a serial or portfolio entrepreneur (Carbonara et al., 2020; Parker, 2014; Plehn-Dujowich, 2010). For example, Carbonara et al. (2020) use a Vietnamese database to conclude that human capital and a higher quality of the new business indicate a greater likelihood that a business owner will become a serial or portfolio entrepreneur. In equilibrium, a high-skilled entrepreneur closes a low-quality business and becomes a serial entrepreneur until a high-quality business is found, whereas a low-skilled entrepreneur closes a low-quality business and enters the labor market. Parker (2014) models the career decisions of serial, portfolio, and first-time entrepreneurs and derives behavioral predictions for each group. Poschke (2013) develops a dynamic model that is consistent with empirical evidence and demonstrates that entrepreneurs are more likely to originate from

the extremes of the education and wage distributions, resulting in a U-shaped relationship with entrepreneurship.

Furthermore, serial entrepreneurs regularly switch industries (Eggers & Song, 2015) since they can use their prior skills to launch a new venture in a different industry (Kuuluvainen, 2010). However, another body of research has found that successful serial entrepreneurs typically stay within their industry and attain economic success with less inventive performance (Dabić et al., 2023). To the best of our knowledge, not enough attention has been paid to this issue in the study of hybrid entrepreneurship, namely the relationship between the industry in which hybrid entrepreneurs build their business ventures and the sector in which they are currently employed (see Demir et al., 2020).

Examples include Steve Wozniak and Sara Blakely, co-founders of Apple and SPANX, who both launched their businesses while working full-time. Steve Wozniak worked at Hewlett Packard, designing scientific calculators, while also creating the Apple I, a homebrew personal computer (Wozniak & Smith, 2006). His two jobs were both in the same industry. Steve brought the prototype to HP before co-founding Apple and begged them to build it, but they refused (Chu, 2017). Similarly to Steve, Sara Blakely began creating her body-slimming women's underwear endeavor while working as a door-to-door fax machine salesperson for the office supply company Danka (Wetzel & Wetzel, 2020).

Urbig et al. (2021) defined this relationship as business proximity, defined as the extent to which the employee's venture is related to that of his or her employer, whereas Kylver (2021) defines it as business related to an individual's current industry and uses his or her concept as a proxy to study industry experience.

Our model recognizes this characteristic of hybrid entrepreneurs by considering the likelihood that the two activities interact as substitutes or complements. Recognizing that committing time to entrepreneurship within one's current industry will help with both activities, whereas developing a venture outside of it may have negative consequences

Another significant aspect is that a share of the individuals who use the hybrid entry mode to entrepreneurship establish their ventures in knowledge-intensive, innovative, and high-technology industries. For example, Apple co-founder Steve Wozniak, eBay founder Pierre Omidyar, and Detroit Automobile Group founder Henry Ford were all workers while launching their ventures (Raffiee & Feng, 2014). Considering that the tech industry does not have low levels of risk, hybrid entrepreneurship is an entry-level option and viable strategy for developing entrepreneurship. To date, research has overlooked the fact that risk exists not only in the self-employment sector but also in the paid employment sector, thus ignoring the role that risk on both sides plays in hybrid entry choice and how hybrid entrepreneurs choose to allocate their working time between entrepreneurship and paid employment.

The studies by Parker (1997) and Ganser-Stickler et al. (2022) are the exceptions. Parker[3] assumed that income risk is concentrated not only in self-employment but also in paid employment. He acknowledged that the assumption that risk is present only in the self-employment sector is not harmless and affects the predicted direction that sectoral risk has on self-employment. He investigated the effects of sectoral risk on self-employment participation using a general equilibrium model based on heterogeneous, risk-averse agents with no switching costs when changing sectors, demonstrating that the sectoral risk effects on the optimal self-employment decision are ambiguous for hybrid entrepreneurs.

In a similar way to Parker, Ganser-Stickler et al. (2022) study how risk in paid employment and in entrepreneurship individually and jointly influence an individual's choices, particularly the decision to enter entrepreneurship in a hybrid mode rather than as a full-time entrepreneur. Using real options reasoning, they argue that the hybrid portfolio created by holding both real options is less valuable than starting a full-time business due to the increased volatility in paid employment. Hence, paid employment risk reduces the individual's likelihood of choosing the hybrid mode over full-time entrepreneurship.

Unlike previous studies, our methodological approach allows us to improve realism by acknowledging how hybrid entrepreneurs allocate more time to entrepreneur-

[3] Unlike Parker's (1997) findings, our paper provides the conditions under which an uninsurable risk (in paid employment or the entrepreneurial sector) has an unambiguous precautionary effect on the optimal time allocated to each occupation, and these conditions are based on the strengths of risk aversion and downside risk aversion.

ship as a multidimensional decision. We do so by assuming that entrepreneurs face two sources of risk and that their preferences depend on two attributes: consumption and leisure. From an economics perspective, we concentrate on comprehending entrepreneurial behavior following entry by analyzing the marginal decisions made by those individuals who continue in both simultaneous activities. Our analysis therefore focuses on both the hybrid entrepreneurs who aim to become full-time entrepreneurs and those who do not. In contrast with the findings of Parker (1997), our paper shows that the effects that the uninsurable risk in paid employment and in the entrepreneurial sector has on the optimal time allocated to each occupation are not ambiguous. We find that the conditions explaining precautionary entrepreneurial time allocation and precautionary paid-employment time allocation are related to the strengths of downside risk aversion.

Our paper also relates to the recent literature on risk. We build on the seminal work of Eeckhoudt and Schlesinger (2006), who developed the idea of preferences for harm disaggregation under a behavioral approach and showed that risk apportionment relates to the differentiable utility function u, which satisfies the additive risk apportionment of order n if $(-1)^{n+1}u^{(n)} \geq 0$,[4]. Working with the behavioral approach enables us to analyze entrepreneurship predominantly as an application of decision theory under risk by employing lottery pairs that perfectly connect with higher-order properties of utility functions that are particularly amenable to experimentation. The main intuition is that decision-makers favor combining relatively good assets with relatively poor assets, where poor is defined by nth-order stochastic dominance. We will apply these concepts to our analysis beginning with the simplest exercise conceivable in Sect. 4.2.1.

[4] The function $u^{(n)}$ denotes the nth order derivative of utility function u. Thus, the mathematical expression $(-1)^{n+1}u^{(n)} \geq 0$ describes an nth-degree risk-averse decision-maker. This expression encompasses the concepts of risk aversion ($n = 2$) and downside risk aversion ($n = 3$), which are used is this paper to describe the behavior of the hybrid entrepreneur (more technically inclined readers should consult Ekern 1980). These concepts have been tested in laboratory experiments by Deck and Schlesinger (2014) and Ebert and Wiesen (2014), among others. The first work tested the direction of third-order risk preferences (prudence), while the second developed a method to measure its intensity or strength.

Our paper is also related to the multiplicative risk apportionment literature (e.g., Eeckhoudt et al., 2009a; Wang and Li, 2010; Chiu et al., 2012) and to the notion of bivariate risk apportionment, first developed by Eeckhoudt et al. (2007) who introduced this notion by acknowledging that decisions under risk are often multidimensional and, therefore, decision-maker preferences depend on several attributes. Jokung (2011) describes this concept as the implications of preferring a bivariate lottery with one bad and one good attribute over a lottery with a 50–50 probability of either two good or two bad attributes. Denuit and Rey (2013) refer to this as cross-risk apportionment, while Crainich et al. (2020) refines the concept of bivariate risk apportionment by highlighting that this preference for dispersing risks is associated with two aspects of an individual's well-being in different states of the world. We use the risk apportionment technology to derive an optimal decision of a hybrid entrepreneur who faces multiple risks in his or her alternative occupations. We provide the conditions under which an uninsurable risk (in paid employment or the entrepreneurial sector) has an unambiguous precautionary effect on the optimal time allocated to each occupation, and these conditions are based on the strengths of risk aversion and downside risk aversion.

3 The basic model of time allocation

We developed an economic model of entrepreneurship in a context of risk where hybrid entrepreneurs choose how to allocate their working time between paid employment (labor) and entrepreneurship (working on a side venture or self-employment). We used a single-agent, single-period occupational choice bivariate model and followed a methodological approach similar to Liu and Menegatti (2019) but added a risk apportionment framework (Chiu et al., 2012; Eeckhoudt et al., 2009b; Eeckhoudt & Schlesinger, 2006) that allows us to represent the decision under risk of changing time allocation between competing activities.

The hybrid entrepreneur's preferences are defined over consumption c and leisure l and represented by a Von Neumann-Morgenstern utility function $u(c, l)$. The utility function is assumed to be continuous, increasing, and concave in both c and l. That is, $u_c > 0$

and $u_l > 0$ (nonsatiation), $u_{cc} < 0$ and $u_{ll} < 0$ (risk aversion) However, u_{cl} can be negative or positive. When $u_{cl} > 0$, this means that the marginal utility of one good increases with the consumption of the other good. Alternatively, when $u_{cl} < 0$, this means that the marginal utility of one good decreases with the consumption of the other (pairwise risk aversion), and both cases are possible in our model.

The decision-maker (DM) is endowed with total time T, which he or she distributes between leisure and productive activities (labor and entrepreneurship). Let L be the working hours that he or she devotes to paid employment, while e describes the working hours devoted to entrepreneurship. The total number of hours devoted to leisure time is $l = T - L - e$.

The individual has wealth level a (exogenous assets that, without loss of generality, we will set to zero) and earns a fixed wage w for each time unit devoted to paid employment. This w is nonrandom (for now) and nonnegative. On the other hand, the income from the entrepreneurial activity is described by a positive random variable $\tilde{\theta}$ that represents the random returns generated by the time unit devoted to entrepreneurship. Therefore, the hybrid entrepreneur has to choose the optimal time allocations for both: the number of hours allocated to paid employment and the number of hours allocated to entrepreneurship. In this context, the hybrid entrepreneur's optimization problem is:

$$\max_{L,e} V(e, L) \tag{1a}$$

$$\text{where,} \quad V(e, L) = Eu(wL + \tilde{\theta}e, T - L - e) \tag{1b}$$

The first- and second-order conditions of the maximization problem are:

$$V_e = E[\tilde{\theta}u_c] - Eu_l = 0 \tag{2}$$

$$V_L = wEu_c - Eu_l = 0 \tag{3}$$

$$V_{ee} = E[(\tilde{\theta})^2 u_{cc}] - 2E[\tilde{\theta}u_{cl}] + Eu_{ll} < 0 \tag{4}$$

$$V_{LL} = w^2 Eu_{cc} - 2wEu_{cl} + Eu_{ll} < 0 \tag{5}$$

$$H = V_{ee}V_{LL} - (V_{eL})^2 > 0 \tag{6}$$

where

$$V_{eL} = wE[\tilde{\theta}u_{cc}] - E[\tilde{\theta}u_{cl}] - wEu_{lc} + Eu_{ll} \tag{7}$$

The joint optimum (e^*, L^*) is obtained when both Eqs. 2 and 3 are satisfied. Furthermore, note that V_{eL} indicates whether the two occupations are substitutes ($V_{eL} < 0$) or complements ($V_{eL} > 0$), and this difference will comprise an important part of the results in this article. To conceptualize this difference for the case of complements, we consider the case where a DM starts a venture that is related to his or her current employment, so any time devoted to the venture will produce positive effects on his or her performance as an employee and any professional improvement and better knowledge of his or her current craft will positively impact his or her venture. On the contrary, when the occupations are substitutes, developing a venture in an entirely different industry will have adverse effects on the current occupation. In this case, we considered a DM whose focus and energy are limited, so focusing on industry A takes away focus and efficiency from industry B.

4 Preliminaries and definitions

4.1 Precautionary behavior

In this paper, we study precautionary entrepreneurial and labor time allocations. We define these concepts as the extra working time allocated to either of those activities when the DM faces an increase in risk.

We determined the precautionary effect by the difference between the optimal choice in problem Eq. 1a[5] and the optimal choice made when we assumed that there is no risk to entrepreneurship, and consequently, the return on entrepreneurship is replaced by its nonrandom expectation ($E[\tilde{\theta}]$), that is,

$$\max_{L,e} u(wL + E[\tilde{\theta}]e, T - L - e) \tag{8}$$

[5] Problem Eq. 1a represents the classical model of self-selection of occupations in which entrepreneurship provides a risky payment while labor provides a secure wage.

Let e^* and L^* be the optimal choice in problem Eq. 1a and e^{**} and L^{**} be the optimal choice in problem Eq. 8. We thus have the following two definitions:

Definition 1 We have precautionary entrepreneurial time allocation when $e^* > e^{**}$ and precautionary entrepreneurial time deallocation when $e^* < e^{**}$.

Definition 2 We have precautionary labor time allocation when $L^* > L^{**}$ and precautionary labor time deallocation when $L^* < L^{**}$.

We will use these definitions in the next section where we study precautionary time allocation induced by different sources of risk (not only risk in the entrepreneurial sector). For now, let us focus on the equilibrium conditions of problem 1(a) that will be represented by Fig. 1. First, note that if $V_{eL} < 0$, from Eq. 6, we can infer that

$$-\frac{V_{LL}}{V_{eL}} < -\frac{V_{eL}}{V_{ee}} < 0 \tag{9}$$

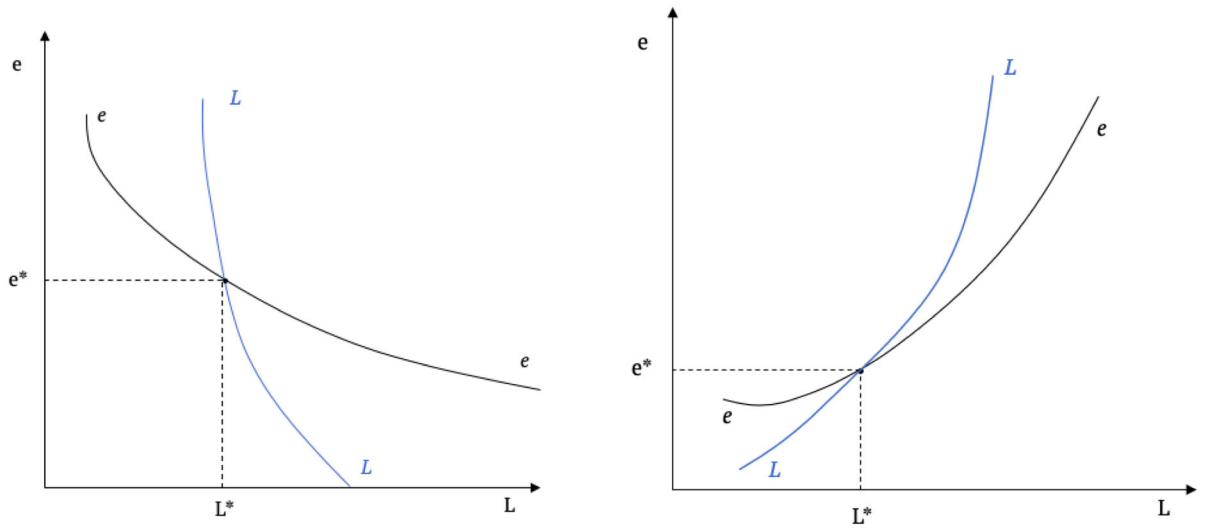
and alternatively, if $V_{eL} > 0$, from Eq. 6, we infer that

$$-\frac{V_{LL}}{V_{eL}} > -\frac{V_{eL}}{V_{ee}} > 0 \tag{10}$$

Now, totally differentiating Eq. 2, we obtain $dV_e = V_{ee}de + V_{eL}dL$, and from this equation we can define the curve ee with slope $\frac{de}{dL} = -\frac{V_{eL}}{V_{ee}}$. Similarly, totally differentiating Eq. 3, we obtain $dV_L = V_{LL}dL + V_{eL}de$, and from this equation we define the curve LL with slope $\frac{de}{dL} = -\frac{V_{LL}}{V_{Le}}$. The intersection of both curves represents the optimal allocation of (e^*, L^*) as shown in Fig. 1.

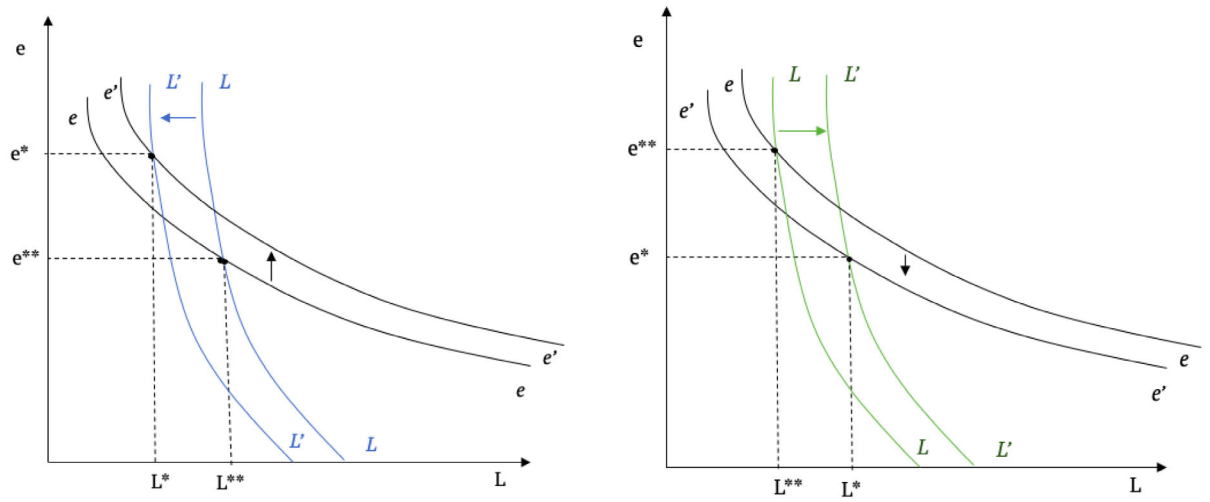
Note that Eqs. 9 and 10 can be rewritten in the following way:

If $V_{eL} < 0$ (the curves ee and LL are downward sloping), $\left.\frac{de}{dL}\right|_{LL} < \left.\frac{de}{dL}\right|_{ee}$ (Eq. 9). Additionally, if $V_{eL} > 0$ (the curves ee and LL are upward sloping), $\left.\frac{de}{dL}\right|_{LL} > \left.\frac{de}{dL}\right|_{ee}$ (Eq. 10). We know that the sign of the cross-partial derivative V_{eL} determines whether the two occupations are substitutes ($V_{eL} < 0$) or complements ($V_{eL} > 0$). Figure 1 illustrates from left to right the cases shown in Eqs. 9 and 10, respectively. Consequently, precautionary behavior in entrepreneurial time allocation is mediated by how both activities interact. If they are substitutes, the precautionary decision to allocate more time to entrepreneurship implies the decision to deallocate time from paid employment (see Fig. 2).



(a) if $V_{eL} < 0$ (b) if $V_{eL} > 0$

Fig. 1 The LL curve and ee curve



(a) $e^* > e^{**}$ and $L^* < L^{**}$ (b) $e^* < e^{**}$ and $L^* > L^{**}$

Fig. 2 Precautionary entrepreneurial and labor time allocation (deallocation) when $V_{eL} < 0$

If we focus on Fig. 2a, we see that sufficient conditions that guarantee an increase in e and a decrease in L—when we go from no-risk to risky entrepreneurship—arise when curve LL moves to the left, curve ee moves to the right, or both curves move at the same time in those directions.[6]

Conversely, when these occupations interact as complements ($V_{eL} > 0$), the precautionary decision to allocate more time to entrepreneurship implies the allocation of more time to paid employment (see Fig. 3), which is when curve LL moves to the right, curve ee moves to the left, or both curves move at the same time in those directions.

As we will see in what follows, we used a behavioral approach to show that hybrid entrepreneurs can diversify risk by choosing time allocation between entrepreneurship and labor. Our results are based on preferences for harm disaggregation (and the strength of downside risk aversion[7]). We will develop a particular set of lotteries to characterize an increase in risk and to analyze the precautionary behavior in this context.

4.2 Risk aversion and downside risk aversion in the behavioral approach

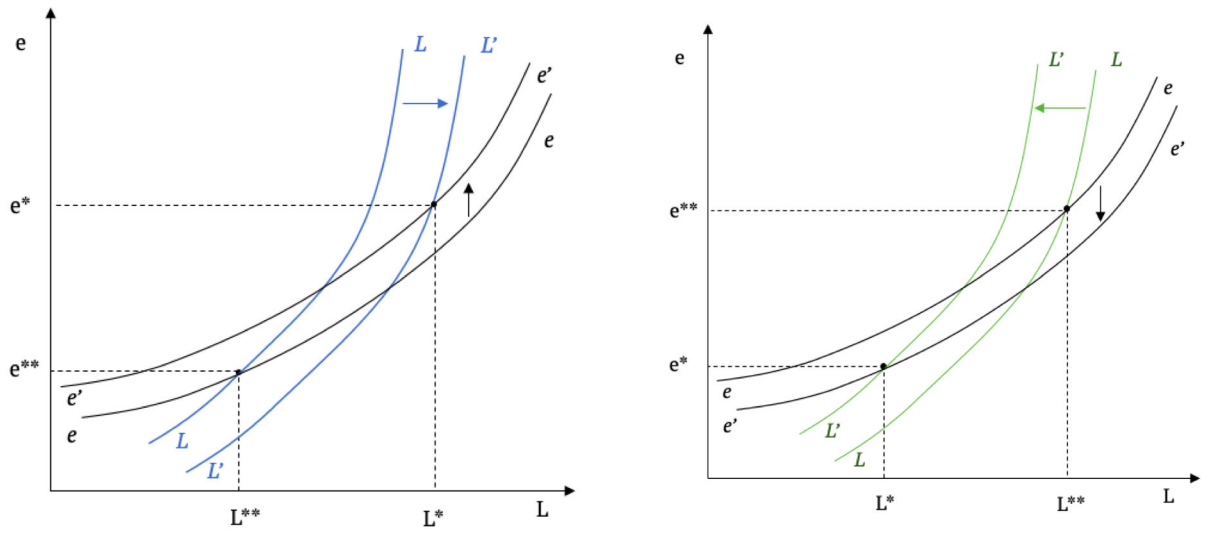
We argued above that the behavioral approach enables us to analyze entrepreneurship predominantly as an application of decision theory under risk by employing lottery pairs that are particularly amenable to experimentation. Thus we will introduce our approach building on the basics of risk aversion. In this subsection, we will introduce the concepts of risk aversion and downside risk aversion via 50–50 lottery pairs using a one-argument utility function. Later, we will extend such concepts to more complex pairs of lotteries that will help us to explain precautionary effects in the time allocation problem of hybrid entrepreneurs.

4.2.1 Risk aversion

In the expected utility space, utility function $u(c)$ is risk averse if its second derivative is negative ($u_{cc} < 0$), and the intensity of risk aversion is measured by $A = -\frac{u_{cc}}{u_c}$ (Arrow, 1965; Pratt, 1964).

[6] In the proof of Proposition 1 we will provide the mathematical conditions required for this comparative statics exercise.

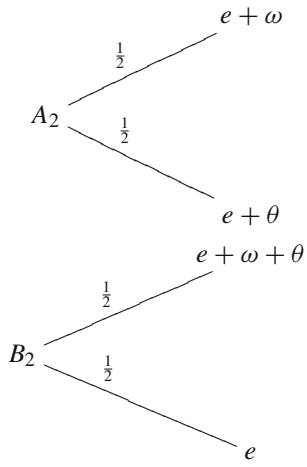
[7] The term strength of downside risk aversion was coined by Menezes and Wang (2004) and represents the disutility attached to a downside risk increase. We provide this term a more intuitive interpretation using the risk apportionment approach.



(a) $e^* > e^{**}$ and $L^* > L^{**}$ **(b)** $e^* < e^{**}$ and $L^* < L^{**}$

Fig. 3 Precautionary entrepreneurial and labor time allocation (deallocation) when $V_{eL} > 0$

An alternative way to define risk aversion is using pairs of lotteries. Let us initially assume that e is the individual's initial wealth, $\theta, \omega > 0$ are fixed monetary amounts, and $\theta > \omega$. Consider two pairs of lottery options A_2 versus B_2.



A decision-maker is risk averse if he or she prefers A_2 to B_2 for arbitrary parameter values e, ω and θ. This means that he or she prefers to disaggregate unavoidable gains ω and θ across states of nature.[8] In the expected utility framework, $A_2 \succeq B_2$ if and only if

$$Eu(A_2) = \tfrac{1}{2}u(e+\omega) + \tfrac{1}{2}u(e+\theta) \geq \tfrac{1}{2}u(e+\omega+\theta) + \tfrac{1}{2}u(e) = Eu(B_2) \quad (11)$$

where $SR = Eu(A_2) - Eu(B_2)$ is the disutility associated with the mean-preserving spreads and represents the strength of risk aversion. Condition Eq. 11 can be written as:

$$[u(e+\theta) - u(e)] - [u(e+\omega+\theta) - u(e+\omega)] \geq 0$$

For a small θ, we have:

$$\theta[u_c(e) - u_c(e+\omega)] \geq 0$$

and then for small ω we obtain:

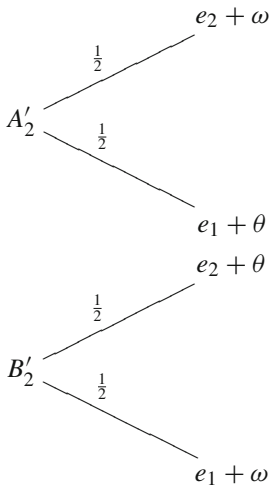
$$-\theta\omega u_{cc}(e) \geq 0$$

which is true if and only if $u_{cc} < 0$.

There is still another way to represent risk aversion,which we present below. Consider now the fol-

[8] An alternative way to represent risk aversion is through the $50-50$ lottery pair: $A_2 = [e-\omega, e-\theta]$ and $B_2 = [e, e-\theta-\omega]$. Thus, a decision-maker prefers to disaggregate unavoidable losses $-\omega$ and $-\theta$ across states of nature.

lowing pair of lotteries where $e_2 > e_1$ and $\theta > \omega$.



With this pair of lotteries, a decision-maker is risk averse if he or she prefers A_2' to B_2'. Note that the lottery A_2' combines good with bad since e_1 is attached to the best state of nature, while e_2 is attached to the worst state of nature. The opposite case occurs with the B_2' lottery in which good is combined with good and bad with bad. Then, e_1 has more one-degree risk than e_2 (e_2 dominates e_1 via first-order stochastic dominance), and ω also has more one-degree risk than θ (θ dominates ω via first-order stochastic dominance). Based on the corollary of Eeckhoudt et al. (2009), the lottery B_2' displays more second-degree risk than A_2' or B_2' is a mean-preserving increase in risk of A_2'. In the expected utility framework, $A_2' \succeq B_2'$ if and only if

$$Eu(A_2') = \tfrac{1}{2}u(e_2+\omega) + \tfrac{1}{2}u(e_1+\theta) \geq \tfrac{1}{2}u(e_2+\theta) + \tfrac{1}{2}u(e_1+\omega) = Eu(B_2') \quad (12)$$

where $SR = Eu(A_2') - Eu(B_2')$ is the disutility associated with the mean-preserving spread and represents the strength of risk aversion. This condition can be expressed equivalently as:

$$[u(e_1+\theta) - u(e_1+\omega)] - [u(e_2+\theta) - u(e_2+\omega)] \geq 0 \quad (13)$$

Defining $\Gamma(e,\omega,\theta) = u(e+\theta) - u(e+\omega)$ and denoting the partial derivative of Γ with respect to e by Γ_e, Eq. 13 can be further written as $\Gamma(e_1,\omega,\theta) - \Gamma(e_2,\omega,\theta) \geq 0$, which is clearly true if and only if $\Gamma_e \leq 0$. Because $\Gamma_e = u_c(e+\theta) - u_c(e+\omega)$, then $\Gamma_e \leq 0$ is equivalent $u_c(e+\omega) \geq u_c(e+\theta)$, which is true if and only if $u_{cc} < 0$, i.e., if the decision-maker is risk averse.

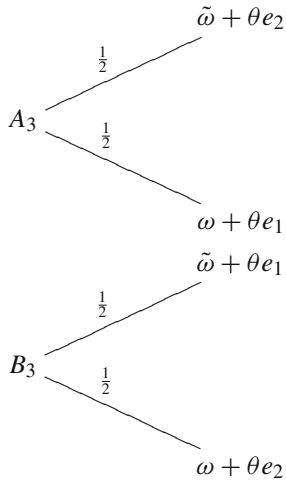
Then, when we have pairs of 50–50 lotteries, the following lemma characterizes a risk-averse decision-maker.

Lemma 1 *In the expected utility framework, assuming that $e_2 > e_1$ and $\theta > \omega$, then $A_2' \succeq B_2'$ if and only if $u_{cc} < 0$.*

4.2.2 Downside risk aversion

In the expected utility space, downside risk aversion (or prudence) is defined by a positive third derivative of the utility function ($u_{ccc} > 0$). This concept was coined by Kimball (1990) in the context of the precautionary demand for saving. Additionally, $u_{ccc} > 0$ implies that a decision-maker has a convex marginal utility function and, therefore, dislikes transferring risk from a richer to a poorer state.

Let us assume that the positive constant ω in the definition of risk aversion is replaced by a random variable $\tilde{\omega}$ with mean $E(\tilde{\omega}) = \omega$. Let A_3 be the $50-50$ lottery $[\tilde{\omega}+\theta e_2, \omega+\theta e_1]$ and B_3 be the $50-50$ lottery $[\tilde{\omega}+\theta e_1, \omega+\theta e_2]$, and let us recall that in our model, $e_2 > e_1$ and $\theta > \omega$. We see that θ is just a constant that multiplies e_1 and e_2 through the states of nature. Thus, θ can take any nonnegative value and in particular, may be chosen such that $\theta = 1$. These pairs of lotteries can be expressed as:



A decision-maker is downside risk averse if he or she prefers A_3 to B_3. Note that lottery A_3 combines good with bad since e_2 is attached to the state in which the random variable ($\tilde{\omega}$) appears, while e_1 is attached

to the state in which the expected value of the random variable (ω) appears. The opposite case occurs with the B_3 lottery in which good is combined with good and bad with bad. Thus, e_1 has more one-degree risk than e_2 (e_2 dominates e_1 via first-order stochastic dominance), and $\tilde{\omega}$ has more two-degree risk than ω ($\tilde{\omega}$ is a mean-preserving spread of ω). If again we use the corollary of Eeckhoudt et al. (2009), then lottery B_3 displays more third-degree risk than A_3, or B_3 has more downside risk than A_3. In the expected utility framework, $A_3 \succeq B_3$ if and only if

$$Eu(A_3) = \tfrac{1}{2}Eu(\tilde{\omega}+\theta e_2) + \tfrac{1}{2}u(\omega+\theta e_1) \geq \tfrac{1}{2}Eu(\tilde{\omega} + \theta e_1) + \tfrac{1}{2}u(\omega+\theta e_2) = Eu(B_3) \quad (14)$$

where $SD = Eu(A_3) - Eu(B_3)$ is the disutility associated with the increase in downside risk and represents the strength of downside risk aversion. Condition Eq. 14 can be expressed equivalently as:

$$[u(\omega+\theta e_1) - Eu(\tilde{\omega}+\theta e_1)] - [u(\omega+\theta e_2) - Eu(\tilde{\omega}+\theta e_2)] \geq 0 \quad (15)$$

Let $\Gamma(e,\omega,\tilde{\omega},\theta) = u(\omega+\theta e) - Eu(\tilde{\omega}+\theta e)$ be defined. Differentiating Γ with respect to e, we obtain $\Gamma_e = u_c\theta - Eu_c\theta$. Observe that Eq. 15 can be expressed as $\Gamma(e_1,\omega,\tilde{\omega},\theta) - \Gamma(e_2,\omega,\tilde{\omega},\theta) \geq 0$ if and only if $\Gamma_e \leq 0$, which is equivalent to $Eu_c \geq u_c$ because θ is nonrandom. By Jensen's inequality, $g(\omega) = u_c$ is a convex function when $g_{\omega\omega} = u_{ccc} > 0$, which is the prudence condition.

Given the previous analysis on pairs of 50–50 lotteries, the following lemma characterizes a downside risk-averse decision maker.

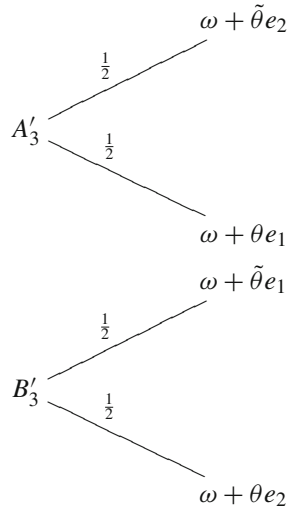
Lemma 2 *In the expected utility framework, assuming that $e_2 \succ e_1$, $\omega > 0$ and $\tilde{\omega}$ being a mean-preserving spread of ω, then $A_3 \succeq B_3$ if and only if $u_{ccc} > 0$.*

The definition of downside risk aversion given above is based on additive risk. In this context, what the literature calls the income effect arises. Thus a prudent decision-maker prefers to suffer risk in better states of nature, and the income effect encourages him or her to increase his or her working hours (and therefore his or her expected wealth) devoted to the risky activity as a way to face increased risk.

As we will show below, in the case of multiplicative risk, we have two opposing effects: an income effect induced by prudence and a substitution effect induced by risk aversion.[9] These two effects are embedded in an index of partial relative prudence introduced by Choi et al. (2001).

4.2.3 Partial relative prudence

Now, consider the pair of lotteries A_3' and B_3', where $\omega > 0, e_2 > e_1$ and $\tilde{\theta}$ is a random variable with $E(\tilde{\theta}) = \theta$.



In this case, ω is just a constant that is added to all states of nature. We see that e_1 has more one-degree risk than e_2 (e_2 dominates e_1 via first-order stochastic dominance) and $\tilde{\theta}$ has more two-degree risk than θ ($\tilde{\theta}$ is a mean-preserving spread of θ). These two degrees of risk multiply each other, giving rise to a behavior that combines risk aversion and downside risk aversion. In the expected utility framework, $A_3' \succeq B_3'$ if and only if

$$Eu(A_3') = \tfrac{1}{2}Eu(\omega+\tilde{\theta}e_2) + \tfrac{1}{2}u(\omega+\theta e_1) \geq \tfrac{1}{2}Eu(\omega+\tilde{\theta}e_1) + \tfrac{1}{2}u(\omega+\theta e_2) = Eu(B_3') \quad (16)$$

where $SRD = Eu(A_3') - Eu(B_3')$ is the disutility associated with the increase in multiplicative risk and represents the strength of risk aversion and downside risk aversion. Condition Eq. 16 can be rewritten as follows:

$$[u(\omega+\theta e_1) - Eu(\omega+\tilde{\theta}e_1)] - [u(\omega+\theta e_2) - Eu(\omega+\tilde{\theta}e_2)] \geq 0 \quad (17)$$

[9] See, for instance, Bonilla and Vergara (2022), who analyzed the problem of precautionary saving under linear and nonlinear risks.

Following the same procedure described above, let us define $\Gamma(e, \omega, \theta, \tilde{\theta}) = u(\omega + \theta e) - Eu(\omega + \tilde{\theta} e)$, where the derivative of Γ with respect to e is equal to $\Gamma_e = u_c\theta - Eu_c\tilde{\theta}$. Condition Eq. 17 can be rewritten as $\Gamma(e_1, \omega, \theta, \tilde{\theta}) - \Gamma(e_2, \omega, \theta, \tilde{\theta}) \geq 0$, which is true if and only if $\Gamma_e \leq 0$. By Jensen's inequality $Eu_c\tilde{\theta} \geq u_c\theta$ whenever $g(\theta) = u_c\theta$ is a convex function. Observe that $g_{\theta\theta} = [\theta e u_{ccc} + 2u_{cc}]e$. An income effect is associated with the term $\theta e u_{ccc}$, which includes prudence, while a substitution effect is associated with the term $2u_{cc}$, which includes risk aversion. These two forces operate in opposite directions.

Then, based on pairs of 50–50 lotteries, the following lemma characterizes a risk-averse and downside risk-averse decision-maker who faces a multiplicative risk.

Lemma 3 *In the expected utility framework, assuming that $u_{cc} < 0$ and u_{ccc}, $\omega > 0$, $\tilde{\theta}$ is a mean-preserving spread of θ, and $e_2 > e_1$, then $A_3' \succeq B_3'$ if and only if $-\theta e \frac{u_{ccc}}{u_{cc}} > 2$.*

A clear example of additive and multiplicative risks is the intertemporal savings model under risk, in which e is savings, $\tilde{\omega}$ is a labor income risk.[10], and $\tilde{\theta}$ corresponds to an interest-rate risk.[11] We assume that a consumer has a separable utility function $u(c_1) + \beta Eu(c_2)$ where $c_1 = a - e$ and $c_2 = \tilde{\omega} + \tilde{\theta} e$ corresponds to consumption in the periods $t = 1$ and $t = 2$, respectively. In this example, a is the initial wealth, and β is the discount factor.[12] Assume that labor income is risky ($\tilde{\omega}$) and interest-rate risk is replaced by its expected value ($E(\tilde{\theta}) = \theta$). From the pair of lotteries A_3 and B_3, it follows that a prudent consumer will increase his savings, that is, he will reallocate wealth from the present to the future to face risk. Thus, the disutility generated by future risk is reduced with precautionary savings. In this case, only the income effect arises, which encourages the consumer to increase his or her savings.

Now, assume that labor income risk is replaced by its expected value ($E(\tilde{\omega}) = \omega$) and an interest-rate risk ($\tilde{\theta}$). From the pair of lotteries A_3' and B_3', it follows that the income effect pushes the prudent consumer to increase savings to increase wealth in the period where he or she faces risk. The substitution effect, on the other hand, induces the risk-averse consumer to save less. This occurs when the consumer's exposure to risk increases as a result of saving and investing in assets with risky interest rates. Thus, interest-rate risk is a multiplicative risk because it multiplies the decision variable, i.e., savings, while labor income risk is an additive risk given that it does not multiply the decision variable.

Our model presents characteristics similar to that of intertemporal consumption under risk. The hybrid entrepreneur faces risk in the entrepreneurial sector and in the labor sector, which can be additive or multiplicative, and he or she must decide how much time to invest in each sector to maximize his expected utility, and consequently, income and substitution effects will emerge in both sectors.

Next, we will use concepts analyzed in this subsection to represent pairs of lotteries that will help us understand the hybrid entrepreneur's precautionary behavior. To do this, we will use a utility function with two arguments, that is, $u(c, l)$, where c is consumption and l is leisure.

4.3 Harm disaggregation in consumption in the behavioral approach

In this subsection, we will introduce the strength of downside risk in entrepreneurship and labor. These strengths represent preferences for harm disaggregation in consumption by changes in the time allocated between entrepreneurship (e) and paid employment (L).

4.3.1 Strength of risk aversion and downside risk aversion on entrepreneurship

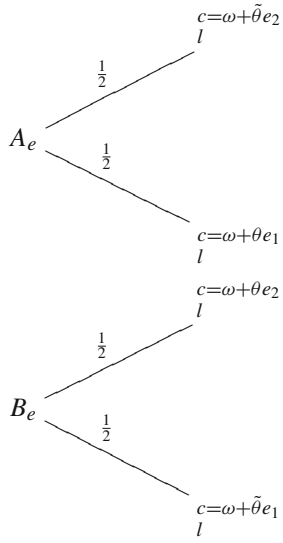
Consider first the traditional case when the source of income risk comes only from the entrepreneurial sec-

[10] Losing a job is the greatest blow to income because the majority of consumers rely on labor, and the likelihood of future unemployment is increased with financial, economic and political slumps (Lugilde et al., 2019) Additionally, empirical evidence has shown a significant increase in wage dispersion in recent years due to the globalization and digitization process (Berlingieri et al., 2017).

[11] See, for instance, Eeckhoudt and Schlesinger (2008).

[12] Vereshchagina and Hopenhayn (2009) utilized this model to explain why self-financed entrepreneurs may believe that it is best to invest in risky ventures with no risk premium.

tor, where $\tilde{\theta}$ is a multiplicative risk. Let us define the strength of risk aversion and downside risk aversion in entrepreneurship (SRD_e) as the difference (in expected utility terms) between 50–50 lotteries A_e and B_e:



where $\omega = wL, e_i$ for $i = 1, 2$ are the time allocated to the entrepreneurial activity and $e_2 > e_1$. Additionally, $\tilde{\theta}$ and $E[\tilde{\theta}] = \theta$ are the risky return on entrepreneurship and the expected return on entrepreneurship, respectively. This pair of lotteries is essentially the same as the pair of lotteries A_3' and B_3', in which time allocated to labor and leisure remain constant through states of nature.

Thus, in the expected utility framework, $A_e \succeq B_e$ if and only if

$$SRD_e \equiv Eu(A_e) - Eu(B_e) \geq 0 \tag{18}$$

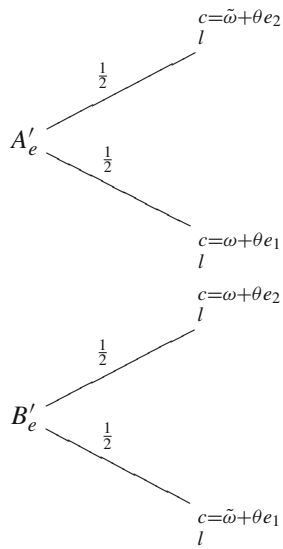
then, by Lemma 3, we know that

$$SRD_e \geq 0 \text{ iff } -\theta e \frac{u_{ccc}}{u_{cc}} > 2 \tag{19}$$

For the SRD_e to be positive, prudence must be strong enough in the sense that the index $-\theta e \frac{u_{ccc}}{u_{cc}}$ is greater than 2. In this problem, we have two opposing forces. On the one hand, risk aversion leads to a decrease in risk-taking, and therefore, hybrid entrepreneurs will want to reduce their risk exposure by decreasing the time allocated to entrepreneurial activities. On the other hand, prudence implies an aversion to downside risk, which induces a precautionary time allocation effect on entrepreneurship, with an opposite effect. We can call this index ($-\theta e \frac{u_{ccc}}{u_{cc}}$) the partial relative prudence index in entrepreneurship as a parallel concept to the partial relative prudence index.

Since $e_2/e_1 > 1$, with lottery A_e, the entrepreneur would be bearing greater risk when he or she is richer ($\tilde{\theta}$ is a mean-preserving spread of θ). As a result, if he or she prefers to bear a greater degree of risk as he or she becomes wealthier, the decision between A_e and B_e will be dictated by the respective strengths of two opposing effects. The first is the substitution effect that makes A_e less attractive because the entrepreneur is risk averse and hence wants to escape from risk. The second is the income effect, which makes A_e more attractive because the entrepreneur is prudent, which means that he or she prefers to bear greater risk when he or she is richer, i.e., with A_e, the harm of a greater risk and a lower wealth are disaggregated. Thus, the substitution effect is dominated by the income effect if and only if the partial relative prudence index is larger than 2.

Now, consider the uncommon case where the source of income uncertainty comes only from the paid-employment sector, where $\tilde{\omega}$ is an additive risk. Let us define the strength of downside risk aversion in entrepreneurship (SD_e) as the difference (in expected utility terms) between 50–50 lotteries A_e' and B_e':



where $\tilde{\omega} = \tilde{w}L$ and $E(\tilde{\omega}) = wL$ are the labor income risk and its expectation, respectively. This pair of lotteries is essentially the same as the pair of lotteries A_3 and B_3, in which time allocated to labor and leisure remain constant through states of nature.

Thus, in the expected utility framework, $A'_e \succeq B'_e$ if and only if

$$SD_e \equiv Eu(A'_e) - Eu(B'_e) \geq 0 \tag{20}$$

then, by Lemma 2, we know that

$$SD_e \geq 0 \text{ if } u_{ccc} > 0 \tag{21}$$

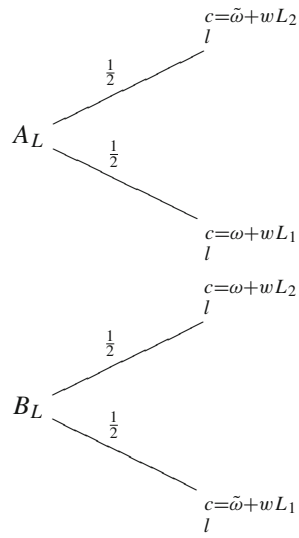
Therefore, in the presence of an additive risk, the income effect prevails (prudent behavior since $u_{ccc} > 0$). This means that a hybrid entrepreneur prefers to attach risk to the state with the greater wealth, that is, to the state in which he or she allocates the most time to entrepreneurial activity (θe_2).

In conclusion, regardless of the source of income risk, we have shown that the strength of downside risk aversion in entrepreneurship (SD_e) and the strength of risk aversion and downside risk aversion in entrepreneurship (SRD_e) entail a preference for combining risk with higher levels of income produced by a concordant increased time allocation.

4.3.2 Strength of risk aversion and downside risk aversion on labor

Now, we will analyze the effect of risk on the allocation of time to labor (or paid employment). The results are similar to those obtained in the previous section. However, we will see that the risk in the entrepreneurial sector becomes an additive risk, while the risk in the paid employment sector becomes a multiplicative risk.

Consider again the traditional case in which the source of income risk comes only from the entrepreneurial sector. Let us define the strength of downside risk aversion in labor (SD_L) as the difference (in expected utility terms) between the 50–50 lotteries A_L and B_L:



where L_i for $i = 1, 2$ are the time allocated to paid employment and $L_2 > L_1$. Additionally, $\tilde{\theta}$ and $E[\tilde{\theta}] = \theta$ are the risky return on entrepreneurship and its expectation. Let $\tilde{\omega} = \tilde{\theta}e$ and $E(\tilde{\omega}) = \theta e$ be the time allocated to entrepreneurship (in this exercise it does not change); then, $\tilde{\omega}$ becomes an additive risk.

Thus, in the expected utility framework, $A_L \succeq B_L$, if and only if

$$SD_L \equiv Eu(A_L) - Eu(B_L) \geq 0 \tag{22}$$

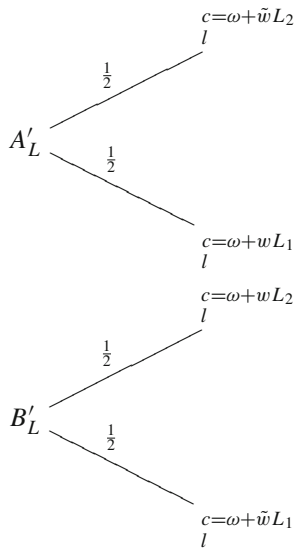
then, by Lemma 2, we have

$$SD_L \geq 0 \text{ iff } u_{ccc} > 0 \tag{23}$$

Therefore, the strength of downside risk aversion in labor (paid employment) is positive only when the individual is prudent. The intuition mimics the previous case: the individual prefers to allocate risk in the good state, which combines risk with higher levels of income that in this case is produced by an increased time allocated to nonrandom paid employment (wL_2). In this case, only the income effect arises.

Now, assume that the source of income risk comes only from the paid employment sector. In this case, the risk is multiplicative because the time allocated to labor changes through states of nature. Thus, the

two variables interact with each other. Let us define the strength of risk aversion and downside risk aversion in labor (SRD_L) as the difference in the following 50–50 lotteries A'_L and B'_L:



Thus, in the expected utility framework, $A'_L \succeq B'_L$ if and only if

$$SRD_L \equiv Eu(A'_L) - Eu(B'_L) \geq 0 \tag{24}$$

and, by Lemma 3, we know that

$$SRD_L \geq 0 \text{ if } -wL\frac{u_{ccc}}{u_{cc}} > 2 \tag{25}$$

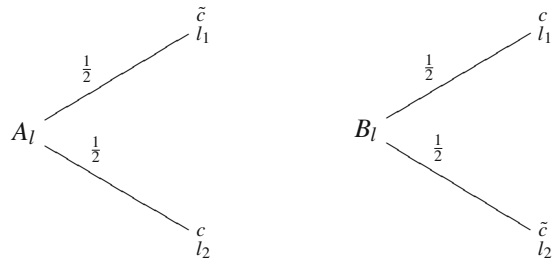
This result tells us that for SRD_L to be positive, the partial prudence index in labor ($-wL\frac{u_{ccc}}{u_{cc}}$) must be greater than 2. Again, this is very similar to the condition found in the previous section, and both conditions are in line with traditional results from the risk literature.

The income effect encourages hybrid entrepreneurs to increase the time allocated to labor to increase their wealth. The substitution effect, on the other hand, arises only in the presence of a multiplicative risk and encourages hybrid entrepreneurs to reduce their level of time allocated to paid employment. This is because he or she allocates time to the paid employment sector with a risky return, which increases his or her exposure to risk. Ultimately, we have a positive net precautionary effect on the time dedicated to paid employment when the income effect dominates the substitution effect.

4.4 Harm disaggregation in leisure in the behavioral approach

In this section, we address the strength of downside risk aversion on leisure. To do this, we take the comparative statics analysis to the second argument of the utility function and hold constant the first argument of the utility function (consumption).

Consider the pair of lotteries A_l and B_l, where $l_1 > l_2$ and $\tilde{c}$ is random consumption with $E(\tilde{c}) = c$.



where l_i for $i = 1, 2$ are the time allocated to leisure.

In this pair of lotteries, consumption is random because the hybrid entrepreneur faces risk in the entrepreneurial sector or in the paid employment sector. On the other hand, $l_1 > l_2$ when $e_2 > e_1$ or $L_2 > L_1$.

Next, we will derive the condition behind the definition of the effect of the strength of downside risk aversion on leisure that encompasses the different combinations of risk and leisure described in the previous paragraph.

Note that lottery A_l combines good with bad since l_1 is attached to the state in which the random consumption ($\tilde{c}$) appears, while l_2 is attached to the state in which its expected value (c) appears. The opposite case occurs with the B_l lottery in which good is combined with good and bad with bad. Thus, l_2 has more one-degree risk than l_1 (l_1 dominates l_2 via first-order stochastic dominance), which is reflected in the second argument of the utility function, while $\tilde{c}$ has more two-degree risk than c ($\tilde{c}$ is a mean-preserving spread of c), which is reflected in the first argument. In the expected utility framework, $A_l \succeq B_l$ if and only if

$$\begin{aligned} Eu(A_l) = \tfrac{1}{2}Eu(\tilde{c}, l_1) + \tfrac{1}{2}u(c, l_2) \geq \tfrac{1}{2}u(c, l_1) \\ + \tfrac{1}{2}Eu(\tilde{c}, l_2) = Eu(B_l) \end{aligned} \tag{26}$$

where $SD_l = Eu(A_l) - Eu(B_l)$ is the disutility associated with the one-degree increase in risk (leisure) and mean-preserving increase in risk (consumption) and represents the strength of the effect of downside risk

aversion on leisure. Condition Eq. 26 can be rewritten as:

$$[Eu(\tilde{c}, l_1) - u(c, l_1)] - [Eu(\tilde{c}, l_2) - u(c, l_2)] \geq 0 \quad (27)$$

Denote $\Gamma(\tilde{c}, c, l) = Eu(\tilde{c}, l) - u(c, l)$. Differentiating Γ with respect to l, we obtain $\Gamma_l = Eu_l - u_l$. Condition Eq. 27 can be rewritten as $\Gamma(\tilde{c}, c, l_1) - \Gamma(\tilde{c}, c, l_2) \geq 0$ if and only if $\Gamma_l \geq 0$, which is equivalent to $Eu_l \geq u_l$. By Jensen's inequality, $g(c) = u_l$ is a convex function whenever $g_{cc} = u_{lcc} > 0$.

The results above show that regardless of the source of income uncertainty, for the strength of the effect of downside risk aversion on leisure (SD_l) to be positive, u_{lcc} has to be positive. Following Eeckhoudt et al. (2007); Liu and Menegatti (2019), we can interpret the condition $u_{lcc} > 0$ as "cross-prudence in leisure". Therefore, higher levels of leisure help entrepreneurs to mitigate the detrimental effects on consumption created by the presence of risky returns on entrepreneurship or in the paid employment sector (with risky wages). Hybrid entrepreneurs can raise their levels of leisure by decreasing the time devoted to either of these two alternatives.

SD_l entails the strength of the preference for combining bad outcomes from income risk affecting consumption (c) with higher levels of leisure (l) by reducing the time allocated to entrepreneurship or the time allocated to paid employment.

Based on pairs of 50–50 lotteries, the following lemma characterizes a cross-prudence decision maker.

Lemma 4 *In the expected utility framework, assuming that $l_1 > l_2$, $c > 0$ and $\tilde{c}$ is a mean-preserving spread of c, then $A_l \succeq B_l$ if and only if $u_{lcc} > 0$.*

5 Optimal time allocations

Based on the definitions and the preliminary results developed in the previous section, we now analyze how individuals allocate their time between occupations in three scenarios: first, under risky returns on entrepreneurship (the usual approach in the occupational choice literature), under risky labor (earning a random wage), and under risky returns on both occupations, entrepreneurship, and paid employment. In each scenario, we analyze cases where these two occupational choices interact as substitutes or complements. Additionally, recall that e^* and L^* denote the optimal solutions under uncertainty (problem Eq. 1a) and e^{**} and L^{**} denote the optimal solutions under certainty (problem Eq. 8).

5.1 Risky returns on entrepreneurship

Proposition 1 *Let e and L be substitute occupational choices ($V_{eL} < 0$) if the hybrid entrepreneur is risk averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship that exceeds 2, ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$). Then, under risky entrepreneurship, when $SRD_e > SD_l > SD_L$, the hybrid entrepreneur will decide to make a precautionary entrepreneurial time allocation and a precautionary labor time deallocation. The opposite occurs when $SRD_e < SD_l < SD_L$, i.e., the hybrid entrepreneur will decide to make a precautionary entrepreneurial time deallocation and a precautionary labor time allocation.*

Proof See Appendix A. □

Proposition 1 includes four conditions: $SRD_e > 0$, $SD_L > 0$, $SD_l > 0$ and $SRD_e > SD_l > SD_L$. In this scenario, the hybrid entrepreneur faces a multiplicative risk in the entrepreneurship sector. Thus, condition $SRD_e > 0$ means that the income effect prevails over the substitution effect ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$), giving rise to a precautionary effect in time allocated to entrepreneurship. On the other hand, condition $SD_l > 0$ tells us that the entrepreneur is cross-prudent ($u_{lcc} > 0$) and thus encouraged to increase the level of leisure and, therefore, reduce the time allocated to entrepreneurship. However, condition $SRD_e > SD_l$ prevents the precautionary time allocation for leisure from reversing the precautionary effect on the time allocated to entrepreneurship ($e^* > e^{**}$). In this same scenario, risky returns on entrepreneurship become an additive risk in the labor sector. Therefore, condition $SD_L > 0$ tells us that the hybrid entrepreneur is prudent ($u_{ccc} > 0$) and hence encourages to increase the time allocated to labor. Nevertheless, condition $SD_l > SD_L$ causes the precautionary time allocated for leisure to reverse the precautionary effect on the time allocated to labor ($L^* < L^{**}$).

A similar argument can be given for condition $SRD_e < SD_l < SD_L$. As the effect of downside risk aversion on leisure is greater than the effect of risk aversion and downside risk aversion on entrepreneurship

($SRD_e < SD_l$), then the precautionary time allocation to leisure prevails over the precautionary effect on time allocated to entrepreneurship, which reduces time allocated to entrepreneurship ($e^* < e^{**}$). On the other hand, as the effect of downside risk aversion on labor exceeds that of downside risk aversion on leisure, precautionary effect on time allocated to labor dominates the effect of precautionary time allocation to leisure ($L^* > L^{**}$).

Proposition 2 *Let e and L be complementary occupational choices ($V_{eL} > 0$). If the hybrid entrepreneur is risk averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship that exceeds 2, ($-\theta e\frac{u_{ccc}}{u_{cc}} > 2$). Then, under risky entrepreneurship when $SRD_e > SD_l$ and $SD_L > SD_l$, the hybrid entrepreneur decides to make a precautionary entrepreneurial time allocation and a precautionary labor time allocation. The opposite occurs when $SRD_e < SD_l$ and $SD_L < SD_l$, i.e., the hybrid entrepreneur will decide to make a precautionary entrepreneurial time deallocation and a precautionary labor time deallocation.*

Proof See Appendix B. □

Proposition 2 includes five conditions: $SRD_e > 0$, $SD_L > 0$, $SD_l > 0$, $SRD_e > SD_l$ and $SD_L > SD_l$. The interpretation of the first three conditions is the same as that for intuition for Proposition 1. Conditions $SRD_e > SD_l$ and $SD_L > SD_l$ prevent the time precautionarily allocated to leisure to reverse the precautionary effect on the time allocated to entrepreneurship ($e^* > e^{**}$) and labor ($L^* > L^{**}$), respectively. The opposite occurs when $SRD_e < SD_l$ and $SD_L < SD_l$, i.e., the hybrid entrepreneur decides to make a precautionary leisure time allocation, reducing the time allocated to entrepreneurship ($e^* < e^{**}$) and labor ($L^* < L^{**}$).

Thus, the results from Proposition 2 indicate that when occupational choices interact as complements, the decision to allocate more time to entrepreneurship does not imply the decision to deallocate time from labor but rather to allocate more time to that occupational choice as long as the effects of risk aversion and downside risk aversion on entrepreneurship and the effect of downside risk aversion on labor are both higher than the effect of downside risk aversion on leisure.

Propositions 1 and 2 show that precautionary entrepreneurial and labor time allocation are mediated by the sign of V_{Le} and depend on the comparison between the relative magnitudes of the effects of the strength of downside risk aversion in entrepreneurship, labor, and leisure. Therefore, the usual sufficient conditions for risk aversion and DARA used in entrepreneurship literature (Bonilla & Vergara, 2021; Cressy, 2000; Hvide & Panos, 2014; Kihlstrom & Laffont, 1979) are insufficient when the entrepreneur is a hybrid entrepreneur. In this case, the strength of downside risk aversion becomes the key factor in inducing precautionary behavior.

5.2 Risky returns on labor

We will now study what happens to the optimal time allocations in each occupation when the source of risk comes from the paid-employment sector. In this case, the hybrid entrepreneur's optimization problem is

$$\max_{L,e} Eu(\tilde{w}L + \theta e, T - L - e) \tag{28}$$

where $\tilde{w}$ is a positive random variable describing the random wage and θ is a nonrandom return on entrepreneurship.

The analysis is analogous to the previous case where the source of risk comes from risky entrepreneurship $\tilde{\theta}$. Therefore, the following two propositions mirror Propositions 1 and 2 in the previous section. However, in this case, e^* and L^* are the optimal choices in problem Eq. 38, while e^{**} and L^{**} are the optimal choice for the following:

$$\max_{L,e} u(E[\tilde{w}]L + \theta e, T - L - e) \tag{29}$$

Proposition 3 *Let e and L be substitute occupational choices ($V_{eL} < 0$). If the hybrid entrepreneur is risk averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in labor that exceeds 2, ($-wL\frac{u_{ccc}}{u_{cc}} > 2$). Then, in risky labor when $SD_e > SD_l > SRD_L$, the hybrid entrepreneur decides to make a precautionary entrepreneurial time allocation and a precautionary labor time deallocation. The opposite occurs when $SD_e < SD_l < SRD_L$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time*

deallocation and a precautionary labor time allocation.

Proof See Appendix C. □

Proposition 4 *Let e and L be complementary occupational choices ($V_{eL} > 0$). If the hybrid entrepreneur is risk averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in labor that exceeds 2, ($-wL\frac{u_{ccc}}{u_{cc}} > 2$). Then, in risky labor when $SD_e > SD_l$ and $SRD_L > SD_l$, the hybrid entrepreneur will decide to make both a precautionary entrepreneurial time allocation and a precautionary labor time allocation. The opposite occurs when $SD_e < SD_l$ and $SRD_L < SD_l$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time deallocation and a precautionary labor time deallocation.*

Proof See Appendix D. □

The results of Propositions 3 and 4 confirm our results from Propositions 1 and 2 by providing the sufficient conditions for precautionary entrepreneurial and labor time allocation (or deallocation). As noted previously, these conditions depend on the relative magnitude of the effect of downside risk aversion on entrepreneurship, leisure, and labor. However, regardless of the source of income risk (whether entrepreneurship or labor), the results remain the same. We know that occupations can interact as substitutes or as complements and that hybrid entrepreneurs can mitigate the effects of risky income from either labor or entrepreneurship through precautionary behavior. Our results show that a hybrid entrepreneur will always prefer to allocate more time to entrepreneurship in the substitute case or to both entrepreneurship and labor in the complementary case if it will help him or her better apportion the detrimental effects of risk. Therefore, hybrid entrepreneurs will self-insure to increase the expected value of their random income and, in the case of complements, to increase the deterministic part of their income.

5.3 Risky returns on both entrepreneurship and labor

Finally, considering the recent experience with the pandemic that affected both product and labor markets, we analyze the case of random returns on both occupations. We assume that these two risks are independent. Our case is where entrepreneurship is associated with the economic cycle and the level of intrinsic competition within the industry, while the risk associated with the returns on labor is connected with specific individual characteristics such as hierarchical position in the organization and years of experience.

In this case, the maximization problem of a hybrid entrepreneur becomes

$$\max_{L,e} Eu(\tilde{w}L + \tilde{\theta}e, T - L - e) \tag{30}$$

The optimal decision (e^*, L^*) is obtained when the first-order conditions of problem Eq. 30 are satisfied and the second-order conditions hold.

Moreover, similar to the previous sections, to study precautionary entrepreneurial and labor time allocation or deallocation by comparing the optimal choices in Eq. 30 to those in the following problem:

$$\max_{L,e} u(E[\tilde{w}]L + E[\tilde{\theta}]e, T - L - e) \tag{31}$$

In the next propositions, let us call e^* and L^* the optimal choices in Eq. 30 and e^{**} and L^{**} the optimal choices in Eq. 31.

Proposition 5 *Let e and L be substitute occupational choices ($V_{eL} < 0$). If the hybrid entrepreneur is risk averse ($u_{cc} < 0$), cross-prudent in leisure ($u_{lcc} > 0$), prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship and labor that exceeds 2, ($-\theta e\frac{u_{ccc}}{u_{cc}} > 2$ and $-wL\frac{u_{ccc}}{u_{cc}} > 2$). Then, under simultaneously risky entrepreneurship and labor, when $SRD_e > SD_l > SRD_L$, the hybrid entrepreneur will decide to make a precautionary entrepreneurial time allocation and a precautionary labor time deallocation. The opposite occurs when $SRD_e < SD_l < SRD_L$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time deallocation and a precautionary labor time allocation.*

Proof See the Appendix E. □

Proposition 6 *Let e and L be complementary occupational choices ($V_{eL} > 0$). If the hybrid entrepreneur is risk averse ($u_{cc} < 0$), cross-prudent in leisure ($u_{lcc} > 0$), prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship and labor that exceeds 2, ($-\theta e\frac{u_{ccc}}{u_{cc}} > 2$ and $-wL\frac{u_{ccc}}{u_{cc}} > 2$). Then, under simultaneously risky entrepreneurship and*

labor, when $SRD_e > SD_l$ and $SRD_L > SD_l$, the hybrid entrepreneur will decide to make both a precautionary entrepreneurial time allocation and a precautionary labor time allocation. The opposite occurs when $SRD_e < SD_l$ and $SRD_L < SD_l$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time deallocation and a precautionary labor time deallocation.

Proof See the Appendix E.

Propositions 5 and 6 demonstrate that the precautionary time allocation behavior of hybrid entrepreneurs depends first on how the occupations interact with each other and second on the relative magnitude of the effect of downside risk on entrepreneurship, labor, and leisure. The analysis of this particular case (independent risks in both sectors) does not differ from the previous analyses, where precautionary time allocation to sectors is a way to self-insure against risk. In particular, higher downside risk aversion in entrepreneurship tends to make a hybrid entrepreneur more willing to devote additional hours to entrepreneurship to avoid, at all costs, bad states of nature from that activity and to increase the expected payment from entrepreneurship. This means that, if the two activities are substitutes, then time allocation to labor will decrease, while if they are complements, time allocation to labor will increase. A summary of the results is presented in the following table: (Table 1)

6 Conclusion and implications

This study aims to shed light on hybrid entrepreneurship, a trend that will become more prevalent in the near future. Technological innovation and labor contract flexibility will further expand hybrid entrepreneurship.

Traditional literature on entrepreneurship studying the transition from labor to self-employment has been based on the occupational discrete choice model. However, some recent literature (see Folta et al., 2010) and the disruptive effect of the pandemic that made employment relationships more flexible and granted more freedom in the use of working time through remote technologies highlight the importance of studying the intermediate case of hybrid entrepreneurship to better understand how the productive sector and the labor market interact in the post-COVID-19 economy. This is particularly important when the economy is hit by a negative and unprecedented shock, and policy-makers need to make policy decisions to help the economy bounce quickly back from the economic downturn and regain healthy economic growth.

We developed a bivariate choice model in which entrepreneurship and paid employment can interact as substitutes or complements, providing the conditions for precautionary time allocation to entrepreneurship in the face of increased risk. We also studied different cases, such as when the risk comes solely from the entrepreneurial sector (the classical case), when risk comes solely from the labor sector (rare, blackboard experimental thought), and when risk comes from both sectors, which is the new and more realistic perspective of hybrid entrepreneurship.

One of the key findings in our bivariate model of hybrid entrepreneurship is that many of the results rest on conditions of downside risk aversion. In particular, when the strength of downside risk aversion in entrepreneurship is high enough, if risk increases, a hybrid entrepreneur will allocate more time to entrepreneurship (precautionary behavior) as a way to avoid, at all costs, bad entrepreneurship results.

Our findings have implications for entrepreneurship scholars and policymakers. First, we connect an entrepreneurial economic model with the risk apportionment approach based on decision theory and demonstrate that hybrid entrepreneurs can self-insure in response to income risks by allocating more time to entrepreneurship. Our results therefore contribute to the longstanding puzzle identified by Parker et al. (2005) regarding why self-employed individuals work longer hours for lower average compensation than their employed counterparts do.

Second, our findings show that hybrid entrepreneurs who have precautionary time allocation behavior are willing to allocate more time to entrepreneurship in scenarios of increased risk. Therefore, boosting hybrid entrepreneurship during recessions would be a viable economic policy for economic recovery through entrepreneurial innovation, especially considering the countercyclical dynamic between self-employment and the business cycle (see Fossen, 2021).

Third, our findings have a practical application for organizations that employ hybrid entrepreneurs. It is critical for such organizations to understand how

Table 1 e^* and L^* under labor income risk

Source of risk	V_{eL}	e	L	Sufficient condition	Risk preferences
Entrepreneurial sector	< 0	$e^* > e^{**}$	$L^* < L^{**}$	$SRD_e > SD_l > SD_L$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-\theta e \frac{u_{ccc}}{u_{cc}} > 2$
	> 0	$e^* > e^{**}$	$L^* > L^{**}$	$SRD_e > SD_l$ and $SD_L > SD_l$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-\theta e \frac{u_{ccc}}{u_{cc}} > 2$
Paid employment sector	< 0	$e^* > e^{**}$	$L^* < L^{**}$	$SD_e > SD_l > SRD_L$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2$
	> 0	$e^* > e^{**}$	$L^* > L^{**}$	$SD_e > SD_l$ and $SRD_L > SD_l$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2$
Both sectors	< 0	$e^* > e^{**}$	$L^* < L^{**}$	$SRD_e > SD_l > SRD_L$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0, -\theta e \frac{u_{ccc}}{u_{cc}} > 2$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2$
	> 0	$e^* > e^{**}$	$L^* > L^{**}$	$SRD_e > SD_l$ and $SRD_L > SD_l$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0, -\theta e \frac{u_{ccc}}{u_{cc}} > 2$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2\ u_{cc} < 0,$

these risks affect their employees' time devoted to both occupations, especially if they believe that this type of worker is more imaginative, motivated, and entrepreneurial, positively impacting the work of their businesses through emerging opportunities for innovation (Marshall et al., 2019; Sessions et al., 2021; Urbig et al., 2021).

Our analysis, however, has certain limitations. First, we do not account for entrepreneurial inclinations; that is, some people have a natural proclivity for entrepreneurship and hence begin in paid work only as a temporary activity while "getting things right" to transition into entrepreneurship. As a result, these types of personalities are not adequately represented in the rational marginal analysis that we have just constructed. Another limitation is the exclusive utilization of 50–50 lotteries, which restricts the analysis to several alternative potential distributions.

Future research should empirically explore the hybrid entrepreneur's precautionary time allocation behavior with data. For example, increased risk can be measured using the EPU index as a way to test the implications of our model in the real world. Additionally, since entrepreneurial choice theory is primarily focused on 50–50 lotteries; a far more technically complex study involving various types of lotteries is left for future research.

The new economy has incorporated significant new flexibility in the labor market that must be considered in any serious economic policy that aims to boost the entrepreneurial sector, and our paper provides an initial analytical conceptualization of this issue.

Appendix

Appendix A

Proof of Proposition 1

Proof We want to show that under the previous assumptions, when $V_{eL} < 0$ and the returns on entrepreneurship are risky, then a sufficient condition for $e^* > e^{**}$ and $L^* < L^{**}$ is $SRD_e > SD_l > SD_L$. This is exactly what is represented in Fig. 2a, which is the case in which we replace $E[\tilde{\theta}]$ with $\tilde{\theta}$ to incorporate risky entrepreneurship, and the curve LL moves to the left while the curve ee moves to the right. We will now determine the conditions for such movements.

Let us suppose that $e^* > e^{**}$. Then, substituting e^{**} into the FOC of Eq. 2 implies:

$$E\tilde{\theta}u_c(wL+\tilde{\theta}e^{**}, T-L-e^{**}) - Eu_l(wL+\tilde{\theta}e^{**}, T-L-e^{**}) > 0 \tag{32}$$

while the FOC of problem Eq. 8 guarantees that

$$E[\tilde{\theta}]u_c(wL + E[\tilde{\theta}]e^{**}, T - L - e^{**}) - u_l (wL + E[\tilde{\theta}]e^{**}, T - L - e^{**}) = 0 \tag{33}$$

Using Eqs. 32 and 33 together, we see that

$$E\tilde{\theta}u_c - E[\tilde{\theta}]u_c > Eu_l - u_l \tag{34}$$

Given Eq. 19 and Lemma 4, this occurs if $SRD_e > SD_l$, which is the first part of the sufficient condition of the proof.

Now, let us focus on L. We know that $L^* < L^{**}$ is satisfied when:

$$wEu_c(wL^{**}+\tilde{\theta}e, T-L^{**}-e) - Eu_l(wL^{**}+\tilde{\theta}e, T-L^{**}-e) < 0 \tag{35}$$

while the FOC of problem Eq. 8 guarantees that

$$wu_c(wL^{**} + E[\tilde{\theta}]e, T - L^{**} - e) - u_l(wL^{**} + E[\tilde{\theta}]e, T - L^{**} - e) = 0 \tag{36}$$

Using Eqs. 35 and 36 together, we see that

$$wEu_c - wu_c < Eu_l - u_l \tag{37}$$

Given Eq. 23 and Lemma 4, this occurs if $SD_L < SD_l$.

In summary, under risky entrepreneurship and when $V_{eL} < 0$, precautionary entrepreneurial time allocation and precautionary labor time deallocation occur when:

$$SRD_e > SD_l > SD_L \tag{38}$$

Following the same procedure, we show straightforwardly that precautionary entrepreneurial time deallocation and precautionary labor time allocation occur when $SRD_e < SD_l < SD_L$, which is the case illustrated in Fig. 2b. □

Appendix B

Proof of Proposition 2

Proof We want to show that under the previous assumptions, when $V_{eL} > 0$ and the returns on entrepreneurship are risky, then a sufficient condition for $e^* > e^{**}$ and $L^* > L^{**}$ is $SRD_e > SD_l$ and $SD_L > SD_l$. This is exactly what is represented in Fig. 3a, which is the case where we replace $E[\tilde{\theta}]$ with $\tilde{\theta}$ to incorporate risky entrepreneurship, and the curve LL moves to the right while the curve ee moves to the left. We will now determine the conditions for such movements, and since the proof is homologous with the previous proof, we will present it very succinctly. Thus, when $e^* > e^{**}$ and $L^* > L^{**}$, we know that

$$E\tilde{\theta}u_c - E[\tilde{\theta}]u_c > Eu_l - u_l \tag{39}$$

$$wEu_c - wu_c > Eu_l - u_l \tag{40}$$

Similar to the proof of Proposition 1 and in accordance with Eqs. 19 and 23 and Lemma 4, this occurs as long as

$$SRD_e > SD_l \text{ and } SD_L > SD_l \tag{41}$$

Straightforward calculations show that precautionary entrepreneurial time deallocation and precautionary labor time deallocation occur when $SRD_e < SD_l$ and $SD_L < SD_l$, which is the case illustrated in Fig. 3b. □

Appendix C

Proof of Proposition 3

Proof We want to show that under the previous assumptions, when $V_{eL} < 0$ and the return on labor is risky, then a sufficient condition for $e^* > e^{**}$ and $L^* < L^{**}$ is $SD_e > SD_l > SRD_L$. Note that for $e^* > e^{**}$ and $L^* < L^{**}$ to be satisfied, we only need that

$$\theta Eu_c - \theta u_c > Eu_l - u_l \tag{42}$$

$$E\tilde{w}u_c - E[\tilde{w}]u_c < Eu_l - u_l \tag{43}$$

In accordance with Eq. 21 and Lemma 4, we know that Eq. 42 occurs when $SD_e > SD_l$, and from Eq. 25 and Lemma 4, we know that Eq. 43 occurs if $SRD_L < SD_l$. To summarize in the same way as for the proof of Proposition 1, this occurs as long as $SD_e > SD_l > SRD_L$.

In this scenario, the hybrid entrepreneur faces an additive risk in the entrepreneurship sector. Thus, condition $SD_e > 0$ tells us that the entrepreneur is prudent ($u_{ccc} > 0$) and thus encouraged to increase the time allocated to entrepreneurship. Therefore, condition $SD_e > SD_l$ means that the precautionary entrepreneurship effect is stronger than the precautionary leisure effect, and as a result, $e^* > e^{**}$. Otherwise, as the entrepreneur faces a multiplicative risk in the labor sector, condition $SRD_L > 0$ tells us that the income effect prevails over the substitution effect, that is, $-wL\frac{u_{ccc}}{u_{cc}} > 2$. Then, condition $SD_l > SRD_L$ means that the precautionary leisure effect is stronger than the precautionary labor effect, and as a result, $L^* < L^{**}$.

We can make a similar argument to explain condition $SD_e < SD_l < SRD_L$, where $SD_e < SD_l$ implies that $e^* < e^{**}$, while $SD_l < SRD_L$ implies that $L^* > L^{**}$. □

Appendix D

Proof of Proposition 4

Proof The proof and interpretation of the results is similar to that of Proposition 2, except that we use conditions Eqs. 21 and 25 and Lemma 4. Thus, condition $SD_e > (<) SD_l$ implies that $e^* > (<) e^{**}$, while condition $SRD_L > (<) SD_l$ implies that $L^* > (<) L^{**}$. □

Appendix E

Proof of Proposition 5

Proof We want to show that under the above assumptions, when $V_{eL} < 0$ and the return on both labor and entrepreneurship is risky, then a sufficient condition for

$e^* > e^{**}$ and $L^* < L^{**}$ is $SRD_e > SD_l > SRD_L$. Note that for $e^* > e^{**}$ and $L^* < L^{**}$ to be satisfied, we only need that

$$E\tilde{\theta}u_c - E[\tilde{\theta}]u_c > Eu_l - u_l \tag{44}$$

$$E\tilde{w}u_c - E[\tilde{w}]u_c < Eu_l - u_l \tag{45}$$

Condition Eq. 44 was already proved in Proposition 1, while condition Eq. 45 was proven in Proposition 3. Then, from both conditions, we deduce that $SRD_e > SD_l > SRD_L$.

In this new scenario, the hybrid entrepreneur faces a multiplicative risk in both the entrepreneurship and labor sectors. Thus, condition $SRD_e > 0$ tells us that the income effect prevails over the substitution effect when risk increases in the entrepreneurial sector ($-\theta e\frac{u_{ccc}}{u_{cc}} > 2$), while $SRD_L > 0$ means that the income effect prevails over the substitution effect when risk increases in the labor sector ($-wL\frac{u_{ccc}}{u_{cc}} > 2$). As we saw before, $SRD_e > SD_l$ implies that $e^* > e^{**}$, i.e., the precautionary effect in time allocated to entrepreneurship is stronger than the precautionary effect in time allocated to leisure. Additionally, $SD_l > SRD_L$ implies that $L^* < L^{**}$, i.e., the precautionary effect in time allocated to leisure is stronger than the precautionary effect in time allocated to labor. The opposite occurs whenever $SRD_e < SD_l < SRD_L$. □

Appendix F

Proof of Proposition 6

Proof Analogous to the proofs of Propositions 2 and 4. □

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