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Estimating damages from bidding rings in first-price auctions[☆]

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ABSTRACT

Bidding rings typically coordinate to rig auctions and keep prices low. Despite bid rigging being pervasive, measuring its damages (i.e., the revenue loss suffered by the auctioneer) is a challenge for antitrust authorities. Indeed, most of the methods to quantify damages compare outcomes of auctions affected by the collusive behavior with unaffected auctions, requiring data that is hard to obtain. We propose a model-based method to estimate damages. Its main advantages are that only information on affected auctions is required and that the underlying assumptions of the economic model are explicit, so they can be challenged and eventually modified for damage reassessment. In a Monte Carlo exercise, we show that our methodology performs well in moderate-size samples. We apply our method to data from the Ohio milk cartel and estimate damages similar to those found in previous studies, even when we discard information from non-affected markets.

1. Introduction

Auctions and procurements are widely used mechanisms in the private and public sectors to buy and sell goods and services. Collusion among bidders, however, is a permanent concern for auctioneers and antitrust authorities, as the benefits of handling an auction critically depend on bidders behaving competitively.[1] In the US, most of Section 1 violations of the Sherman Act cases are related to collusion in auctions or procurements; the evidence of collusion is overwhelming.[2] According to the D.O.J.'s website, since 2000, around 400 cases involving a bid-rigging accusation were filed.

From the perspective of antitrust authorities, it is of paramount importance to use publicly available bidding data to (1) "flag" bidders whose bids are inconsistent with competitive bidding, and (2) quickly estimate potential damages if anticompetitive behavior is suspected. These analyses should help authorities "police" the market, especially before they launch a costly full investigation. For instance, it might not even be worthwhile to start an investigation if the damages are smaller than the cost of an investigation.

Related to the second issue, we propose a tractable yet general method to estimate the damages caused by collusive behavior using bid data from affected markets only, with no need of having additional information from non-affected markets. We use structural analysis of auction data methodologies to recover players' underlying valuations, and then we compute numerically counterfactual competitive bids. The comparison of the data with the estimated counterfactual behavior gives us the damages.

From a theoretical point of view, auctions are games of incomplete information and their equilibria are well understood. This makes auctions attractive for a model-based empirical analysis, the so called "structural approach". The key insight here is that equilibrium bidding functions are strictly monotonic and a one-to-one mapping from observed bids to unobserved valuations can be identified from observed bids (Guerre et al., 2000). And once the unobserved valuations have been recovered, a counterfactual competitive equilibrium can be numerically solved (Hubbard et al., 2013).

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[1] A different but related problem arises when the auctioneer and one bidder somehow collude. See Celentani and Ganuza (2002), Huang and Li (2015), Wang (2020) and Chandel and Sarkar (2023) for models of this sort.

[2] See Comanor and Schankerman (1976), Feinstein et al. (1985), Lang and Rosenthal (1991), Porter and Zona (1993), Froeb et al. (1993), Baldwin et al. (1997), Bajari (2001), Porter and Zona (1999), Pesendorfer (2000), Marshall and Meurer (2004), Asker (2010), Harrington (2008), Marshall and Marx (2012); and Conley and Decarolis (2016).

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The main advantages of our method are that it is simple to implement, it does not require additional information on markets unaffected by the collusive agreement, and it assumptions are explicit, so that they can be challenged and eventually modified for damage reassessment. For example, to apply the methodology, an explicit assumption must be made on how the cartel picks its serious bidder. It could do it randomly, or it could choose the ring member with the highest valuation (e.g., by running an internal auction).[3]

We envision that the method we propose can be used by antitrust agencies both for an initial and quick assessment of damages and for a more in-depth analysis once a full investigation is underway. To provide a proof-of-concept, we apply our method to the data from the Ohio milk cartel from Porter and Zona (1999) and we find the estimated damage to be around 6%, which is similar to what Porter and Zona (1999) find.

Markets like timber, highway constructions, off-shore wildcat drilling, and school milk programs among others have been extensively studied in the empirical auction literature; see Athey and Haile (2006) and the references therein. The main focus of the empirical literature on collusion in auctions has been on the possibility of distinguishing competitive vs. collusive behavior (either explicit or tacit) from economic data.[4]

Motivated by particular bid-rigging cases, several papers also attempt to quantify the auctioneer's revenue loss (or excessive costs in procurement cases) associated with collusion. Most of this literature (we briefly review it below) consists of reduced-form empirical analysis, where the actual performance of the rigged market is compared to a non-rigged market. The comparator could be the same market in a different period, a similar market not suspected of collusion, or unsuspected firms acting in the same market where collusion is suspected. This approach is widely used to assess damages for different anti-competitive conducts (see Rubinfeld (2012)).

Two advantages of the comparator-based techniques are that they are relatively easy to implement and simple to communicate, which might be relevant in litigation contexts. Their main disadvantage, on the other hand, is that the reliability of the results critically hinges on the validity of the comparator market and the possibility of controlling for those additional variables that may explain differences between the two markets. The availability of information on relevant control variables and/or non-affected markets conditions the possibility of using this reduced-form approach. This difficulty can be partially overcome when a diff-in-diff approach can be followed; see, for example, the European Commission 2013 guide on antitrust damage assessment (European Commission, 2013) for a thorough description of different damage-assessment methods and Zona (2011) for a detailed critique of the benchmark approach.[5]

Although structural methods are widely used in antitrust analysis (see for example (Hausman et al., 1994), Shapiro (1996), and Werden (1997) in the context of merger simulations, and Zona (2011) in the context of imperfect competition models to assess damages in price-fixing cases), there is scant literature on this type of methodologies for damage assessment in the context of bidding rings. In this paper, we give a step towards filling this gap.

In a nutshell, we propose a three-step method: In the first step, we follow (Laffont and Vuong, 1996; Guerre et al., 2000) to recover bidders' valuations from observed bids. This is possible because the equilibrium bidding functions are strictly increasing, and therefore there is a one-to-one mapping from observed bids to underlying unobserved valuations. In the second step we follow (Hubbard et al., 2013) and, starting from recovered valuations, we numerically find the equilibrium bidding functions under the assumption that all players behave competitively. This is our counterfactual or *but-for* scenario. As a final step, we simply identify differences between the data and the counterfactual scenario and calculate revenue losses.

To apply our methodology we require that the identity of the cartel members is known and the functioning of the cartel has been established. The identity of the cartel members is necessary to properly recover the valuation of the serious bidder of the cartel (i.e., the one making the more attractive offer among the cartel members), as the first-order equation that characterizes his/her behavior is different from the one of non-colluded players, and also to ignore phony bids in the estimation process. The functioning of the cartel – e.g., whether the cartel members follow a phases-of-the-moon scheme to coordinate or the cartel is efficient – is also key to correctly recovering underlying valuations from the observed bids.

Moreover, to accurately assess economic damages it is also relevant to know if the non-ring players' behavior is being affected by the cartel's existence. Naturally, if non-colluded firms are aware of the cartel's existence, their bidding choices will be different than if they do not suspect its existence. Starting from observed bids, the assumption about the non-ring firm's knowledge will affect the recovered underlying valuations and the counterfactual estimates.

In this respect, we propose a methodology for assessing damages under two polar assumptions: that all firms know about the cartel's existence or that all non-ring firms assign probability zero to its existence. In the first case, total damages could be decomposed into direct and "umbrella damages" (i.e., those related to the less-aggressive bidding behavior of non-cartel firms). In the second one, there would be no umbrella damage.

In our view, there is no obvious assumption to be made. Although a cartel will benefit if non-cartel firms know about the cartel since they will be less aggressive, this benefit may not compensate for the inherent larger risk of being discovered when more people know about the cartel.

In terms of the information required, we need to observe both winning and non-winning bids, but only of the affected market (unlike with comparator-based techniques).

To validate our methodology, we perform Monte Carlo exercises where we can compute the equilibrium behavior of firms under collusion and competition and compute true damages. Starting from the collusive-scenario generated data, we apply our methodology to estimate underlying valuations, counterfactual competitive bids, and damages. The results show that our method is robust and provides accurate results.

We then apply our methodology to the school districts' milk procurement markets in Ohio in the 80 s, where collusion was established. A cartel of three firms with their main plants in the Cincinnati area operated interruptedly between 1980 and 1991. Porter and Zona (1999) thoroughly analyze these markets by comparing bidding participation decisions and bid levels of defendants and non-defendants.

Consistent with the confession of two of the cartel members, they find that cartel members were more likely to submit bids even if

[3] In the first case, we would refer to an inefficient cartel or a phases-of-the-moon cartel (in reference to the 'Electrical Conspiracy' in the 1950s which used the phases of the moon to coordinate and select the serious bidder). In the second case, we would refer to an efficient cartel.

[4] See Porter and Zona (1993), Baldwin et al. (1997), Bajari (1997), Porter and Zona (1999), Bajari and Ye (2003), Aryal and Gabrielli (2013), Conley and Decarolis (2016), Kawai and Nakabayashi (2022), Chassang and Ortner (2019), Schurter (2020); and Harrington (2008) for a review of several methodologies.

[5] Two different approaches are used when only time-series data from the affected market is available. In the forecasting approach prices from the non-infringement period are regressed against control variables and the estimates are used to forecast but-for prices in the infringement period, and these prices are compared to actual prices to calculate damages. The alternative is the dummy approach, where all data is used to explain prices and a dummy variable is included to estimate the change in the level of prices during the infringement period. See McCrary and Rubinfeld (2014) and Deng (2020) for discussions on the advantages of each method and Boswijk et al. (2019) for an analysis of the effect that misdating the infringement period has on damages estimated with the two approaches.

their plants were far away from the school district and that these (phony) bids were insensitive to distance (unlike serious bids). They also estimate a reduced-form price equation including as controls market structure variables, auction-specific variables, and a variable that captures how affected by the ring the school district is. They find that collusion raised prices by 6.5% when considering all school districts where cartel firms had plants within 75 miles.

To apply our methodology to the milk-procurement market, we adapt it to consider endogenous entry. We find that the damages in terms of revenue are around 6%. Moreover, when we restrict the sample to those school districts where at least two members of the cartel were potential bidders, we find very similar results. This is an interesting robustness check for our methodology, as with this subsample there is no unaffected market being used as a comparator.

In our view, measuring the damages from anti-competitive practices with structural analysis makes the analysis more reliable and transparent, since all assumptions behind the theoretical model are explicit and can eventually be challenged and damages re-assessed under different assumptions.[6] Moreover, in many collusion cases pertaining to auctions an appropriate competitive comparator may not be available, so a structural approach becomes the only option.

Related literature. Several papers have assessed damages in bid-rigging cases, most of them performing reduced-form estimations following the before-after approach. Hendricks and Porter (1989) first propose a methodology to assess damages estimating a relationship between costs estimates and prices in non-rigged contracts, and then applying this relationship to calculate but-for prices in rigged contracts. With a richer dataset, Froeb et al. (1993) follow the same logic to estimate price overcharges between 23% and 30% in a frozen-perch-fillets case in the Eighties. The available information includes a post-conspiracy period, so they follow a before-after methodology using the price of fresh perch fillets as an additional control. In the context of an English auction, Nelson (1993) follows Hendricks and Porter's (1989) reduced-form approach and estimates damages between 17% and 28% in the sales of used police cars in the city of New York.

Kwoka (1997) estimates damages between 22% and 32% on the rigging of real-estate properties in the DC area. His calculations use the observed price at the formal English auction and the price resulting from a knock-out auction the cartel members held right after the formal one. Interestingly, his calculation of damages explicitly considers the fact that the distribution rule of ring profits generated incentives to bid below one's own valuation. Therefore, damages cannot be simply assimilated to the observed difference between the two auctions.

Closer to our paper, Asker (2010) estimates efficiency costs and damages in the context of a stamp dealers' cartel. The cartel used an internal knockout auction to allocate the good among the cartel members before participating in an English auction. Interestingly, the sharing rule among ring members induced overbidding in the knockout auction that was carried to the target auction. As a consequence, for some auctions, the presence of the ring generated damages to non-ring members, efficiency losses, and a gain for the auctioneer. In 19% of the auctions the ring won the price was 7% higher than without the ring; and in 27% of the auctions won by the ring the price was, on average, 17% lower than without the ring.

Our methodological proposal is similar in spirit to Asker (2010), in the sense that in the first stage the underlying distribution of valuations is recovered from the observed bids using the econometric model derived from the corresponding economic model, and in the second stage counterfactual data–assuming competition among all players–is generated from the recovered valuations. There are several obvious differences, though, as our focus is on first-price auctions and therefore the counterfactual differs. Also, in our setup point valuations can be recovered, and therefore damages and efficiency losses can be computed for each actual auction, so it is unnecessary to simulate auctions from the recovered valuation distributions.

Caoui (2022) also uses structural estimation to address the issue of umbrella damages in first-price auctions. In the context of an all-inclusive cartel of milk procurement in the Dallas area, he uses a comparator market to estimate a distribution of underlying valuations and then numerically simulate competitive procurement auctions. He finds that umbrella damages–i.e., those suffered in procurement processes won by non-ring firms–are as large as 35% of the damages in the cases the cartel wins.

The rest of the paper is organized as follows. In Section 2 we first present a fairly standard asymmetric first-price auction setup and then describe our methodological proposal to assess collusion damages in terms of revenue and efficiency losses. In Section 3 we describe our Monte Carlo experiments and present the results we obtain, comparing theoretical and estimated damages. In Section 4, we present the results of our methodology as applied to the Ohio Milk data, and in Section 5 we conclude.

2. Model setup and damage-assessment methodology

2.1. Model and notation

For simplicity, we consider homogeneous first-price sealed bid auctions with n_0 low-valuation bidders –type 0– and n_1 high-valuation bidders –type 1. We assume that participation in each auction is exogenous and known to all bidders. In every auction a type k bidder draws his/her valuation, i.i.d. across all other bidders, from $F_k(v)$ with common support $[\underline{v}, \overline{v}]$. A subset of bidders $W = W_0 \cup W_1$ form a collusive ring. We denote by w_0 and w_1 the number of type-0 and type-1 firms in the cartel and by $F_c(v)$ the distribution function of the valuation of the serious ring player, which will depend on the number and type of cartel members and its functioning. E.g., if the cartel is formed only by type-1 players and it is efficient, then $F_c(\cdot) = F_1(\cdot)^{w_1}$, while if the serious player is randomly selected, then $F_c(\cdot) = F_1(\cdot)$.

As discussed in the Introduction, the damages that a collusive agreement may generate depend on whether the non-ring players are aware of the cartel's existence. The equilibrium behavior both of ring and non-ring players would be affected by this assumption. We consider a Bayesian game where, beyond the private information that players have about their own valuations, we assume Nature determines with probability p whether the ring is conformed and, naturally, p is common knowledge. Logically, for the purpose of assessing damages we assume the cartel is formed.

The true value of p may depend on the specifics of each case, and our proposed methodology is silent about it: $p = 1$ implies non-ring members are perfectly aware of the cartel's existence, and $p = 0$ that they are convinced there is no cartel. We present here these two polar cases and leave for the appendix the intermediate cases.[7]

Case p=0. We denote the symmetric-type bidding strategy of a k-type firm that does not belong to the cartel by $s_{k0}(v|n_0, n_1)$ and the strategy of the firm that makes the serious offer of the cartel by $s_{c0}(v|n_0, n_1, |W_0|, |W_1|)$. The maximization problem for a type $k \in \{0, 1\}$ firm i with valuation v_{ki} that does not belong to the cartel is

$$\max_{b_{ki}} (v_{ki} - b_{ki}) F_k(s_{k0}^{-1}(b_{ki}))^{n_k - 1} F_{k'}(s_{k'0}^{-1}(b_{ki}))^{n_{k'}}; \quad k' \neq k \in \{0, 1\} \qquad (P_{k0})$$

[6] For example, in our damage-estimation process, we have to be explicit about assumptions on whether non-ring firms know about the cartel and the cartel functioning. Regarding the latter, we consider two polar cases: that the cartel is efficient (i.e., that the cartel member with the highest valuation makes the serious offer and that the internal functioning of the cartel does not distort its offer) or that the serious bidder is randomly chosen. Alternative cases could be considered with minor adjustments.

[7] An alternative model that could be easily accommodated in our methodology is one where some non-colluded players know about the ring's existence and others do not know.

and for the serious cartel bidder with valuation v_c is

$$\max_{b_c}(v_c - b_c)F_1(s_{10}^{-1}(b_c))^{n_1-w_1}F_0(s_{00}^{-1}(b_c))^{n_0-w_0}. \qquad (P_{c0})$$

The assumption that non-members of the cartel are unaware of the ring's existence is reflected in the formulation of problem (P_{k0}), where $s_{c0}(v)$ plays no role, and in the powers of $F_k(\cdot)$ and $F_{k'}(\cdot)$, which are $n_k - 1$ and $n_{k'}$ respectively.[8]

The first-order conditions of these two problems can be written as:[9]

$$v_{ki} = b_{ki} + \frac{1}{\frac{(n_k-1)f_k(s_{k0}^{-1}(b_{ki}))}{s'_{k0}(b_{ki})F_k(s_{k0}^{-1}(b_{ki}))} + \frac{n_{k'}f_{k'}(s_{k'0}^{-1}(b_{ki}))}{s'_{k'0}(b_{ki})F_{k'}(s_{k'0}^{-1}(b_{ki}))}}; \quad k \neq k' \in \{0,1\} \qquad (1)$$

$$v_c = b_c + \frac{1}{\frac{(n_1-w_1)f_1(s_{10}^{-1}(b_c))}{s'_{10}(b_c)F_1(s_{10}^{-1}(b_c))} + \frac{(n_0-w_0)f_0(s_{00}^{-1}(b_c))}{s'_{00}(b_c)F_0(s_{00}^{-1}(b_c))}}. \qquad (2)$$

We denote the distribution function of non-colluded type-k bids and the cartel serious bidder by $G_k(\cdot)$ and $G_c(\cdot)$, respectively (for simplicity we omit the dependence on n_0, n_1, w_0, and w_1). Following the identification argument of Guerre et al. (2000) and Campo et al. (2003), note that if the equilibrium strategies are strictly increasing, then $G_k(b) = F_k(s_{k0}^{-1}(b))$ and $g_k(b) = f_k(s_{k0}^{-1}(b))/s'_{k0}(s_{k0}^{-1}(b))$ for $k \in \{0,1\}$. This observation allows us to write the previous first-order conditions as

$$v_{ki} = b_{ki} + \frac{1}{\frac{(n_k-1)g_k(b_{ki})}{G_k(b_{ki})} + \frac{n_{k'}g_{k'}(b_{ki})}{G_{k'}(b_{ki})}}; \quad k \neq k' \in \{0,1\} \qquad (3)$$

$$v_c = b_c + \frac{1}{\frac{(n_1-w_1)g_1(b_c)}{G_1(b_c)} + \frac{(n_0-w_0)g_0(b_c)}{G_0(b_c)}}. \qquad (4)$$

Although a limitation, it is common in both the theoretical and empirical literature to assume common support of valuations, which leads to common support of bids (see footnote). A notable exception is the paper by Hubbard and Kirkegaard (2019) that studies a model with "bid-separation".[10]

An important implicit assumption in the previous equations is that firms have prior knowledge of the cost distribution of rival firms and base their bidding behavior on this knowledge. Competitive firms are not *learning* about the empirical distribution of bids from cartel members.

Case p=1. We denote by $s_{kE}(v|n_0,n_1,w_0,w_1)$ the equilibrium strategy of a non-ring type-k firm when an efficient cartel is formed and by $s_{kI}(v|n_0,n_1,w_0,w_1)$ when the cartel is inefficient. Similarly, denote by $s_{cE}(v|n_0,n_1,w_0,w_1)$ and $s_{cI}(v|n_0,n_1,w_0,w_1)$ the equilibrium bidding strategy of the serious cartel bidder in the two cases.

The problem for a type k non-ring firm with valuation v_i is

$$\max_{b_{ki}}(v_{ki} - b_{ki})F_k(s_{kj}^{-1}(b_{ki}))^{n_k-w_k-1}F_{k'}(s_{k'j}^{-1}(b_{ki}))^{n_{k'}-w_{k'}}F_c(s_{k'j}^{-1}(b_{ki})) \qquad (P_{k1})$$

and for the serious player of the cartel with valuation v_c is

$$\max_{b_c}(v_c - b_c)F_1(s_{1j}^{-1}(b_c))^{n_1-w_1}F_0(s_{0j}^{-1}(b_c))^{n_0-w_0}; \qquad (P_{c1})$$

where $k' \neq k \in \{0,1\}$ and $j \in \{E,I\}$.

Following a similar argument to the $p = 0$ case (and invoking GPV's identification argument), we can write the first-order conditions for a competitive type k firm as

$$v_{ki} = b_{ki} + \frac{1}{\frac{(n_k-w_k-1)g_k(b_{ki})}{G_k(b_{ki})} + \frac{(n_{k'}-w_{k'})g_{k'}(b_{ki})}{G_{k'}(b_{ki})} + \frac{g_c(b_{ki})}{G_c(b_{ki})}}; \quad k \neq k' \in \{0,1\}, \qquad (5)$$

while Eq. (4) is still valid for the serious bidder of the ring.[11]

2.2. Methodology to estimate damages

The methodology consists of four steps:

(i) Non-parametrically estimating de distribution functions of bids separately for each type of competitive player and the cartel serious offer (G_0, g_0, G_1, g_1, G_c, and g_c). We use the boundary-corrected kernel method proposed by Hickman and Hubbard (2015).[12]

(ii) Recovering valuation distributions: We know from Guerre et al. (2000) and Campo et al. (2003) that the IPV-FPA model is non-parametrically identified. The first-order conditions can be conveniently re-written in terms of bids' distributions rather than valuations' distributions, resulting in Eqs. (3) and (4) when $p = 0$ or (4) and (5) when $p = 1$.

For all non-serious bidders of the cartel it is not possible to follow a similar argument, as we do not know how they choose their cover bids. This issue is irrelevant if the serious bidder of the cartel is also the ring player who would necessarily do the most competitive offer (among cartel members) in the counterfactual scenario. This will be the case if the cartel is efficient and either all cartel members are of the same type or the serious player is of the weak–and more aggressive–type.

When this is not the case, we must entertain the possibility that non-serious bidders could win the auction in the counterfactual scenario. To allow this possibility, we generate random draws from the recovered valuations and assign them to the non-serious cartel players.

(iii) Computing counterfactual bids: We consider two different avenues. The first one, more involved, is valid for the two cases of $p = 0$ and $p = 1$, and the second one, much simpler, only for the case of $p = 0$.

Option 1. The counterfactual equilibrium bids are fully characterized by the system of first-order differential equations summarized in (3) and the boundary conditions $s_k(\underline{v}) = \underline{v}$, $k \in \{0,1\}$. Based on the estimated valuations of the previous step, we solve numerically for the counterfactual equilibrium using the algorithm proposed in Hubbard et al. (2013) and Hubbard and Paarsch (2009).[13] When non-ring bidders are unaware of the cartel's existence (i.e., $p = 0$), their bids must be identical in the two scenarios. Therefore, we replace the computed counterfactual bids with the original ones.

Option 2 (only if $p = 0$). Given that non-ring players do not change their bidding behavior, their observed bids and recovered valuations can be used to approximate the competitive bidding behavior of the ring members. We choose to fit a polynomial function that subsequently we apply to the ring player's valuations to calculate their counterfactual bids. This is a relatively simple option (which might be relevant in

[8] The case of $p = 0$ can be considered as a limit case for positive but very small values of p, as it is hard to reconcile that a cartel is formed and p is indeed zero.

[9] The existence and uniqueness of an equilibrium that satisfies these first order conditions together with boundary conditions $s_k(\underline{v}) = \underline{v}$ and $s_k(\overline{v}) = \overline{b}$ for some $\underline{v} < \overline{b} < \overline{v}$ and $k \in \{0,1\}$ has been established in Maskin and Riley (2000) and Lebrun (1996, 1999).

[10] We are thankful to a referee for pointing us out this important issue.

[11] Note that F_c in problem (P_{k1}) and G_c in Eq. (5) are different depending on whether the cartel is efficient or not. We are implicitly assuming that competitive players either know the nature of the cartel (and therefore can deduce F_c from F_k) or they learn G_c through the repeated auctions.

[12] It is always the case that the kernel density estimator is biased at the boundaries. There are several "solutions" proposed in the literature. We follow (Hickman and Hubbard, 2015), who in turn use the generalized reflection method proposed by Zhang et al. (1999) and further improved by Karunamuni and Zhang (2008). The main idea of the method is to create "pseudo data" on both sides of the boundary. If the rate at which the pseudo data vanishes is proportional to the sample size, then the kernel density estimator is uniformly consistent. In other words, (Karunamuni and Zhang, 2008) proposed an exact procedure that gets around the boundary issues.

[13] An alternative methodology is proposed in Fibich and Gavish (2011) and followed by Caoui (2022). As emphasized by Hubbard et al. (2013), Fibich and Gavish's (2011) algorithm is based on a fixed-point iteration approach that involves a change of variables, thereby requiring a transformation of the problem. The method we use in this paper is based on approximating the inverse bidding functions by means of Chebyshev polynomials; thus the spirit of each method is different. For an extensive discussion about these approaches, see Hubbard and Paarsch (2014).

the context of litigation) and, as we will show in our Monte Carlo experiments, it is also quite accurate.

(iv) Computing damages: Once the relevant counterfactual bid has been obtained, it is trivial to identify the winner in the counterfactual scenario and compare winning bids. Let b_ℓ^{*data} and b_ℓ^{*cf} be the winning bids in auction ℓ in the data and counterfactual scenarios, respectively. Then

$$\text{Auctioneer's Damages of Collusion} = \frac{\sum_{\ell=1}^{L}(b_\ell^{*cf} - b_\ell^{*data})}{\sum_{\ell=1}^{L} b_\ell^{*data}} \tag{6}$$

Moreover, in the case of $p = 1$ in which all players change their behavior in the counterfactual scenario, we can decompose total damages as the sum of direct damages and umbrella or indirect damages. Let $\mathbb{1}_\ell^{data}$ be an indicator function that assumes the value one when the winning firm in the data is a member of the cartel; then we can define umbrella or indirect damages as

$$\text{Indirect Damages} = \frac{\sum_{\ell=1}^{L}(1 - \mathbb{1}_\ell^{data}) \times (b_\ell^{*cf} - b_\ell^{*data})}{\sum_{\ell=1}^{L}(1 - \mathbb{1}_\ell^{data}) \times b_\ell^{*data}}, \tag{7}$$

and direct damages simply as the difference between total damages and indirect damages.

The methodology we propose is flexible and can (and must) be adjusted depending on how the cartel works. In particular, we consider two main cases: that the cartel members take turns to make the serious offer (a non-efficient cartel) vs. that the cartel selects the most efficient member to bid seriously (an efficient cartel). Also, note that Eqs. (4) and (5) are derived assuming that the internal agreement of the cartel does not distort the bidding incentives of the serious bidder. If this were the case, the first-order condition should be derived explicitly considering how the sharing rule of the cartel affects the bidding behavior.[14]

Other possible extensions to this methodology include more general first-price auctions models. In particular the case in which private values are affiliated could be relevant for some empirical applications. The key insight is to adapt the identification and estimation arguments. We refer the reader to Aryal et al. (2021) for further details.

3. Monte Carlo experiments

3.1. Experiment design

To assess the performance of our proposed methodology, we conduct a number of Monte Carlo simulations. Based on the numerical method proposed by Hubbard et al. (2013), we can solve both the collusive and the non-collusive scenarios and, at the end of the process, compute *true* damages. Starting from the data generated under collusion, we then apply our proposed methodology to estimate damages and compare them to the true ones.

We implement two sets of experiments, one for the case non-ring players are aware of the collusive agreement ($p = 1$), and one when they are not ($p = 0$). We describe in detail the case of $p = 1$, and then the few modifications needed for the $p = 0$ case. We consider a setup with $n_0 = 2$, $n_1 = 4$, $w_0 = 0$, $w_1 = 3$, $F_0(v) = v^{-1/3}$, and $F_1(v) = v^{-1/5}$. Both distributions are truncated between 0.1 and 0.9 to satisfy the assumptions in Guerre et al. (2000).

3.1.1. The case of $p = 1$

(A) Equilibrium behavior

(1) Given $F_0(v)$ and $F_1(v)$ and assuming an inefficient cartel is formed, follow (Hubbard et al., 2013) to compute the equilibrium bidding functions $s_{0I}(v|\cdot)$, $s_{1I}(v|\cdot)$, and $s_{cI}(v|\cdot)$ for a game with n_0 type-0 players, $n_1 - w_1$ type-1 players, and one serious cartel bidder.[15]

(2) Given $F_0(v)$, $F_1(v)$ and assuming an efficient cartel of w_1 type-1 firms is formed, follow (Hubbard et al., 2013) to compute the equilibrium bidding functions $s_{0E}(v|\cdot)$, $s_{1E}(v|\cdot)$, and $s_{cE}(v|\cdot)$ for a game with n_0 type-0 players, $n_1 - w_1$ type-1 players, and one serious cartel bidder.

(B) Data-generating process

(1) Based on $F_0(v)$ and $F_1(v)$, generate valuations for all participants in a thousand auctions.

(2) For the inefficient cartel case, randomly pick one of the cartel firms as the serious cartel bidder. For the efficient cartel case, identify for each auction the higher valuation among cartel bidders to determine the serious cartel bidder.

(3) Use valuations and the bidding functions deduced in (A) to calculate "bids" for all firms in all auctions in three scenarios: "true-counterfactual", "true efficient-cartel", and "true inefficient-cartel".

(4) Identify the winner in each auction and compute total revenue in the thousand auctions for each of the three scenarios. Then, simply by difference with the counterfactual scenario, obtain "true" revenue losses for the efficient and inefficient cartel cases.

(5) Repeat 1000 times.

(C) Estimated damages (for each of the thousand repetitions)

(1) Starting from the "inefficient-cartel bids" or the "efficient-cartel bids", follow (Hickman and Hubbard, 2015) to estimate the distribution of bids of type-0 firms, type-1 firms, and for the serious bidder of the cartel in the two scenarios.

(2)
- *Efficient cartel.* Use Eqs. (4) and (5) to obtain pseudo-valuations for the serious cartel firm and all non-cartel bidders respectively.
  Let $\hat{V}_{0E}$ and $\hat{V}_{1E}$ be the vectors of recovered valuations.
- *Inefficient cartel.* Use Eqs. (4) and (5) to obtain pseudo-valuations for the serious cartel firm and all non-cartel bidders respectively. Use recovered valuations of type-1 competitive firms to generate, randomly and with replacement, additional valuations for non-serious cartel bidders.
  Let $\hat{V}_{0I}$ and $\hat{V}_{1I}$ be the vectors of recovered valuations.

[14] Note also that if the cartel includes all firms of type k, then G_k cannot be directly estimated in step (i). This would be problematic when $p = 0$, because G_k is required in step (ii) (see Eq. (3)). In that case, we need to adapt our proposed method: first we estimate $G_{k'}$ as in step (i); second, we recover v_c as in step (ii) from Eq. (4) (note that $n_k = w_k$ and therefore G_k is irrelevant); third, (a) if the cartel is inefficient, given $G_{k'}$, we compute counterfactual (competitive) bids for type k players and then estimate G_k from the simulated bids, and (b), if the cartel is efficient, from the recovered distribution $\tilde{F}_c$ (which is the distribution of the first-order statistic) we can recover the parental distribution $F_k = \tilde{F}_c^{1/w_k}$, simulate valuations for all cartel bidders and compute counterfactual competitive bids as mentioned above. If $p = 1$ and the cartel is efficient, the only caveat is that the recovered distribution of valuations for the cartel is actually the distribution of the first-order statistic of F_k. In our empirical application we do not face this problem, and in the Monte Carlo exercises we assume the cartel does not include all players of one particular type.

[15] Note that when the cartel is inefficient and, given the assumption that all cartel firms are type-1, $s_{1I}(v|\cdot) = s_{cI}(v|\cdot)$; i.e. the game is one with only two types of players (n_0 type-0 and $n_1 - w_1 + 1$ type-1).

(3) Follow (Hubbard et al., 2013) to compute the (counterfactual) competitive equilibrium bidding functions for two games with n_0 type-0 players and n_1 type-1 players –one game based on the efficient cartel data and the second one based on the inefficient cartel. Then use $\hat{V}_{0E}$, $\hat{V}_{1E}$, $\hat{V}_{0I}$, $\hat{V}_{1I}$ and their corresponding equilibrium bidding functions to obtain "counterfactual-estimated bids" for all players in the two scenarios (efficient and inefficient cartels).
(4) Identify the winner in each auction and compute total revenue in the thousand auctions for the two counterfactual-estimated scenarios. The difference with the revenue in the "data" gives the "estimated" revenue losses for the efficient and inefficient cartel cases.

3.1.2. The case of $p = 0$

A few changes with respect to the process described for $p = 1$ must be considered

(A) Equilibrium behavior and (B) data generating process. For non-ring players, we use their counterfactual bids obtained for $p = 1$ for the three $p = 0$ scenarios: counterfactual, efficient cartel, and inefficient cartel. For the serious bidder of the cartel, we use its corresponding valuation (that may vary between the cases of efficient and inefficient cartels) and numerically solve Eq. (4) to calculate the bids in the two collusive scenarios.
(C) In the estimation process, the only modification with respect to the $p = 1$ case is that we must use Eq. (3)–rather than (5)– to recover pseudo-valuations for all non-ring firms. To compute counterfactual bids, we follow (Hubbard et al., 2013) as in the $p = 1$ case (Option 1 as described above), and also we use the polynomial approach mentioned earlier (Option 2).

3.2. Results

In this section we report our results by comparing the revenue and damages distributions for the different cases considered: efficient or inefficient cartel and $p = 0$ or $p = 1$.

3.2.1. The case of $p = 0$

In Table 1 we report the results for the efficient and inefficient cartel when non-ring players are unaware of the cartel's existence and, therefore, they do not change their bidding behavior in the counterfactual scenario. The two cartels considered – efficient and inefficient – are formed by three type-1 players.

The first part of the table shows the distribution of the "true" revenue in the competitive scenario and the two cartel cases considered, and their corresponding revenue losses (expressed as a percentage of total revenue in the collusive scenario). Logically, damages are larger when the serious bidder is picked randomly (i.e., the cartel is inefficient). As a percentage of total revenue, median damages are 17.7% and 7.8% in the cases of inefficient and efficient cartel, respectively.

In the second and third parts of the table we report the estimated total counterfactual revenue and damages following the proposed methodology. In the second part the counterfactual bids are calculated numerically solving the competitive equilibrium (starting from the recovered valuations), and in the third part we follow a simpler method that consists of estimating a polynomial relation between observed bids and recovered valuations of type-1 competitive players and "applying" the polynomial to the cartel's valuations to calculate counterfactual bids for the cartel (these are the two options described in the third step of our methodology).

The two methods perform remarkably well. This can be noted by comparing the true counterfactual revenue with the four lines that correspond to the different estimations of the counterfactual revenue (i.e., for the two methods and for the efficient and inefficient cartel), or comparing the distribution of estimated damages with the true ones. Although estimated median damages with the two methods are very similar (and very close to the true ones), the comparison of the two distributions of estimated damages shows that the option of adjusting a polynomial is more precise.

3.2.2. The case of $p = 1$

In Table 2 we present the results for the case that all players are aware of the cartel's existence and, therefore, all of them change their behavior in the counterfactual scenario.

Three facts stand out from a quick comparison of the two tables: First, for the inefficient cartel damages are much larger when $p = 1$. This is expected because non-ring firms are much less aggressive when they are aware of the cartel's existence and the cartel is inefficient (and the cartel itself is less aggressive as it takes into account that rivals are less aggressive).

Second, it is apparent that revenue and damages estimates are much noisier when $p = 1$ (as compared to the $p = 0$ case). This is not surprising, since all counterfactual bids must be recovered from the numerical solution of the counterfactual equilibrium and not just those of the cartel.

And third, despite being noisier, the results of Table 2 also show that the proposed methodology performs remarkably well recovering median revenues and damages: Estimated counterfactual revenues are 370 for both, efficient and inefficient cartel, very close to the true value of 365.

3.3. Additional results: Direct and indirect damages

An often-discussed issue in antitrust is who is entitled to collect damages and whether damages associated to "umbrella pricing" can be claimed. . In the context of auctions, umbrella pricing emerges naturally if non-ring firms are aware of the cartel's existence, as they would bid less aggressively when facing a cartel. On the contrary, if non-ring firms are unaware of the existence of the cartel, they would behave as if there were no cartel and there would be no umbrella pricing-effect.

A closely related issue is whether those auctioneers that sold to non-cartel firms can claim damages. They are obviously hurt if non-ring firms are aware of the cartel ($p = 1$), because they bid less aggressively. But even if they are unaware of the cartel and there is no "umbrella pricing", there are damages associated with those auctions won by a non-cartel firm that would have been won by a more aggressive cartel firm in the counterfactual scenario.[16] We refer to the latter as *Indirect Damages*, as opposed to *Direct Damages* associated to auctions won by a cartel member.

Table 3 shows the relevance of direct and indirect damages in the context of our Monte Carlo experiment under the assumption of $p = 0$. As expected, direct damages are much larger than indirect damages since non-ring players are not changing their behavior in the counterfactual scenario. Note also that estimated damages distributions are quite close to the true ones.

[16] The two arguments why a non-direct purchaser of the cartel may suffer damages make perfect economic sense, but in litigation it may be hard for them to successfully claim damages. See Maier-Rigaud (2014) for a discussion (and a critique) on why purchasers outside a vertical chain of purchasers are not considered in damage claims. In the recent judgement of *Ohio vs. Amex* merchants not buying from *Amex* were denied standing to sue for damages. They alleged that in a counterfactual scenario with *Amex* behaving competitively *Visa* would have had lower prices too, since in the factual scenario they were pricing under the umbrella of *Amex* (anticompetitive) prices. The situation in Europe, however, could be different after the ECJ judgement in *Kone AG and Others v. ÖBB Infrastruktur* (Case C-557/12); see Franck (2015) for a discussion of the ECJ ruling and on the desirability of allowing umbrella claims and the risk of over-deterrence.

Table 1
True and estimated revenue and damages (p = 0).

	5p	25p	50p	75p	95p
Data					
Revenue - Inefficient cartel	303.66	307.13	309.53	312.08	315.67
Revenue - Efficient cartel	332.77	335.87	338.13	340.20	342.94
Revenue counterfactual	358.82	362.18	364.60	366.69	369.48
Damages - Inefficient cartel (%)	16.28	17.12	17.67	18.31	19.47
Damages - Efficient cartel (%)	7.35	7.63	7.82	8.00	8.26
Estimation: Option 1 - Numerical solution					
Inefficient cartel					
Estimated revenue - Counterfactual	352.88	361.63	366.41	371.15	378.74
Estimated damages (%)	13.66	17.01	18.37	19.81	22.16
Efficient cartel					
Estimated revenue - Counterfactual	355.21	362.66	366.62	371.17	379.65
Estimated damages (%)	5.44	7.46	8.41	9.58	12.09
Estimation: Option 2 - Polynomial					
Inefficient cartel					
Estimated revenue - Counterfactual	357.71	362.10	365.63	368.71	373.53
Estimated damages (%)	16.20	17.27	18.05	18.88	20.00
Efficient cartel					
Estimated revenue - Counterfactual	358.34	362.26	365.11	368.34	372.45
Estimated damages (%)	7.01	7.54	7.98	8.50	9.35

Note: Damages are measured as a percentage of the total revenue, including those auctions in which there are no damages.

Table 2
True and estimated revenue and damages (p = 1).

	5p	25p	50p	75p	95p
Data					
Revenue - Inefficient cartel	262.23	264.86	267.04	269.02	272.2
Revenue - Efficient cartel	332.55	335.59	337.69	339.78	342.49
Revenue counterfactual	358.82	362.18	364.60	366.69	369.48
Damages - Inefficient cartel (%)	34.89	35.83	36.43	37.17	38.27
Damages - Efficient cartel (%)	7.53	7.79	7.94	8.11	8.36
Estimation					
Inefficient cartel					
Estimated revenue - Counterfactual	335.37	360.56	370.02	376.76	385.17
Estimated damages (%)	24.33	34.71	38.70	41.60	45.14
Efficient cartel					
Estimated revenue - Counterfactual	345.29	362.33	370.67	377.18	384.72
Estimated damages (%)	2.23	7.21	9.84	11.69	13.83

Table 3
Direct and indirect damages (% of revenue, p = 0).

	5p	25p	50p	75p	95p
Direct damages					
Inefficient cartel					
True	30.4	32.6	34.1	35.7	38.2
Estimated	30.2	32.8	34.7	36.8	39.7
Efficient cartel					
True	18.7	18.8	18.8	18.8	18.8
Estimated	17.1	18.3	19.2	20.2	22.0
Indirect damages					
Inefficient cartel					
True	12.5	13.4	14.1	14.8	15.8
Estimated	12.6	13.6	14.3	15.2	16.3
Efficient cartel					
True	1.1	1.2	1.3	1.4	1.6
Estimated	1.1	1.2	1.4	1.6	1.8

Note: Direct and indirect damages are measured as a percentage of the total revenue of the subset of auctions won by the cartel and by non-cartel players respectively, including those auctions in which there are no damages.

Table 4 presents similar results for the case of $p = 1$. The results are much noisier, since all bids change in the counterfactual scenario and the estimation process involves solving numerically the counterfactual equilibrium. Notably, in the case of an inefficient cartel, umbrella (or indirect) damages are as large as direct damages (compare second and sixth rows), while in the case of an efficient cartel they are about 57% of the direct damages (compare fourth and eighth rows). In the case of an efficient cartel, we overestimate indirect damages and somewhat underestimate direct damages.

3.4. Additional results

3.4.1. Misspecified models

Whether non-cartel firms are aware of the cartel's existence is an assumption in our proposed methodology. The assumption is certainly debatable, and it may be valid or not depending on the particular case under scrutiny. In the literature the two assumptions are commonplace. To test for the presence of a cartel, Aryal and Gabrielli (2013) assume that non-ring firms are unaware of the cartel's existence. On the other hand, to estimate umbrella damages, Caoui (2022) assumes that all firms know about the cartel. Moreover, he correctly argues that assuming they are unaware is inconsistent with firms learning about the other firms' cost distributions.

We consider two different types of mistakes: to assume that $p = 0$ when all firms are aware of the cartel's existence, or to assume $p = 1$ when non-ring members are unaware of the cartel's existence. In Table 6 we report median estimated damages when the model is misspecified together with the correctly estimated damages (in bold).

Table 4
Direct and indirect damages (% of revenue, p = 1).

	5p	25p	50p	75p	95p
Direct damages					
Inefficient cartel					
True	33.3	35.3	36.5	37.9	40.7
Estimated	20.5	34.0	37.0	39.5	43.3
Efficient cartel					
True	17.0	17.2	17.4	17.6	17.8
Estimated	8.2	11.7	13.3	14.6	16.3
Indirect damages					
Inefficient cartel					
True	34.4	35.6	36.4	37.2	38.6
Estimated	24.2	34.5	39.3	42.7	46.3
Efficient cartel					
True	2.0	2.2	2.3	2.5	2.6
Estimated	−2.3	4.1	7.6	10.3	13.6

Note: Direct and indirect damages are measured as a percentage of the total revenue of the subset of auctions won by the cartel and by non-cartel players respectively.

Table 5
Well and misspecified cases - median damages.

True \ Estimation	Inefficient cartel		Efficient cartel	
	p = 0	p = 1	p = 0	p = 1
p = 0	**18.1%**	39.3%	**8.0%**	15.6%
p = 1	20.7%	**38.7%**	7.9%	**9.9%**

In the first row we show the case when the true model is $p = 0$ and in the second when it is $p = 1$.

Two different errors occur when the model is misspecified. First, by using the incorrect first-order conditions – compare Eqs. (3) and (5) – the pseudovaluations of competitive firms will be underestimated if the correct model is $p = 1$, or overestimated if it is $p = 0$. Second, and quantitatively more important, we will miscalculate counterfactual bids for the cartel and for competitive firms.

When the true model is $p = 1$ (second row), this second effect is particularly severe: we would impose that all non-ring firms do not change their behavior in the counterfactual scenario and therefore underestimate damages. This effect is particularly large when the cartel is inefficient, because the bidding adjustment between the data and the counterfactual scenario is larger.

When the true model is $p = 0$ (first row), we would overestimate competitive firms' valuations and, in the second stage, we will solve a counterfactual equilibrium among "stronger" firms and obtain more aggressive counterfactual bidding functions that will be applied to overestimated valuations, leading to an overestimation of total damages. Again, this bias is larger when the cartel is inefficient (see Table 5).

These results confirm the intuition that estimated damages considering $p = 0$ and $p = 1$ could be considered as lower and upper bounds of true damages. But they also show that the distance between the bounds can be significant, especially when the cartel is inefficient. It would be therefore relevant to assess the validity of the assumptions. Note that, if the model is correctly specified, the distribution of valuations $F_k(v)$ can be recovered both from $G_k(b)$ and from $G_c(b)$. Therefore, the comparison of the two recovered valuation distributions could be exploited to test the assumption about p.[17]

3.4.2. Missidentified cartel

In this subsection we estimate the effect on estimated damages of incorrectly identifying the cartel members.[18] Logically, there are different mistakes that can be made and each one may have different effects on damages. We consider here one in which a cartel members is incorrectly identified as a competitive player, and we calculate damages for the cases of efficient and inefficient cartels and when $p = 0$ and $p = 1$.

When $p = 0$, a first order effect is that we would underestimate by a third the number of affected auctions for which damages are computed (recall that in this case we assume competitive players do not change their bids in the counterfactual scenario). A reinforcing effect is that since the size of the cartel is smaller, the difference between actual and counterfactual bids recovered for the cartel players will be smaller than the real difference. We observe in Table 6 that damages are severely underestimated for the case of $p = 0$.

But there is an additional effect that goes in the opposite direction that is particularly relevant for the $p = 1$ case. The fact that the size of the cartel considered to recover pseudovaluations is smaller than the true one implies that the recovered pseudovaluations will be larger than the true ones (see Eq. (5)). Therefore, computed counterfactual bids will be larger and bidding functions "more aggressive", potentially leading to overestimated damages. This is what we observe in the case of an efficient cartel and $p = 1$.[19]

4. Empirical illustration

4.1. Preliminaries

In this section we apply the proposed methodology to real data from the well-known bid-rigging case in school milk-procurement contracts in Ohio during the 1980's. Porter and Zona (1999) analyze this market and find strong economic evidence of collusion among three firms in the Cincinnati area. Moreover, two of the firms confessed to the agreement and describe the functioning of the cartel as one of respecting incumbency, with the other firms submitting phony complementary bids. The presence of a cartel including these three firms is also identified by Wachs and Kertész (2019), who develop a network-based method to detect cartels in auction markets.

Each school district awards an annual contract for the supply of school milk and other products. The process of soliciting bids is done independently by each school district every year between May and August. In response to these solicitations, interested dairies in a position to supply school milk submit bids. Typically, the low bidder is selected to supply milk in half-pints to the schools during the following school year.

As described in Porter and Zona (1999), several features of the market may facilitate collusion: The policy of making public all bids and the identity of bidders and the fact that competition is only on prices both simplify the monitoring of any agreement. Also, that there are multi-contact markets and the auctions do not occur at the same time help the firms adjust their behavior to comply with the agreement. And, more importantly, the set of potential competitors in a given auction is quite stable and known to all firms, since transportation costs limit competition to local dairies. For a detailed discussion about the institution, and the collusion scheme and description of the data, we refer the interested reader to Porter and Zona (1999).

[17] Additional tests could be performed depending on the available information. For example, if there is information about similar markets that were not affected by collusion, the bid distributions of non-cartelized players in affected and unaffected markets could be compared to validate if $p = 0$ is reasonable. Also, if the cartel is inefficient, formed by only type-1 firms, and $p = 0$, then the bid distribution of type-1 competitive players $G_1(b)$ must be first-order stochastically dominated by $G_c(b)$.

[18] We thank two anonymous referees for suggesting this additional exercise.

[19] To develop the experiment we need to make an assumption on how non-serious members of the cartel make their offers. For simplicity, we assume that non-serious bidders choose their bids from a uniform distribution between the minimum valuation and the serious bid from the cartel. For the previous exercises this was irrelevant because phantom bids were not used, but in these two exercises there are phantom bids that will be mistakenly considered as serious bids. Logically, this assumption has no theoretical grounds and a different assumption may affect estimated damages.

Table 6
Incorrectly defined cartel - median damages.

	p = 0		p = 1	
	Efficient cartel	Inefficient cartel	Efficient cartel	Inefficient cartel
True	7.8%	17.7%	7.9%	36.5%
Incorrect				
Numerical solution	2.6%	6.3%	10.0%	21.5%
Polynomial solution	1.6%	6.2%	n.a	n.a

4.2. Modeling choices

We closely follow Porter and Zona's (1999) description of the cartel functioning and assume that the cartel operates *only* in those auctions where three copulative conditions are verified: that at least two members of the cartel are among the potential bidders, that one of the members of the cartel is the incumbent of that school district, and that the year of the auction is not 1983 or 1989 (years in which the cartel broke down). To be consistent with the theoretical model where firms draw i.i.d. costs for each auction and the fact that the cartel functioned respecting incumbency, we estimate the model assuming that the cartel is inefficient. Further, we assume that non-serious cartel firms choose not to participate or to submit a phony bid independently of their costs.

To account for the cost advantage of firms that have processing plants close to the school district, we use the Independent Private Value (IPV) paradigm with ex-ante asymmetric bidders. For each auction, we classify as strong or type-1 bidders (i.e., with a cost advantage) those who have a plant within 75 miles from the school district and as weak or type 0-bidders those that are farther away ((Porter and Zona, 1999) estimate an effect of distance on bidding prices of roughly one cent per pint per hundred miles, which is approximately 10% of total incremental cost).[20]

For the empirical exercise we need to depart from the simplified model presented in Section 2 and used in Section 3. In particular, the assumption that entry is exogenous and the number of bidders is known to all players might be problematic. In our data we observe fluctuations in the number of bidders in a particular school district through the years and, as discussed in Porter and Zona (1999), bid preparation costs are relevant. Both elements point to a model with endogenous entry.

We consider a model of selective entry as in Samuelson (1985), where firms first learn their private costs and then decide whether to enter the auction and incur the bid-preparation costs. We further assume that entry decisions are not observed at the moment of submitting the bids (as in Hubbard and Paarsch (2009) and Gentry and Li (2014)), and therefore the bidding function depends only on the number of potential bidders. We define the set of potential bidders for a contract as those firms that participated in the corresponding school district at least once in our sample period.

We denote by w_k the number of potential players of the cartel of type k, by n_k^* the number of potential bidders of type-k, and by $c_{ik} \in [\underline{c}, \overline{c}]$ the private cost of firm i that is drawn i.i.d. from $F_k(c)$. Let $s_k(c_{ik})$ be the bidding equilibrium strategy of non-cartel type k firms, $s_c(c_c)$ the bidding equilibrium strategy of the serious bidder of the cartel, and $G_k(b_{ik})$ and $G_c(b_c)$ their respective distribution functions (for brevity, we are omitting that all bidding functions and the distributions of bids are conditional on the number of potential bidders of each auction).

To perform our estimations we assume $p = 0$. This assumption is reasonable if we consider that cartel activity is usually kept secret by its illegal nature. Moreover, we find that, for those firms competing against the cartel and also bidding in non-cartelized markets, their average bids are not statistically different, justifying the assumption of $p = 0$.

Provided all competitive firms follow a type-symmetric strictly-increasing bidding strategy and they enter the auction if their cost is below some threshold c_k^*, the maximization problem for a competitive firm with cost c_{ki} is

$$\max_{b_{ki} \le \bar{b}} (b_{ki} - c_{ki})(1 - F_k(s_k^{-1}(b_{ki})))^{n_k^*-1}(1 - F_{k'}(s_{k'}^{-1}(b_{ki})))^{n_{k'}^*} - K; \quad k \neq k' \in \{0, 1\} \tag{Pk'}$$

and for the serious cartel bidder of cost c_c is

$$\max_{b_c \le \bar{b}} (b_c - c_c)[1 - F_1(s_1^{-1}(b_c))]^{n_1^*-w_1}[1 - F_0(s_0^{-1}(b_c))]^{n_0^*-w_0} - K \tag{Pc'}$$

where $\bar{b}$ is the maximum price firms can offer and K is the bid preparation cost that, we assume, is identical for all firms.[21] Naturally, the cost thresholds that determine participation are such that a type-k player with cost c_k^* is indifferent between participating or not. Given the assumption that entry is not observed, it is irrelevant whether or not entry costs are random. Moreover, we do not need to estimate c_k^* or K for the purpose of assessing damages.

Following the identification argument of Guerre et al. (2000) and Campo et al. (2003), the first-order conditions that characterize the bidding function for the non-cartelized firms who enter and for the serious member of the cartel can be rewritten as

$$c_{ki} = b_{ki} - \frac{1}{(n_k^*-1)\frac{g_k(s_k^{-1}(b_{ki}))}{1-G_k(s_k^{-1}(b_{ki}))} + n_{k'}^*\frac{g_{k'}(s_{k'}^{-1}(b_{ki}))}{1-G_{k'}(s_{k'}^{-1}(b_{ki}))}}; \quad k \neq k' \in \{0,1\} \tag{(3)$'$}$$

$$c_c = b_c - \frac{1}{(n_1^*-w_1)\frac{g_1(s_1^{-1}(b_c))}{1-G_1(s_1^{-1}(b_c))} + (n_0^*-w_0)\frac{g_0(s_0^{-1}(b_c))}{1-G_0(s_0^{-1}(b_c))}}. \tag{(4)$'$}$$

As described above, given the assumption that $p = 0$, to calculate counterfactual bids for the cartel we have two options: to solve for the system of differential equations that characterize the equilibrium, or to estimate a polynomial relation between recovered costs and observed bids for competitive players and use it to obtain cartel firms' counterfactual bids. The Monte Carlo experiment results suggest that the estimations of the second option are more precise (see Table 1). To account for the fact that auctions differ in the number of potential bidders of each type, the fitted polynomial includes these variables.

Finally, to explicitly consider the endogenous entry problem in the counterfactual scenario, we estimate participation probabilities for our full sample of competitive bidders and use them to determine whether non-serious bidders of the cartel participate. In case they participate, their costs are randomly generated from the recovered distribution of costs. Given our assumption that $p = 0$, entry decisions made by non-cartel firms are identical in the data and counterfactual.

[20] Logically, to classify firms as *strong* or *weak* it is necessary to observe variables that presumably affect costs, as the plant to school distance in milk procurement. In other cases, such as the much-studied highway construction in California, other variables like the workload already committed may be an important determinant of costs.

[21] Note that the maximization problems are stated in terms of the potential number of competitors and the distribution functions of costs $F(\cdot)$, rather than truncated distributions. To see why this is correct, consider as an heuristic proof the case of two symmetric firms that enter the auction iff $c \le c^*$. Assume upon entry firm 2 follows the strategy $s^*(c)$. Then, if $c_1 \le c^*$, firm 1 maximizes $(b_1 - c_1) \times [1 - F(c^*) + F(c^*) \times Pr(s^{*-1}(b_1) \le c_2)]$; but this probability can be rewritten as $[1 - F(c^*) + F(c^*) \times (1 - F(s^{*-1}(b_1))/F(c^*))]$ or simply as $[1 - F(s^{*-1}(b_1))]$.

4.3. The data

We obtained our data from Robert Porter, who shared a database cleaned by Wachs and Kertész (2019). It consists of 7004 observations from 496 different school districts for the period 1980–1991 that the cartel lasted (with the exception of years 1983 and 1989 when the agreement broke down). It includes information of all submitted bids (per-pint prices), identifying the submitting firm and the winning one for every auction. We enlarged the dataset by including for each observation the distance between the submitting firm's closest plant and the school district.

Our data set unfortunately does not include all relevant information analyzed in Porter and Zona (1999). It lacks, for example, information on whether the bidder is a processor or a distributor, information of volumes to be supplied, and also potentially relevant information on whether a district is already on the route of a particular firm. For model tractability, we abstract from auction-specific unobserved auction heterogeneity. The issue could be addressed following the methodologies described in Li and Vuong (1998) and Krasnokutskaya (2011).

The original dataset is reduced once we eliminate one duplicate observation, 174 observations from 127 contracts with missing information, 239 contracts for which there is only one potential bidder, 66 observations from year 1980 for which we cannot identify whether they are affected (because we lack information on incumbency), and an additional 25 contracts (64 observations) in which all potential bidders are members of the cartel.[22] Once we further exclude 176 phony bids by cartel firms, we end up considering 6284 serious offers corresponding to 3339 contracts. The potentially affected auctions – i.e., those where two or more potential bidders are from the cartel, the incumbent is a cartel member, and the year of the auction is not 1983 or 1989 – are 149. In 125 of these auctions a cartel member won.

Table 7 provides some descriptive statistics for all bids, for the winning bids, and for the number of potential bidders. We distinguish between ring firms (2.4% of the serious bids) and type 0 and type 1 competitive firms (26.1% and 71.5% of the serious bids respectively). Average bids are very similar for the three groups (around 0.1 cents per pint), and type 0 and type 1 competitive firms both have similar success rates close to 52%. This figure rises to 84% for the serious offers of cartelized firms.

When we consider all contracts in the sample, there are 4.3 potential bidders on average, while this number rises to 5.6 when considering only those contracts where the cartel participated as a ring (as defined above).

Table 7
Summary statistics.

Variable	Mean	Std. Dev.	Min.	Max.	N
All bids					
bid (c/pint)	0.102	0.011	0.073	0.163	6284
distance (miles)	51.6	47.0	0.0	268.6	6284
Type-0 competitive bids					
bid (c/pint)	0.101	0.012	0.073	0.163	1643
distance (miles)	116.9	37.1	75.2	268.6	1643
Type-1 competitive bids					
bid (c/pint)	0.102	0.011	0.073	0.160	4493
distance (miles)	28.3	20.6	0.0	74.7	4493
Cartel bids					
bid (c/pint)	0.100	0.009	0.081	0.130	148
distance (miles)	35.9	39.3	0.0	147.5	148
Winning bids					
all bids (c/pint)	0.100	0.011	0.073	0.159	3339
type-0 competitive bids (c/pint)	0.099	0.011	0.073	0.155	863
type-1 competitive bids (c/pint)	0.100	0.011	0.073	0.159	2351
cartel bids (c/pint)	0.101	0.009	0.081	0.130	125
Potential number of bidders - All auctions					
all bidders	4.27	1.63	2	9	3339
type-0 bidders	1.24	1.28	0	6	3339
type-1 bidders	3.03	1.53	0	7	3339
cartel bidders	0.11	0.53	0	3	3339
Potential number of bidders - Affected auctions					
all bidders	5.62	1.28	3	8	149
type-0 bidders	1.32	1.52	0	6	149
type-1 bidders	4.30	1.58	0	7	149
cartel bidders	2.46	0.64	0	3	149

Table 8
Estimated damages.

Criterion	# Contracts	(%)	95% Bootstrap CI
Different winning bids	135	6.13	[4.56; 7.23]
Potentially affected auctions	149	5.58	[4.16; 6.59]
Direct damages	125	6.29	[4.33; 7.09]
Indirect damages	10	4.06	[3.87; 12.42]
Indirect damages (as a % of total damages)	24	4.78	[3.60; 20.39]

Note: Confidence intervals based on 1000 bootstrap samples at the auction level with replacement.

4.4. Results

As described in our methodological section, to calculate damages we simply compare observed and counterfactual bids in all auctions. In Table 8 we report the estimated damages as a percentage of actual revenue. We present several measures of damages.

In the first line, we report damages as a percentage of total revenue in the 135 auctions in which there were actually damages; i.e., those auctions for which the winning bids differ in the data and the counterfactual scenario. Estimated average damages are 6.13%, with a 95% confidence interval between 4.56% and 7.23%. In the second line of the table we report damages as a percentage of the amount paid in the 149 potentially affected auctions.

In the third line we report direct damages of 6.29% of the total revenue of the auctions won by the ring. This measure underestimates total economic damages. However, it is interesting to report it, as it might correspond to the damages that can be recouped in court if only directly affected purchasers are allowed to collect damages.

The fourth line reports the damages associated with those auctions in which the winner is not a cartel firm, but in the counterfactual one of the cartel firms would have won. These damages, naturally, are smaller than the direct damages. Finally, the last line reports the fraction that indirect damages represent of total damages, which in this case is almost 5%. Estimated total damages are consistent with Porter and Zona's (1999) findings, they report average damages around 6.5%.

The damages we find, however, can be considered a lower bound on the true damages. As mentioned above, we are excluding from our sample 25 contracts where the only potential bidders were members of the cartel (that is 20% of the total number of contracts the cartel won). Presumably, in these contracts the cartel profited the most.

As a robustness check for our methodology, we present in Table 9 the same set of results reported in Table 8, but performing all estimations using a subsample that includes only those school districts where at least two members of the cartel are potential players and excluding years 1983 and 1989 when the cartel broke down. This robustness check is relevant, as in many antitrust cases it may not be possible to have information on unaffected markets.

[22] For these contracts we cannot model the bidders' behavior without incorporating additional features to the model (e.g., a probability that a too-high offer would be rejected). Our results must therefore be considered as a lower bound to the true damages, since we are excluding those auctions where presumably the cartel was more damaging.

Table 9
Estimated damages (subsample of affected auctions).

Criterion	# Contracts	(%)	95% Bootstrap CI
Different winning bids	136	6.57	[4.2; 9.73]
Potentially affected auctions	149	6.05	[3.86; 8.89]
Direct damages	125	6.68	[3.94; 9.56]
Indirect damages	11	5.39	[4.94; 14.61]
Indirect damages (as a % of total damages)	24	6.67	[3.51; 21.82]

Note: Confidence intervals based on 1000 bootstrap samples at the auction level with replacement.

From the comparison of the two tables it is apparent that the results are quite robust although, as expected, the confidence intervals are wider when estimations are performed with the smaller sample.

5. Conclusion

We propose a conceptually simple structural method to empirically assess the damages and efficiency costs associated with bidding rings in (repeated) first-price auctions. We use well-established empirical methods (Guerre et al., 2000) that allow us to recover, from observed bids, the underlying unobserved valuations of all participating firms. Following (Hubbard et al., 2013), we use the estimated valuations to numerically solve for the counterfactual-competitive scenario. Then it is a matter of simple calculations to find the collusive damages. These damages can be decomposed between those related to auctions won by the cartel (direct damages) and those won by other bidders (indirect or umbrella damages).

The proposed structural method has the advantages of being relatively easy to implement, quite intuitive for courts to follow, and not as data intensive as other damage assessment methodologies (e.g., before-after and diff-in-diff estimations). We illustrate our methodology with a series of Monte Carlo experiments where a subset of strong players collude. We study four cases, depending on whether or not the cartel is efficient (i.e., it picks as the serious bidder the most efficient firm or it picks it randomly), and depending on whether non-ring players are aware or not of the cartel's presence. We show how damages change in the different cases and, more importantly, we can compare the estimated damages with the true ones. The methodology works remarkably well in all cases.

The methodology has a few limitations that have to be kept in mind. As with any structural model, the reliability of the results critically depends on the validity of the model. As shown in the Monte Carlo exercises in Section 3.4, it is important that the researcher has accurate information about the cartel in at least three aspects: (i) whether the cartel is an efficient one or a phases-of-the-moon cartel (and/or if the functioning of the cartel distorts its bidding strategy, in which case the equilibrium conditions must be adjusted), (ii) whether non-ring members were aware of the cartel's existence or not (if this is not clear, then the two polar cases of $p = 0$ and $p = 1$ can be taken as lower and upper bounds on true damages), and (iii) which firms belong to the cartel. As shown in Table 6, this is particularly important because the size of the bias can be large for some cartel configurations, and even the direction of the bias cannot be predicted in other cases.

Another limitation has to do with the use of non-parametric methods when the sample size is small. If this is the case, we can estimate marginal densities using kernel density estimation, but this would be problematic if we need to include several conditioning variables. An alternative would be to consider the use of parametric methods. Conceptually our method still applies when using parametric (or semiparametric) estimation methods to obtain bids densities and distributions instead of non-parametric methods. In this case there would be a risk of misspecification, but the assessment of the damages can be done and the results will be reliable as long as the parametric assumptions are sound.

Finally, it is important to bear in mind that one of the virtues of the proposed methodology is that it does not require data from unaffected markets. Moreover, if for some reason costs are different in affected and unaffected auctions, then only affected auctions should be considered in the estimation process.[23]

We apply our methodology to the well-known case of collusion in the provision of milk for schools in Ohio. The case was analyzed in Porter and Zona (1999), who fully describe the market and show that the behavior of three firms–in terms of participation decisions and the level of bidding–in the Cincinnati area is consistent with the hypothesis of collusion. Their reduced-form estimation of damages is consistent with our findings.

The empirical exercise performed is just one example, and therefore the estimated damages need not be relevant in other contexts. Several of the empirical decisions made are related to the empirical exercise we perform (e.g., the existence of two-types of firms and the assumptions of an inefficient cartel and endogenous entry) and may need to be adapted in different contexts.[24] The logic of the exercise, however, is quite general and widely accepted in antitrust analysis.

Declaration of competing interest

None.

Data availability

Data will be made available on request.

Appendix A

In this Appendix we extend the proposed methodology for intermediate values of $p \in (0, 1)$.[25] The setup of the problem is one where non-cartelized firms know that there might be a cartel (and which firms would conform it and whether the cartel would be efficient or not) and attach a probability p to this event. This probability p is common knowledge, so even if the cartel is not formed, the bidding behavior of the firms that could have formed the cartel is indirectly affected by p, because the strategies of competitive firms is directly affected by p. Logically, the firms that may form the cartel have private information on whether the cartel is formed or not.

We present first the optimization problems and their first order conditions and then discuss the practical limitations of considering intermediate values of p.

Case $p \in (0, 1)$.

We denote by: (i) $s_{jkp}(v|n_0, n_1, W_0, W_1, p)$ the equilibrium strategy of a non-ring type-k firm –$k \in \{0, 1\}$– when a j-cartel is potentially formed ($j \in \{E, I\}$ indicates whether the cartel is efficient or inefficient) with probability p; (ii) $s_{jcp}(v|n_0, n_1, W_0, W_1, p)$ the strategy of the serious bidder of the cartel firm when the cartel is indeed formed; and (iii) $\bar{s}_{jkp}(v|n_0, n_1, W_0, W_1, p)$ the strategy of a type k firm that would have been part of the cartel that was not formed.

The problems for a type k non-ring firm, the serious player of the cartel, and a type k firm that would have been partel of the cartel that it is not formed are respectively:

$$\max_b p(v_i - b)F_k(s^{-1}_{jkp}(b))^{n_k - w_k - 1} F_{k'}(s^{-1}_{jk'p}(b))^{n_{k'} - w_{k'}} F_c(s^{-1}_{jcp}(b)) +$$
$$(1 - p)(v_i - b)F_k(s^{-1}_{jkp}(b))^{n_k - w_k - 1} F_{k'}(s^{-1}_{jk'p}(b))^{n_{k'} - w_{k'}} F_k(\bar{s}^{-1}_{jkp}(b))^{w_k} F_{k'}(\bar{s}^{-1}_{jk'p}(b))^{w_{k'}}$$

[23] We thank an anonymous referee for pointing out this issue.

[24] As explained in Zona (2011), though, it is not enough to have consistency between the modeling assumptions and the empirical decisions; the modeling assumptions also need to be consistent with the economic facts of the case to conform with the Daubert criteria.

[25] We thank two anonymous referees that suggested developing the general case of $p \in (0, 1)$.

$$\max_b (v_c - b) F_1(s_{j1p}^{-1}(b))^{n_1 - w_1} F_0(s_{j0p}^{-1}(b))^{n_0 - w_0}$$
$$\max_b (v_c - b) F_k(s_{jkp}^{-1}(b))^{n_k - w_k} F_{k'}(s_{jk'p}^{-1}(b))^{n_{k'} - w_{k'}} F_k(\bar{s}_{jkp}^{-1}(b))^{w_k - 1} F_{k'}(\bar{s}_{jk'p}^{-1}(b))^{w_{k'}}$$

where $k' \neq k \in \{0, 1\}$ and $j \in \{E, I\}$.

Using the identification argument of Guerre et al. (2000), after some algebraic manipulations the three first order conditions can be written in terms of the bids distributions.[26] For a competitive type k firm the first order condition is:

$$v_i = b + \frac{pG_k(b)^{n_k - w_k - 1} G_{k'}(b)^{n_{k'} - w_{k'}} G_c(b) + (1-p) G_k(b)^{n_k - w_k - 1} G_{k'}(b)^{n_{k'} - w_{k'}} G_{\overline{k}}(b)^{w_k} G_{\overline{k}'}(b)^{w_{k'}}}{p\{A\} + (1-p)\{B\}}$$

where

$$\begin{aligned} A = {} & \left(n_k - w_k - 1\right) G_k(b)^{n_k - w_k - 2} g_k(b) G_{k'}(b) G_c(b) \\ & + G_k(b)^{n_k - w_k - 1} \left(n_{k'} - w_{k'}\right) G_{k'}(b)^{n_{k'} - w_{k'} - 1} g_{k'}(b) G_c(b) \\ & + pG_k(b)^{n_k - w_k - 1} G_{k'}(b)^{n_{k'} - w_{k'}} g_c(b) \end{aligned}$$

and

$$\begin{aligned} B = {} & \left(n_k - w_k - 1\right) G_k(b)^{n_k - w_k - 2} g_k(b) G_{k'}(b)^{n_{k'} - w_{k'}} G_{\overline{k}}(b)^{w_k} G_{\overline{k}'}(b)^{w_{k'}} \\ & + G_k(b)^{n_k - w_k - 1} \left(n_{k'} - w_{k'}\right) G_{k'}(b)^{n_{k'} - w_{k'} - 1} g_{k'}(b) G_{\overline{k}}(b)^{w_k} G_{\overline{k}'}(b)^{w_{k'}} \\ & + G_k(b)^{n_k - w_k - 1} G_{k'}(b)^{n_{k'} - w_{k'}} w_k G_{\overline{k}}(b)^{w_k - 1} g_k(b) G_{\overline{k}'}(b)^{w_{k'}} \\ & + G_k(b)^{n_k - w_k - 1} G_{k'}(b)^{n_{k'} - w_{k'}} G_{\overline{k}}(b)^{w_k} w_{k'} F_{\overline{k}'}(b)^{w_{k'} - 1} g_{\overline{k}'}(b) \end{aligned}$$

For the serious firm of the cartel the first order condition is:

$$v_c = b + \frac{1}{\frac{(n_1 - w_1) g_1(b)}{G_1(s_{j1p}^{-1}(b))} + \frac{(n_0 - w_0) g_0(b)}{G_0(s_{j0p}^{-1}(b))}}$$

And for a type k firm of the cartel, when the cartel is not formed, the first order condition is:

$$v_c = b + \frac{G_k(b)^{n_k - w_k} G_{k'}(b)^{n_{k'} - w_{k'}} G_{\overline{k}}(b)^{w_k - 1} G_{\overline{k}'}(b)^{w_{k'}}}{C + D + E + F}$$

where we use

$$\begin{aligned} C &= \left(n_k - w_k\right) G_k(b)^{n_k - w_k - 1} g_k(b) G_{k'}(b)^{n_{k'} - w_{k'}} G_{\overline{k}}(b)^{w_k - 1} G_{\overline{k}'}(b)^{w_{k'}} \\ D &= G_k(b)^{n_k - w_k} \left(n_{k'} - w_{k'}\right) G_{k'}(b)^{n_{k'} - w_{k'} - 1} g_{k'}(b) G_{\overline{k}}(b)^{w_k - 1} G_{\overline{k}'}(b)^{w_{k'}} \\ E &= G_k(b)^{n_k - w_k} G_{k'}(b)^{n_{k'} - w_{k'}} \left(w_k - 1\right) G_{\overline{k}}(b)^{w_k - 2} g_{\overline{k}}(b) G_{\overline{k}'}(b)^{w_{k'}} \\ H &= G_k(b)^{n_k - w_k} G_{k'}(b)^{n_{k'} - w_{k'}} G_{\overline{k}}(b)^{w_k - 1} w_{k'} G_{\overline{k}'}(b)^{w_{k'} - 1} g_{\overline{k}'}(b) \end{aligned}$$

Appendix B. Discussion

Although the intermediate case of $p \in (0, 1)$ is theoretically sound and the previous conditions could be used to recover valuations, from an empirical perspective there is an obvious challenge. To estimate $G_{\overline{k}}(b)$, $g_{\overline{k}'}(b)$, $G_{\overline{k}'}(b)$ and $g_{\overline{k}'}(b)$, we would need to observe bids of *potential cartel members* in a period (or market) where the cartel was not active. Moreover, that period (or market) should be one in which all firms share a belief p that there might be an active cartel. All in all, the extreme cases of $p = 0$ and $p = 1$ seem to be the more plausible ones, and they can be used to estimate lower and upper bounds on damages.

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[26] We denote by $G_{\overline{k}}(b)$ and $g_{\overline{k}}(b)$ the distribution and density functions of bids of type k firms that would have been cartel members, when the cartel was not formed. I.e., the distribution and density functions of bids that correspond to the strategy $\bar{s}_{jkp}(v)$. Similarly we define $G_{\overline{k}'}(b)$ and $g_{\overline{k}'}(b)$.

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