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Discrete-continuous models of residential energy demand: A comprehensive review☆

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ABSTRACT

This paper reviews forty years of research applying econometric models of discrete-continuous choice to analyze residential demand for energy. The review is primarily from the perspective of economic theory. We examine how well the utility-theoretic models developed in the literature match data that is commonly available on residential energy use, and we highlight the modeling challenges that have arisen through efforts to match theory with data. The literature contains two different formalizations of a corner solution. The first, by Dubin and McFadden (1984) and Hanemann (1984), models an extreme corner solution, in which only one of the discrete choice alternatives is chosen. While those papers share similarities, they also have some differences which have not been noticed or exposited in the literature. Subsequently, a formulation first implemented by Wales and Woodland (1983) and extended by Kim et al. (2002) and Bhat (2008) models a general corner solution, where several but not all of the discrete choice alternatives are chosen. Seventeen papers have employed one or another of these models to analyze residential demand for fuels and/or energy end uses in a variety of countries. We review issues that arose in these applications and identify some alternative model formulations that can be used in future work on residential energy demand.

1. Introduction

Household energy use accounted for about 37% of total energy consumption in the US in 2021.[1] Energy pricing matters greatly for energy and climate policy. Household energy demand functions are crucial for both predicting demand responses to price changes and calculating welfare impacts. But, household demand for energy is not a simple thing to model. Households do not consume energy

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[1] Broken down, residential energy use accounted for 22%, and light-duty vehicle use for about 15% (US Energy Information Agency, 2022).

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directly; they consume it through the use of appliances which are long-lived. Household energy use thus combines decisions on appliance ownership along with appliance utilization. Appliance ownership and utilization satisfy end uses which are the primitives of consumer preferences. One wants to model energy appliance ownership and utilization in a unified manner.[2] For welfare evaluation, that unified analysis should derive from a budget-constrained utility maximization. For the past forty years, this has been accomplished through the application of economic models of discrete-continuous choice (DCC).

This paper reviews the application of DCC models to analyze the residential demand for energy. The review is primarily from the perspective of economic theory. We examine how well the utility-theoretic models developed in the literature match data that is commonly available on residential energy use, and we highlight the modeling challenges that have arisen through efforts to match theory with data.

Appearing simultaneously, the papers by Dubin and McFadden (1984) and Hanemann (1984) were important contributions to the development of the DCC literature.[3] In both papers, a consumer chooses only one of the discrete alternatives available – what is known as an *extreme* corner solution. While the papers shared similarities in their approach to model formulation, how they generated a theoretically consistent model was different, although this has not previously been noticed or explicated. We show here that, in fact, the Dubin-McFadden formulation is not fully consistent with utility maximization.

The alternative to an extreme corner solution is a *general* corner solution, where the consumer may choose more than one – but not all – of the discrete alternatives available. This was formulated in a utility-theoretic manner by Hanemann (1978) and Wales and Woodland (1983), but their models were at first estimable only with very few choice alternatives. Bhat (2005, 2008) formulated a utility-theoretic model of DCC involving a general corner solution that is more practical to estimate and has been used in applications with large choice sets. We explicate here some extensions of Bhat's formulation. An extreme corner solution is estimated directly from the discrete choices and the continuous choice (the demand function). With a general corner solution, the demand function is not used in the estimation; once the model is estimated, the demand function requires significant additional computation. Both types of models have been used to analyze residential energy demand depending on the type of data available.

The utility-theoretic approach to modeling the connection between discrete and continuous choices was once a holy grail. Since then, interest has waned. One factor was the trend against parametric statistical modeling. Another was data limitations. It has become more common to estimate DCC models where the deterministic component of the model is not utility-theoretic and the interconnection of the two choices comes about through the stochastic component of the model rather than the deterministic component.[4] Nevertheless, estimation of DCC models not derived from a utility-theoretic choice runs into difficulties when one seeks to deduce welfare implications from the fitted choice models – for example, to measure the economic cost to consumers when prices rise or supply is rationed, issues that arise in energy markets.

Besides household demand for energy, DCC models have been applied to transportation demand by consumers and producers - for example, choice of vehicle ownership and mileage travelled, or truck route selection and tonnage shipped. Another application is consumer choice, such as brand choice and frequency of consumption. However, in order to focus more concretely on issues that arise, this paper deals exclusively with the literature on residential energy demand, which is comprehensively reviewed.[5]

The paper is organized as follows. Section 2 covers conceptual preliminaries associated with the formulation of a utility-theoretic DCC. Section 3 exposits the formulation of utility-theoretic models for extreme corner solution based on Hanemann (1984). Section 4 introduces the extreme corner solution model developed by Dubin and McFadden (1984) (DM), and applied by them to residential electricity demand. Section 5 explains how Hanemann and DM followed different routes to an extreme corner solution. It reviews issues that have arisen when their models have been applied to residential energy demand, including issues when the available data do not fit assumptions implied by the theoretical model.

Some residential energy demand studies employ data where households use not one energy source (an extreme corner solution) but several energy sources simultaneously (a general corner solution). Section 6 exposits the formulation of utility-theoretic models for general corner solutions from Hanemann (1978) and Wales and Woodland (1983) to Bhat (2008) and beyond. Section 7 reviews studies estimating general corner solutions of residential energy demand, noting issues that arise. Section 8 reviews issues when recovering the demand function from a general corner solution of demand and performing welfare evaluations. Section 9 offers some concluding observations.

2. Utility-theoretic preliminaries

The starting point is a set of commodities whose individual demands the researcher wishes to model – we refer to them as the *inside goods*. Often, they are related in that they provide similar flows of utility services to the consumer. However, because they provide similar flows of services, the consumer chooses to consume only some of them, not all of them. Following Hanemann (1984), we distinguish between two cases: an *extreme corner* solution in which the consumer selects at most one inside goods, and a *general corner* solution in which the consumer selects more than one but not all of them. Before describing these two types of corner solution, we introduce several concepts relevant for formulating a utility model.

[2] Whether a unified model is always justified for residential energy appliances is discussed in the conclusions.

[3] Important precursors include Hausman and Wise (1978) and King (1980).

[4] In addition to formulating a utility-theoretic model of consistent discrete and continuous choice, Dubin and McFadden (1984) also described alternative econometric approaches useful when the stochastic components connect the discrete and continuous choices.

[5] Table 1 lists the papers covered.

The first concept is an *outside good*. The inside goods do not account for all of the consumer's utility: she also consumes other goods. Her consumption of other goods necessarily affects her choice of the inside goods because they compete for her income and also for her preference, since some of them they may be complements or substitutes. Utility theoretic welfare evaluation requires paying attention to the consumer's overall choice portfolio and budget constraint. There are two common ways to do this. One is to assume that all other goods besides the inside goods can be aggregated to form a Hicksian composite commodity.[6] The alternative is to assume that the inside goods are weakly separable so that the consumer's utility function can be written $u = u(\varphi(x_1,\ldots,x_{N-1}),\ldots.)$ and then invoke two stage budgeting where the second stage involves a utility maximization of $\varphi(x_1,\ldots,x_{N-1})$ subject to the constraint that $\sum_{i=1}^{N-1} x_i = \bar{y}_{N-1}$ where $\bar{y}_{N-1}$, total expenditure on the goods in $\varphi(.)$, must be taken (often implausibly) as exogenous. The composite commodity approach is a strong assumption. Problems with weak separability are discussed by LaFrance (1991, 1993). We use the composite commodity approach, taking the consumer's utility function to be $u = u(x_1,\ldots,x_{N-1},x_N)$.[7] We assume that this utility function is quasi-concave, increasing and continuously differentiable. The consumer's choice is:[8].

$$\text{Maximize}_{x_1,.,x_{N-1},x_N}\ u\left(x_1,.,x_{N-1},x_N\right)\text{subjectto} \tag{1}$$

$$\sum_{j=1}^{N} p_j x_j = y \tag{2}$$

$$x_j \geq 0\ j = 1,\ldots,N \tag{3}$$

This maximization generates a set of ordinary demand functions, $x_i = h_j(p_1,.,p_N,y)$, $i = 1,\ldots,N$, and an indirect utility function $u = v(p_1,\ldots,p_N,y)$. To distinguish them from other functions introduced below, we refer to them as the *unconditional demand functions* and the *unconditional indirect utility function*.

Another concept is an *essential* good: this is a commodity such that, because of the structure of the consumer's preferences, no matter how high its price or how low her income, she always chooses to consume some positive quantity of that good. The utility function needs to be formulated such that x_N in the composite commodity approach, or $\varphi(.)$ in the weak separability approach, are an essential good.[9] An example is the Bergson family of utility functions, $u = \sum_{i=1}^{N} a_i x_i^c \quad 0 \leq c < 1$, which rules out selecting any $x_i = 0$.

While the consumption of an essential good is never zero, the quantity consumed can come *arbitrarily close* to zero. It seems desirable to rule this out for the composite commodity/outside good. Instead, one might want to impose a *lower threshold* that bounds consumption of this good away from zero. The threshold consumption amount (i) is estimated from the data, and (ii) can be made a function of socio-demographic or other variables. To do this we need to employ the procedure known as *translation*, which works as follows.[10] Let $u^*(x_1,\ldots,x_N)$ be some given utility function with standard properties that generates ordinary demand functions $x_i = h_i^*(p_1,.,p_N,y)$ and an indirect utility function $u = v^*(p_1,\ldots,p_N,y)$. Let a utility function $u(.)$ be created as a translation of $u^*(.)$:[11].

$$u(x_1,\ldots,x_N) = u^*(x_1 - \gamma_1,\ldots,x_N - \gamma_N) \tag{4}$$

Then, the demand functions and indirect utility function associated with $u(.)$ take the form:

$$\begin{aligned} x_i &= h_i\left(p_1,.,p_N,y\right) = \gamma_I + h_i^*\left(p_1,.,p_N,y - \sum_{j=1}^{N}\gamma_j p_j\right) \\ u &= v\left(p_1,.,p_N,y\right) = v^*\left(p_1,.,p_N,y - \sum_{j=1}^{N}\gamma_j p_j\right) \end{aligned} \tag{5}$$

The formulas in (5) hold generally, regardless of whether the γ_i are positive or negative. However, x_i is only what we will call a *subsistence good* if γ_i is *positive*. In that case, the quantity of x_i consumed is always at least γ_i.[12] What happens when some of the γ_i are negative – which makes them *non-essential goods* – is discussed in Section 6.

In the standard utility formulation, the consumer cares only for quantities of the different goods. The utility formulation developed by Lancaster (1966) and extended by Maler (1971) recognizes that the consumer may also care about some other items which she does not get to choose but which do affect her utility. An example is *attributes* (characteristics, quality variables) of the available

[6] In that case, consumption of the inside goods is denoted $(x_1,\ldots,x_{N-1})$ while outside good consumption is x_N.
[7] It would be natural also to take x_N as the numeraire and set its price, p_N, to unity.
[8] We dispense with using a subscript for the individual consumer.
[9] In the weak separability approach, $\varphi(.)$ would not be an essential good if none of $(x_1,\ldots,x_{N-1})$ were being consumed.
[10] Pollak (1971).
[11] Note that $u(.)$ inherits the standard properties of a utility function.
[12] If all the γ_i are positive and income is greater than $\sum_j \gamma_j p_j$, one can think of the consumer as first purchasing the threshold quantities of the goods $(\gamma_1,\ldots,\gamma_N)$ and then dividing up the residual income, $y - \sum_j \gamma_j p_j$.

commodities.[13] We represent those by $q = (q_1,\ldots,q_{N-1})$, where q_j is a vector of attributes of the j^{th} commodity. We also include attributes of the household and its home – for example, household size and age composition, home age and size, etc.; these attributes, represented by a vector z, may determine the importance to the household of particular commodity/fuel attributes. The utility function is then written $u(x,q,z)$; this formulation replaces $u(x)$ in (1). The resulting ordinary demand functions are denoted $x_i = h_j(p_1,.,p_N,q,z,y)$, $i = 1,\ldots,N$ and the indirect utility function is $u = v(p_1,\ldots,p_N,q,z,y)$. It may be reasonable to assume that, for every element q_{jk} in q_j,

$$x_j = 0 \quad \rightarrow \quad \frac{\partial u(x,q,z)}{\partial q_{jk}} = 0 \tag{6}$$

which implies that attributes of good j do not matter if that good is not actually consumed, a property called *weak complementarity* by Maler (1974).[14] This becomes a restriction on $u(x,q,z)$.

The last concept is the notion of a *random utility model.* This arises when one assumes that, although a consumer's utility function is deterministic for him, it contains some components that are unobservable to the econometric investigator and are treated by the investigator as random variables. The unobservables could be characteristics of the consumer and/or attributes of the commodities (i.e., elements of z or q). This combines two ideas: the idea of a variation in tastes among individuals in a population and the idea of unobserved variables in econometric models. For now, we introduce two sets of random variables, ε which is associated with unobserved attributes of the alternatives and η which is associated with unobserved characteristics of the consumer. The utility function in (1) now becomes $u(x,q,z;\varepsilon,\eta)$, the unconditional ordinary demand functions become $x_i = h_i(p_1,.,p_N,q,z,y;\varepsilon,\eta)$ and the unconditional indirect utility is $u = v(p_1,.,p_N,q,z,y;\varepsilon,\eta)$.[15]

3. Utility-theoretic models of an extreme corner solution

An extreme corner solution can arise under two circumstances. First, there may be some logical or physical reason why the x_j's are *mutually exclusive* in consumption, so that it is not possible to consume more than one of $(x_1,\ldots,x_{N-1})$. This imposes no particular restriction on the form of the utility function. Instead, it adds to (3) a set of constraints of the form.

$$x_i x_j = 0,\ all\ i \ddagger i,\ i,j = 1,\ldots,N\text{-}1 \tag{7}$$

Otherwise, the goods must be seen by the consumer as *perfect substitutes*, so that it is only worth selecting one good at any time.[16] A utility function with this property is

$$u = u*\left(\sum_{j=1}^{N-1}\psi_j x_j, x_N\right) \tag{8}$$

where $\psi_j > 0$ and $u^*(\bullet,\bullet)$ is a bivariate utility function which Hanemann (1984) chooses so that the first good is essential.[17] The ψ_j terms are a measure of the overall quality or attractiveness of each x_j and it is natural to write them as functions of q_j.

$$\psi_j = \psi_j\,(q_j, z,\ \varepsilon_j) \tag{9}$$

where ε_j, a scalar, accounts for randomness (unobservable variation) in the attractiveness of x_j. Thus, the utility function becomes

$$u = u\left(x_1,.,x_{N-1},x_N,q,z;\varepsilon,\eta\right) = u*\left(\sum_{j=1}^{N-1}\psi_j\left(q_j,z,\varepsilon_j\right)x_j, x_N;\eta\right) \tag{10}$$

This formulation interacts quantity, x_j, and quality, ψ_j, multiplicatively, in a manner known as *scaling* – the number of units consumed is scaled by their quality.[18] By construction, (10) satisfies weak complementarity, (6). Because of the distinctive formulation of the utility function in (10), and because it generates an extreme corner solution for $x_1,\ldots,x_{N-1}$, the unconditional demand functions and the unconditional indirect utility function resulting from the maximization of (10) subject to (2) and (3) take distinctive forms.

Suppose that the consumer had already decided to consume discrete alternative j. For example, in the context of home heating fuel choice, with the alternative fuels being natural gas ($j = 1$), electricity ($j = 2$) and fuel oil ($j = 3$), suppose the consumer has chosen

[13] The consumer chooses the attributes *indirectly* through her choice of which commodities to consume and not consume. But, she has to take as given the attributes associated with each particular commodity.

[14] In the context of valuation of environmental amenities, this excludes the possibility of an existence (non-use) value for q_i. Whether weak complementary is desirable – or useful in the context of energy demand – is discussed further below.

[15] For the individual consumer ε is a set of fixed constants, but for the investigator it is a random vector with some joint density function $f_\varepsilon(\varepsilon)$. Similarly, η is a random vector with some joint density $f_\eta(\eta)$.

[16] An example would be if the consumer saw space heating with electricity and gas as perfect substitutes.

[17] Let $h_I^*(p_I,p_{II},y)$and $h_{II}^*(p_I,p_{II},y)$denote the ordinary demand functions when $u^*(\bullet,\bullet)$ is maximized subject to a budget constraint and non-negativity conditions, and $v*(p_I,p_{II},y)$ denotes the indirect utility function. Hanemann (1984) chooses a bivariate indirect utility – (3.16a) - such that $h_I^*(p_I,p_{II},y)$is a semi-log demand function, and also considers the case yielding a log-log demand – (3.21a).

[18] This is also known as repackaging (Fisher and Shell, 1971). Another formulation analyzed by Hanemann, eq. 3.25) (1984) is Willig's (1978) *cross-product repackaging*, whereu $= u*(\sum_{j=1}^{N-1}x_j, x_N + \sum_{j=1}^{N-1}\psi_j x_j;\eta)$; this too satisfies weak complementarity.

natural gas. Conditional on that fuel choice, the consumer maximizes (8) or (10) subject to (2) and, instead of (3), subject to:

$$x_j > 0,\ x_N > 0,\ and\ x_i = 0\ all\ i \ddagger j\ and\ i \ddagger N \tag{11}$$

Combining (11) with (10) simplifies the utility function to

$$u = \overline{u}_j(x_j, x_N, q_j, z; \varepsilon_j, \eta) = u * (\psi_j(q_j, z, \varepsilon_j) x_j, x_N; \eta) \tag{12}$$

combining (11) with (2) simplifies the budget constraint to:

$$p_j x_j + p_N x_N = y \tag{13}$$

We refer to $\overline{u}_j()$ in (12) as the *conditional direct utility function*, conditional on the choice of discrete alternative j; and we call the maximization of (12) subject to (13) the consumer's *conditional* utility maximization. The resulting demand functions,

$$x_j = \overline{h}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) \tag{14a}$$

and

$$x_N = \overline{h}_N(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) \tag{14b}$$

are known as the *conditional demand functions,* and the resulting indirect utility function,

$$u = \overline{v}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) \tag{14c}$$

is the *conditional indirect utility function,* conditional on the choice of the j^{th} discrete alternative.

The combination of perfect substitutability among the choice alternatives in (8) and (10) and attribute scaling leads these conditional functions to take the following distinctive form[19] [20].

$$x_j = \overline{h}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = h_I^*(p_j/\psi_j, p_N, y; \eta)/\psi_j \tag{15a}$$

$$x_N = \overline{h}_N(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = h_{II}^*(p_j/\psi_j, p_N, y; \eta) \tag{15b}$$

$$u = \overline{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = v * (p_j/\psi_j, p_N, y; \eta)\ . \tag{15c}$$

Note that the conditional functions satisfy Roy's identity:

$$\begin{aligned} x_j = \overline{h}_j\left(p_j, p_N, q_j, z, y; \varepsilon_j, \eta\right) &= -\frac{\partial\overline{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta)/\partial p_j}{\partial\overline{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta)/\partial y} \\ x_N = \overline{h}_N\left(p_j, p_N, q_j, z, y; \varepsilon_j, \eta\right) &= -\frac{\partial\overline{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta)/\partial p_N}{\partial\overline{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta)/\partial y} \end{aligned} \tag{16}$$

There is a set of conditional functions (14a,b,c) conditional on the choice of each discrete alternative $j = 1,\ldots,N\text{-}1$. Two distinctive features of the conditional demand functions in (15a,b) should be noted. Using the example of choosing electricity versus gas as the fuel for home heating, these are:

P1. If the consumer has chosen to use electricity for home heating, conditional on that choice the quantity of electricity demanded for home heating depends on the price of electricity but *not* on the prices of gas or heating oil.

P2. Because of the weak complementarity property, if the consumer has chosen to use electricity for home heating, the quantity of electricity demanded for home heating depends on attributes associated with electricity but not on attributes associated with gas or heating oil.[21]

The discrete choice of which fuel to use for home heating is represented by a set of N-1 binary valued indicators, $\delta_j = 1$ if $x_j > 0$ and $\delta_j = 0$ if $x_j = 0$. The discrete choice can be formulated in terms of the conditional indirect utility functions – the consumer chooses whichever discrete alternative has the highest conditional utility:

$$\delta_j(p_1, \ldots, p_N, q_1, \ldots, q_{N-1}, z, y; \varepsilon_1, \ldots, \varepsilon_{N-1}, \eta) = \begin{cases} 1 \text{if} \overline{v}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) \geq \overline{v}_i(p_i, p_N, q_i, z, y; \varepsilon_i, \eta);\ i = 1, \ldots, N-1 \\ 0 \text{otherwise} \end{cases} \tag{17}$$

[19] Muellbauer (1974) demonstrates how the indirect utility function reflects scaling in the direct utility function.

[20] With cross-product repackaging, the analog of (15c) is $u = v * (p_j - \psi_j, p_N, y; \eta)$(Hanemann, 1984, eq. 3.26c).

[21] These propositions apply to the *continuous* choice. The discrete choice of whether to use electricity for home heating depends on the prices of and attributes of both electricity and gas.

Thus, the consumer chooses electricity for home heating if her conditional utility associated with using electricity exceeds that associated with using gas or heating oil. Substituting (15c) into (17) yields:

$$\delta_j(p_1,\dots,p_N,q_1,\dots,q_{N-1},z,y;\varepsilon_1,\dots,\varepsilon_{N-1},\eta) = \begin{cases} 1\text{if}\,\dfrac{p_j}{\psi_j} \le \dfrac{p_i}{\psi_i}; i = 1, N-1 \\ \\ 0\text{otherwise} \end{cases} \tag{18}$$

Hence, the consumer chooses the alternative with the lowest quality adjusted price (p_j/ψ_j).[22] The discrete choice indicators combine with the conditional demand functions to generate the unconditional demand functions[23].

$$x_i = h_i(p_1,.,p_N,q_1,.,q_{N-1},z,y;\varepsilon_1,.,\varepsilon_{N-1},\eta) = \delta_i(p_1,.,p_N,q_1,.,q_{N-1},z,y;\varepsilon_1,.,\varepsilon_{N-1},\eta)\times \overline{h}_i(p_i,p_N,q_i,z,y;\varepsilon_i.\eta) \qquad i = 1,.,N-1. \tag{19}$$

Similarly, the unconditional indirect utility function is given by

$$\begin{aligned} v\left(p_1,.,p_N,q_1,.,q_{N-1},z,y;\varepsilon_1,.,\varepsilon_{N-1},\eta\right) &= \sum_{i=1}^{N-1}\delta_i(p_1,.,p_N,q_1,.,q_{N-1},z,y;\varepsilon_1,.,\varepsilon_{N-1},\eta)\times \\ &\qquad \overline{v}_i(p_i,p_N,q_i,z,y;\varepsilon_i,\eta) \\ &= \max_i[\overline{v}_i(p_i,p_N,q_i,z,y;\varepsilon_i,\eta)]. \end{aligned} \tag{20}$$

The behavioral implication of (19) is:

P3. In an extreme corner solution there are three own-price elasticities: (a) the own-price elasticity of demand for gas for home heating conditional on using gas, $\overline{h}_i()$; (b) the own-price elasticity of choosing natural gas for home heating instead of choosing electricity (δ_j); and (c) the own-price elasticity of the unconditional demand for gas (x_i) for home heating. With cross-price elasticity, there is the cross price elasticity of choosing gas for home heating with respect to the price of electricity. Conditional on choosing gas, the conditional demand for gas does not depend on the price of electricity. The unconditional demand for gas for home heating depends on the price of electricity only via the discrete choice decision.

The quantities conditionally demanded (14a,b), the discrete choice indicators (17), the quantities unconditionally demanded (19), and the utility attained by the consumer through her extreme corner solution choice (20), are all random variables whose distributions derive from the distributions of ε and η. The practical challenge is to identify specifications that can yield tractable expressions. Without going into detail, we summarize the approach generally used in the literature. To simplify things we drop the random variables associated with characteristics of the consumer (η) while focusing on unobserved attributes of the choice alternatives (ε).

The common formulation for the attractiveness parameters in (8) is:

$$\psi_j\left(q_j,z,\varepsilon_j\right) = \exp\left(\alpha_j + \sum_k \phi_{kj}q_{kj} + \sum_l \tau_{lj}z_l + \varepsilon_j\right) \equiv \exp\left(\lambda_j + \varepsilon_j\right) \tag{21}$$

The discrete choice indicators, (18), are binomial random variables with means, $E\{\delta_j\} = \pi_j$, given by

$$\pi_j = \Pr\{p_j/\psi_j(q_j,z,\varepsilon_j) \le p_i/\psi_i(q_i,z,\varepsilon_i) \quad i = 1,.,N-1\} \tag{22}$$

Next, we introduce the sets $A_j \equiv \{\varepsilon | p_j/\psi_j(q_j,z,\varepsilon_j) \le p_i/\psi_i(p_i,z,\varepsilon_i) \quad i = 1,.,N-1\}$. From f_ε one can construct $f_{\varepsilon|\varepsilon_j \in A_j}$ the conditional joint density of $\{\varepsilon_1,\dots,\varepsilon_{N-1}\}$ given that alternative j is selected. If the ε_j's are independently and identically distributed (iid) according to the Extreme Value distribution, $EV(\mu,0)$, $\mu > 0$, this takes a convenient form

$$f_{\varepsilon_j|\varepsilon\in A_j}(\varepsilon_j) = \beta_j e^{-\varepsilon_j/\mu}\exp\left[-\beta_j e^{-\varepsilon_j/\mu}\right]\Big/\mu \tag{23a}$$

with

$$\beta_j \equiv e^{-\lambda_j/\mu}\sum_{i=1}^{N-1} e^{\lambda_i/\mu}. \tag{23b}$$

Then, the probability density of $\overline{x}_j, f_{x_j|\varepsilon\in A_j} = \Pr\{\overline{x}_j = x | \varepsilon_j \in A_j\}$, can be obtained from $f_{\varepsilon|\varepsilon_j\in A_j}$ by a change of variable based on (15a). From (19), the probability density of the quantities unconditionally demanded is given by

$$\Pr\{h_i(p_1,.,p_N,q_1,.,q_{N-1},z,y;\varepsilon) = x\} = \Pr\{\overline{h}_i(p_i,p_N,q_i,z,y;\varepsilon_i) = x | \varepsilon \in A_i\}\Pr\{\varepsilon \in A_i\} \tag{24}$$

The unconditional demand function provides the basis for the likelihood function used to estimate the model. Given observations on H individuals (households) and let j^* be the index of the discrete alternative chosen by the h^{th} household and $x^*_{j^*h}$ the quantity he

[22] This holds because $v^*()$ in (15c) is decreasing in prices. With cross-product repackaging, the consumer chooses the alternative for which $(p_j - \psi_j)$ is lowest (Hanemann, 1984, eq. 3.26c).

[23] Econometrically, these demand equations may be thought of as a switching regression, as analyzed by Goldfeld and Quandt (1973) and others.

consumes of that discrete alternative, the likelihood function for the h^{th} observation is:

$$L_h = f_{x_{j^*}/\varepsilon \in A_{j^*}}\left(x^*_{j^*h}\right)\pi_{j^*h} \tag{25}$$

Similar to (24), from (20) the probability density of the utility attained by the consumer following utility maximization is given by[24].

$$\Pr\{v(p_1,.,p_N,q_1,.,q_{N-1},z,y;\varepsilon) = u\} = \Pr\{\overline{v}_i(p_i,p_N,q_i,z,y;\varepsilon_i) = u|\varepsilon \in A_i\}\Pr\{\varepsilon \in A_i\} \tag{26}$$

This serves as the basis for calculating welfare measures such as the compensating variation for a change in the product quality attributes, q's. The compensating variation itself is a random variable by virtue of the random terms in (26), and therefore something such as the expected value of the compensating variation would have to be calculated.

What we have summarized here follows Hanemann's (1984) analysis of an extreme corner solution based on the assumption that the choice alternatives are perfect substitutes, as in (8). The conditional indirect utility functions in (14c) and (15c) are the key building blocks. Once those have been specified, the continuous choices follow from (16) and the discrete choices from (17).

4. The Dubin-McFadden model

How does what has been described differ from the Dubin and McFadden's (1984) – DM – model of a DCC? The choice they wished to model was different; the data they had were different; and the pathway by which they formulated their model was different. The end result, however, was similar.

Hanemann (1984) developed his choice model in order to analyze the visitation of recreation sites – which sites were chosen and how often were they visited – seen as the analog of a consumer's choice among differentiated commodities. DM wanted to model households' discrete choice of electricity versus gas for space and water heating. They also wanted to estimate electricity demand, on which there was already an extensive literature.[25] However, existing econometric estimates of residential energy demand had assumed statistical independence between the error term for fuel choice and the error term in the energy demand equation. DM believed that this was wrong due to correlation among unobservables influencing each choice. Such correlation would bias existing estimates of electricity demand. DM also wanted to model both choices in a manner consistent with the hypothesis of utility maximization, so that the two choices were modeled as though made simultaneously by the homeowner.

DM had detailed information on energy use by single-family homes plus information on appliance ownership and other details of the house and the occupants. DM pre-selected the subset of homes where space and water heating were both either electric or gas.[26] Unlike Hanemann, they conceptualized the choice alternatives of electric and gas heating as mutually exclusive in the manner of (7), rather than as perfect substitutes in the manner of (8). For DM, that eliminated the need to specify an underlying direct utility function. Instead, they started by specifying two *conditional* indirect utility functions, one conditional on using electricity for heating both space and water ("heating"), the other conditional on using gas.[27] They used the conditional indirect utility functions to set up the discrete choice between using electricity versus gas for heating in the same manner as in (17)[28] and then applied Roy's identity to derive demand functions for electricity/gas conditional on the choice of electricity/gas for heating in the same way as (16).[29]

A minor difference is that DM used a functional form for the conditional indirect utility functions that led to a linear conditional demand function for electricity, while Hanemann focused on a functional form that led to linear-in-logs conditional demand functions.[30] Another small difference is the method of estimation. Hanemann proposed estimating the model parameters in one step by maximizing the likelihood functions for the unconditional demand functions, based on terms like (25). DM estimated their model in two steps. The first step was maximum likelihood estimation of the discrete choice, based on the probabilities like (22). The second step was a regression based on the conditional demand functions like $\overline{h}_j()$ in (16).

A more substantive issue is the price arguments in the conditional indirect utility functions that served as the starting point for the modeling analyses. DM postulated that, conditional on using electricity for heating, the indirect utility function depends not only on the price of electricity but *also on the price of gas*. However, no other end use listed by DM used gas apart from heating space and heating

[24] Hanemann (1984) develops these formulas for his base model of scaling combined with a bivariate utility function generating a semi-log demand $h^*_I(.)$, as well as for cross-product repackaging and other bivariate utilities.
[25] Two-part tariffs generally do not arise in the consumer context considered by Hanemann (1984). They commonly arise in the electricity demand context of Dubin and McFadden (1984), but were explicitly ignored by them with "electricity treated treated as a commodity available in any quantity at a fixed marginal (=average) price." However, both those models could be applied to discrete-continuous choice of commodities sold under a two-part tariff at the cost of complicating and extending the analysis.
[26] Their ownership data also included other appliances such as central or room air conditioning, freezers, electric ranges, dishwashers, clothes dryers and color TV.
[27] They started with a general formulation of the conditional indirect utility functions (DM, Eq. (1)) which was then refined in a series of special cases (DM, Eqs. (6), (10) and (14)), culminating in a final form (DM, Eq. (18)).
[28] DM, Eqs. (4), (9) and (16).
[29] Their application of Roy's identity appears in DM Eqs. (2) and (3). Their conditional demand functions for electricity and gas are, successively, DM Eqs. (5)–(6) and (11)-(12). Their final demand equation for electricity is DM (15).
[30] In his main formulation (3.16a), ln(x_j) was a linear function of ln(pj) and ln (ψj) but linear in income (y).

water.[31] Therefore, conditional on using electricity for heating, the price of gas should have had no effect on the utility attained by the household. Because the price of gas should not have been an argument in the household's indirect utility function conditional on using electricity, it should also not have been an argument in the household's conditional demand function for electricity, as asserted in Proposition P1 above.

The most substantive difference concerns a mismatch between the utility-theoretic formulation and the data actually available for estimating the continuous choice, as explained below.

But first, we note DM's important econometric contribution to DCC models, which involved the regression in the two-step estimation of their DCC model. Like Hanemann (1984), DM's conditional indirect utility functions had additive error terms, ε_i, with independent extreme value distributions. There were also additive error terms in DM's conditional demand equation, η_i, which DM allowed to be correlated with the ε_i. Consequently the expected quantity of electricity demanded for heating, conditional on electricity being chosen for heating, contained a conditional expectation term, $E\{\eta_i|i \text{ chosen}\}$, that was non-zero (a "Heckman" selection term). DM developed correction formulas for the expected value of the conditional demands.[32] These yielded coefficient estimates markedly different from those obtained by conventional OLS, which supported DM's conjecture that unobserved factors influencing fuel choice were not independent of unobserved factors influencing intensity of fuel use.

5. Theory meets data – extreme corner solution models

Many papers estimating residential or commercial energy demand have appeared that follow DM or Hanemann.[33] Some refined the stochastic specification of the model, others abandoned the utility-theoretic formulation. A problem for some papers that maintained the utility-theoretic formulation was a mismatch between the model and the data – a mismatch not typically found with DCC models for consumer choices of differentiated goods. Mismatch could occur in two ways.

The first way, a problem for DM, is as follows. The discrete choice is a choice of which fuel to use for an appliance or an end use. However, the data on household fuel use does not break usage down by appliance or by end use; it reports total fuel usage across *all* appliances and end uses. To estimate total household usage of a fuel – say, electricity – one needs to add the conditional demand for electricity by households that choose electricity for heating to the conditional demand for electricity (for non-heating uses) by households that choose gas for heating, weighted by dummy variables for whether electricity or gas was chosen for heating, like (19).[34]

DM handled this by assuming that electricity used for purposes other than heating is a fixed quantity for the household. DM calculated that quantity based on physical house characteristics and appliance ownership combined with engineering data on appliance energy utilization rates.[35] This seems inconsistent with treating electricity usage for heating as determined by household preferences. Why is the amount of electricity used to cool a home physically predetermined, but not the amount used to heat a home?

Given that DM intended the conditional indirect utility functions to represent the household's utility from *all* of its uses of electricity, conditional on heating with electricity or gas, the *direct* utility function underlying these conditional indirect utility functions must have non-heating uses of electricity as arguments.[36] Assume the non-heating uses can be bundled into a single composite commodity, "other use of electricity", denoted x_{oe}. Let x_e denote electricity usage for heating space and water, and x_g gas usage for heating space and water. Including an outside commodity, x_N, the unconditional utility function would be:[37].

$$u = u\left(x_e, x_g, x_{oe}, x_N; \omega\right) \tag{27}$$

Let X_e denote the household's total usage of electricity consisting of.

$$X_e = x_e + x_{oe} \tag{28}$$

Let $h_e(\bullet)$ denote the household's unconditional demand for electricity for heating, $h_{oe}(\bullet)$ its unconditional demand for electricity in other end uses, $H_e(\bullet)$ its unconditional total demand for electricity, and $h_g(\bullet)$ its unconditional demand for gas for heating. Similarly, let $\overline{h}_e()$ denote the household's demand for electricity for heating, conditional on its choosing electricity for heating; let $\overline{H}_e()$ denote its total demand for electricity in all uses, conditional on choosing electricity for heating. DM assume that[38].

$$\overline{H}_e(\cdot) = \overline{h}_e(\cdot) + \overline{x}_{oe} \tag{29}$$

[31] DM Table I.

[32] The formulas came to be used widely in the literature on switching regressions; see, for example, Schmertmann (1994) and Bourguignon et al. (2007).

[33] These papers are listed in Table 1. Papers in the table are classified by the type of corner solution modeled, what was being chosen in the discrete choice, what was being chosen in the continuous choice, the estimation method, whether the model was utility-theoretic, and whether it contained an outside good.

[34] The discrete choice makes the choice indicators endogenous, which biases the regression step in two-step estimation. DM showed that consistent parameter estimates can be obtained using the predicted choice probabilities as either instruments or replacements for the discrete choice dummies.

[35] This calculation appears on DM p. 361. It accounted for electricity used in the following non-heating end uses: central and room air conditioners, freezers, electric ranges, dishwashers, clothes dryers and color TV.

[36] All the non-heating end uses of energy listed by DM in their data were fueled by electricity.

[37] Here the random terms are combined into $\omega = (\varepsilon, \eta)$with q and z temporarily suppressed.

[38] This corresponds to DM Eq. (30).

Presumably, the conditional demands come about when the household solves

$$\begin{array}{ll} \text{maximize}_{x_e, x_N} & u\left(x_e, 0, \overline{x}_{oe}, x_N; \omega\right) \\ \text{subjectto} & \mathrm{p}_e\left(x_e + \overline{x}_{oe}\right) + p_N x_N \le y \end{array} \tag{30}$$

with non-negativity restrictions on x_e and x_N. The value of x_e chosen in the maximization, (30), satisfies (29) if and only if the unconditional direct utility function in (30) takes the particular form: [39].

$$u = u(x_e + \overline{x}_{oe}, 0, x_N; \omega) \tag{31}$$

This formulation implies that the household views electricity for non-heating end uses as a perfect substitute for electricity used for heating, which seems implausible. While the electrons used for watching TV and for home heating are perfect substitutes, the utility services generated by those two activities are unlikely to be perfect substitutes.

If the utility function does *not* have the form of (31) and one abandons the notion of a fixed consumption of electricity for non-heating purposes, the consumer's conditional utility maximization is then

$$\begin{array}{ll} \text{maximize}_{x_e, x_{eo}, x_N} & u\left(x_e, 0, x_{oe}, x_N; \omega\right) \\ \text{subjectto} & \mathrm{p}_e\left(x_e + x_{oe}\right) + p_N x_N \le y \end{array} \tag{32}$$

with non-negativity restrictions on x_e, x_{eo} and x_N. This gives rise to conditional demand functions for x_e and x_{oe} (and x_N) and a conditional indirect utility function which take the form:

$$x_e = \overline{h}_e(p_e, p_N, y; \omega) \tag{33a}$$

$$x_{oe} = \overline{h}_{oe}(p_e, p_N, y; \omega) \tag{33b}$$

$$u = \overline{v}_e(p_e, p_N, y; \omega) \tag{33c}$$

Applying Roy's Identity, from (33c) one obtains[40].

$$-\frac{\partial \overline{v}(p_e, p_N, y; \omega)/\partial p_e}{\partial \overline{v}(p_e, p_N, y; \omega)/\partial y} = \overline{h}_e\left(p_e, p_N, y; \omega\right) + \overline{h}_{oe}\left(p_e, p_N, y; \omega\right) = \overline{H}_e\left(p_e, p_N, y; \omega\right) \tag{34}$$

which is the household's total demand for electricity conditional on using electricity for heating, as needed to match the data available.

To summarize, the data mismatch arose when the discrete choice was a choice of fuel for a particular end use but the continuous choice data covered all end uses. The solution is to start with a direct utility function, like (27), covering all end uses; DM's model lacked such a utility function. Given (27), one sets up the utility maximization, (32), to obtain a correct specification of the conditional indirect utility function, (33c), which generates a demand function for total electricity demand, (34). DM's total residential demand (their Eq. 29) does not comport with (34).

The mismatch is highlighted in contrasting studies by Bernard et al. (1996) and Liao and Chang (2002). Both estimated the discrete choice of fuel for residential space and water heating.[41] Liao and Chang had data on fuel consumption by end use and estimated separate demand equations for each fuel in each end use (space heating, water heating). Bernard et al. (1996), like DM, only had data on fuel consumption across *all* end uses; they estimated total residential demand for electricity.

Both papers abandon DM's utility-theoretic link between discrete and continuous choices. For the discrete choice, they postulate what Bernard et al. called a linear approximation to DM's conditional indirect utility functions

$$u_i = X_i \beta_i + \varepsilon_i \tag{35}$$

where $\mathrm{u_i}$ is the household's indirect utility conditional on the choice of i^{th} fuel alternative, ε_i is an alternative specific random disturbance, and X_i is a vector of explanatory variables (some being alternative specific). The treatment of prices in (35) was different. DM had included the annual operating cost of the heating portfolio, the annualized capital cost, and fuel prices, carrying the same coefficients across choice alternatives. Bernard et al. included the annual operating cost and the annualized capital cost, carrying the same coefficients across choice alternatives.[42] Liao and Chang (2002) used the price of electricity as their only price variable, with

[39] To understand the equivalence of (29) and (31), note that (29) casts X_e as a translation of x_e. Then, (31) follows from (4) and (5).

[40] This follows because $\overline{v}(p_e, p_N, y; \omega)$is the value function of the constrained optimization problem in (32); application of the Envelope Theorem then yields (34).

[41] Liao and Chang estimated separate discrete choices for heating space and heating water; fuel alternatives were electricity, gas and oil for space heating, and electricity, gas, and other for water heating. Bernard et al. focused on households that used electricity or natural gas for heating space and/or water. Combining space and water heating, there were nine discrete choice alternatives: gas/gas; gas/electricity; electricity & oil/oil; electricity & oil/electricity; oil/oil; oil/electricity; electricity/electricity; wood/electricity; and wood & electricity/electricity.

[42] For Bernard et al. (1996), fuel prices entered through the operating cost of the fuel alternative. Note that the ratio of operating to capital costs varied by choice alternative.

coefficients differing across choice alternatives.[43]

In both papers, the continuous choice equation (the fuel demand function) followed DM's linear formulation:

$$x = Z\gamma + \eta \tag{36}$$

With (35) and (36), there could be correlation between ε_i and η but no utility-theoretic linkage. Liao and Chang (2002) estimated a set of five demand equations, (36), for each of three fuels used in space heating and two fuels used in heating water. These were conditional demand functions conditional on the fuel being chosen for that end use. Bernard et al. – like DM – estimated one demand equation, for electricity. DM's version of (36)[44] was the unconditional demand function for electricity used in heating.[45] For Bernard et al. (1996), it was the unconditional demand function for electricity across all uses.[46]

Since Bernard et al. (1996), the literature has largely abandoned utility-theoretic formulations for modeling extreme corner solutions and switched to a set-up like (35)-(36), with ad hoc formulations of the right-hand sides.[47] Any linkage between discrete and continuous choices arises purely through correlation among unobservables (ε,η).[48]

Another contrast between Bernard et al. (1996) and Liao and Chiang (2002) illustrates the second data issue. The discrete choice alternatives found by Liao and Chang (2002) in their space heating data were electricity, gas or oil. For Bernard et al. (1996), by contrast, while some households in Quebec used just electricity for space heating, or just gas, or just oil, others used a mix of gas and electricity, or oil and electricity, or wood and electricity.[49] Similarly, in the Norwegian data used by Nesbakken (1999, 2001) and Vaage (2000), while some households used just electricity for space heating, others used wood and electricity, oil and electricity, or even oil, wood and electricity.[50] These authors modeled the choice among fuel alternatives in the manner of an extreme corner solution, as in DM or Hanemann. From a utility-theoretic perspective, an extreme corner solution comes about because either the fuel alternatives are mutually exclusive, as in (7), or they are perfect substitutes, as in (8). Bernard et al. (1996), and Nesbakken (2001), following DM, invoke (7); Vaage (2000), following Hanemann (1984), invokes (8). But neither framing seems to fit the facts well.[51] Heating with electricity, gas, wood and oil etc. may hardly seem perfect substitutes. They also seem unlikely to be mutually exclusive choices. Treating those choices as a *general* corner solution seems a better theoretical framing.[52]

Bernard et al. (1996) also introduced two generalizations of DM's stochastic specification. One generalization was to allow for a nested logit (MNL) error structure of the ε_i in (35) which induced a degree of heteroscedasticity and correlation among them. For example, given the choice of natural gas for space heating, only gas and electricity were fuel options for water heating; given oil and electricity for space heating, only oil and electricity were options for water heating. The second generalization was to express the ε_i as the sum of two parts, a standard extreme value combined with a correlated multivariate normal, thereby producing a hybrid multinomial probit (MNP) model. The more general formulations with stochastic interdependence performed better than standard MNL.[53]

Mansur et al. (2008), Newell and Pizer (2008), Davis and Kilian (2011), Couture et al. (2012), and Risch and Salmon (2017) all followed Bernard et al. in formulating an extreme corner solution model of energy demand like (35)-(36), while mostly using DM's simpler assumption of iid extreme value terms for the ε_i and – sometimes – DM's formulation of the mean of η_i as a linear function of the ε_j, $j \neq i$. The nature of (36) varied. Sometimes it was a single demand function, sometimes multiple demand functions. Sometimes the demand (continuous choice) was aligned with the discrete choice alternatives in (35), as in Liao and Chang (2002), while sometimes not, as with DM and Bernard et al.

[43] With space heating, for example, a higher electricity price lowered the probability of choosing electricity relative to natural gas but raised the probability of choosing oil relative to natural gas.

[44] Their Eq. (30).

[45] While DM assumed electricity consumption not for heating was a fixed quantity, Bernard et al. did not.

[46] There were differences in the treatment of price. Bernard et al. (1996) included all three fuel prices, electricity, gas and oil. DM included fuel prices and the annualized capital cost, while Liao and Chang (2002) included the gas price in demand functions conditional on the use of gas but included the electricity price in every conditional demand function.

[47] Two-step estimation is employed with DM's correction for the conditional expectation in (36).

[48] One could also combine a utility theoretic specification of the demand function(s) in (36), such as PIGLOG or AIDS, together with an ad hoc formulation in (35) plus correlation among ε_i and η.

[49] Typically, electricity signifies electric heaters; wood signifies wood-burning stoves.

[50] Vaage's data is for 1980 and covers all residential uses of energy; Nesbakken's data is for 1990 and covers residential energy for space heating.

[51] This was not an issue for DM because they pre-selected a sample of households that used only one fuel, electricity or gas, for heating both space and water.

[52] A practical formulation for estimating general corner solutions was not known at the time these papers were written.

[53] Bernard et al. noted that, while MNL and MNP are not nested, their MNP formulation can capture whatever structure is represented by an MNL as well as whatever more general structure might be desired.

Davis and Kilian (2011) estimated a discrete choice among three separate fuels (not fuel mixes) for heating residential space (but not water): gas, electricity and oil. Their (36) was a single demand function for total household consumption of gas, conditional on having chosen gas for space heating.[54] Risch and Salmon (2017) similarly estimated a discrete choice for space heating of single-family homes in France in 2006, with gas, electricity and oil as alternatives.[55] Their (36) was a single demand function for something different from the existing literature – overall household fuel use (all fuels, all end uses) per square meter. Couture et al. (2012) focused on household use of fuelwood in the mid-Pyrenees region of France. The discrete choice alternatives in (35) were not to use wood at all; to use wood as the primary fuel for space heating; to use electricity as the primary fuel for heating, with wood as backup fuel; to use gas as the primary fuel with wood as backup; and to use oil as the primary fuel with wood as backup. The continuous choices in (36) were household demand for wood fuel in each of the four cases where wood was being used.[56] Combining those demand functions in the manner of (19), the authors simulated the impact of price changes on total household demand for wood as a fuel.

Rather than estimating a single conditional demand equation for just one fuel among the options, Mansur et al. (2008) and Newell and Pizer (2008) estimated a complete set of conditional demand functions for each fuel in the discrete choice. Mansur et al. (2008) estimated separate models of fuel choice and use in residential buildings and commercial buildings.[57] They divided residential buildings into two groups, depending on whether gas was or was not available. Households in the first group were divided into: homes that used electricity only, homes that used electricity and gas, and homes that used electricity and oil.[58] Where gas was not available homes were divided into: homes that used electricity only; homes that used electricity and oil; and homes that used electricity and other fuels (LPG, kerosene). Commercial buildings were similarly divided into mutually exclusive groups.[59] For each group, a discrete choice was estimated based on an equation like (35). This was followed, for the first residential group, by estimating a system of conditional demand functions like (36) with a Heckman selectivity correction term for the amount of electricity used in the home, for the amount of gas used, and for the amount of oil used, conditional on the particular alternative fuel mix chosen. There was a similar set of estimations for the second residential group and for commercial buildings.

Analyzing energy use in commercial buildings, Newell and Pizer (2008) were unusual in having data with a full breakdown of fuel consumption by end use and by fuel type. They were therefore able to estimate separate demand systems for each of five end uses in commercial buildings.[60] The fuels were electricity, gas, oil and district heat. In the case of cooking, for example, some buildings used just electricity, some used just gas, and some used a combination of electricity and gas – those were the three alternatives in the discrete choice model. Four conditional demand equations were estimated – a demand for the amount of electricity used in cooking conditional on just using electricity, a demand for gas used conditional on just using gas, a demand for electricity used conditional on using both electricity and gas, and a demand for gas used conditional on using both electricity and gas.

Several points about this analysis are of interest. For the discrete choice, the building occupant was conceptualized as a firm selecting its energy input in a cost minimizing manner. Consequently, (35) was formulated as a translog cost function, conditional on the i^{th} fuel being used.[61] Newell and Pizer considered using for conditional input demand equations the share equation derived from the translog cost function but found that a linear form like (36) fit better.[62] However, some of the difficulty in having the same cost function formula account for the discrete and continuous choices could also be due to the sheer heterogeneity of the buildings in their data, which included stores, offices, hotels, churches, and schools. It might otherwise have been possible to estimate DCC models separately for different types of commercial buildings. Also, as noted above, if some consumers choose electricity, others choose gas, and others choose a mix of electricity and gas, this has the appearance of a general corner solution rather than the extreme corner solution modelled by Newell and Pizer. Another question is the relationship between end uses – are some end uses complement or substitutes, rather than being independent commodities as modelled by them?

[54] That total includes gas used to cook and to heat water. David and Kilian (2011) analyzed the economic impact of the deregulation of natural gas in the US in 1989. They estimated the discrete choice from Census data for 2000, and the continuous choice with data for 1980, 1990 and 2000.

[55] Risch and Salmon estimated demand separately for single-family homes and flats. For flats, the discrete choice alternatives were an individual heating system using electricity, and individual system using gas, or a collective (building-wide) system. They used probit rather than logit for (35). There was no correlation among the random terms in (35) and (36) but, in calculating total household fuel use as the dependent variable for their (36), they used the predicted probabilities of choosing each type of space heating fuel from (35). Risch and Salmon's (2017) main interest was the relative explanatory power of alternative sets of explanatory variables in (36) representing physical features of the house and the local climate as opposed to socio-demographic characteristics of the occupants. Household energy use was determined almost completely by the former.

[56] Rather than an extreme corner solution, this household demand for wood could better be modeled as a general corner solution.

[57] Both sets of choices were represented as consumer rather than producer choices.

[58] The fuel choice was not limited to any particular end use such as space heating.

[59] Namely, buildings that used electricity only; buildings that used electricity and gas; buildings that used electricity and oil; and buildings that used electricity and other fuels. There was no information as to whether or not gas was available for commercial buildings.

[60] Namely, space heating; water heating; cooking; miscellaneous; and other end uses that use only electricity, such as lighting, cooling, and office equipment.

[61] For example, if the input was electricity, the input price in (35) was the price of electricity; if the input was a mix of electricity and gas, the price of electricity and the price of gas both appeared in the cost function.

[62] They pointed out that, even if they had imposed the structural link between the fuel choice equation and the fuel demand equation, "the decisions being modeled may occur at different times and under different conditions, making both parameter and structural restrictions suspect."

6. Utility-theoretic models of a general corner solution

The difference in formulating a utility model for a general corner solution versus an extreme corner solution is the starting point from which the model is derived. For an extreme corner solution, as noted, the starting point is the conditional indirect utility functions and the conditional demand functions. For a general corner solution, the starting point is the Kuhn-Tucker first order conditions applied to the unconditional direct utility function. Take (1) as the general formulation of the unconditional direct utility function, and let it now include random terms, denoted ω, along with quality attributes, q, and household characteristics, z. Let the outside good be the numeraire, so that $p_N = 1$. Suppose one observes a consumer purchasing positive quantities of the outside good plus some but not all of the inside goods. Let the inside goods purchased be numbered from 1 to Q, with those not purchased numbered from $Q+1$ to N-1. Let the observed quantities of the purchased goods be $x_i = x_i^* > 0$, $i = 1,\ldots,Q$, while the observed quantities of the remaining inside goods are $x_i^* = 0$, $i = Q+1,\ldots,N-1$, and assume that $y - \sum_{j=1}^{Q} p_j x_j^* \equiv x_N^* > 0$. Let X * denote the vector $(x_1^*,..,x_N^*)$. We can then rewrite (1) to give the utility of the observed consumption bundle as $u = u(X^*, q, z; \omega)$. For this particular consumer, as noted by Hanemann (1978) and Wales and Woodland (1983), we can infer that the following (N-1) equations hold true:

$$\begin{aligned} \frac{\partial u(X^*, q, z; \omega)}{\partial x_i} &- \frac{\partial u(X^*, q, z; \omega)}{\partial x_N} p_i = 0 \qquad \text{where} x_i^* > 0 \\ \frac{\partial u(X^*, q, z; \omega)}{\partial x_i} &- \frac{\partial u(X^*, q, z; \omega)}{\partial x_N} p_i \leq 0 \qquad \text{where} x_i^* = 0. \end{aligned} \tag{37}$$

The probability of observing the consumption bundle X^* – the probability of observing this general corner solution – is the probability that (37) holds.

The tractability of that probability statement depends on the parametric formulation of the utility function $u(X^*, q, z; \omega)$and the probabilistic specification of ω. Finding a tractable formulation was difficult initially if $N > 3$.[63] Moreover, a tractable formula for the probability that the Kuhn-Tucker conditions, (37), hold, does not guarantee the existence of a tractable formula for the unconditional ordinary demand functions emerging when the Kuhn-Tucker conditions are solved. With an extreme corner solution, since the likelihood function is based on the demand functions, as in (25), if estimation is computationally tractable, the demand functions are too. Not so with a general corner solution.

Note that (37) does not require any special feature of the utility function. All that is needed is that $u(x, q, z; \omega)$ not make the inside goods, $x_1,\ldots,x_{N-1}$, either perfect substitutes or essential goods. This allows a potentially wide range of choices, subject to computational tractability. [64]

Subsequently, Lee and Pitt (1986) developed a parallel formulation of the conditions for a general corner solution with the indirect utility function or the ordinary demand functions as primitives. The starting point is the *unconstrained* indirect utility function implied by the maximization of $u(x, q, z; \omega)$subject to the budget constraint (2) but *without the non-negativity constraints* (3), expressed as a function of *normalized* prices, $\rho_i \equiv (p_i/y)$, $i = 1,\ldots, N$:[65].

$$u = H\left(\rho, q, z; \omega\right) = \max_x \left\{ u\left(x, q, z; \omega\right) \middle| \sum_{i=1}^{N} \rho_i x_i = 1 \right\} \tag{38}$$

The demand functions resulting from the maximization in (38),$x_i = D_i(\rho, q, z; \omega)$, satisfy Diewert's (1974) version of Roy's identity for normalized prices:[66].

$$x_i = D_i\left(\rho, q, z; \omega\right) = \frac{\partial H(\rho, q, z; \omega)/\partial \rho_i}{\sum_{j=1}^{N} \rho_j \partial H\left(\rho, q, z; \omega\right)/\partial \rho_j} \qquad i = 1,.,N. \tag{39}$$

Let the observed quantities of the purchased goods be $x_i = x_i^* > 0$, $i= 1,\ldots,Q$, while the observed quantities of the remaining inside goods are $x_i^* = 0$, $i=Q+1,\ldots,N$-1, with $y - \sum_{j=1}^{Q} p_j x_j^* \equiv x_N^* > 0$; the observed normalized prices are $\rho_1,\ldots,\rho_N$. The indirect utility approach proceeds in two steps. The first step is to solve the following (N-Q-1) equations for *shadow* (or virtual or notional or reservation) normalized prices, χ_i, for the non-consumed commodities as functions of the other variables:[67].

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[63] Wales and Woodland (1983) were able to estimate this type of model with N = 3, where these were all inside goods, based on the assumption of weak separability, which obviated the need for a fourth good. Hanemann (1978), for whom N was 35, could not estimate this type of model. Estimability changed with the introduction of the method of simulated moments and related methods, starting with McFadden (1989), see also Train (2009). General corner solutions have now been estimated with N = 62 and even N = 89 inside goods (von Haefen et al., 2004; von Haefen, 2007, 2008).

[64] Wales and Woodland (1983) made (1) a quadratic utility function with multivariate normal random terms. Quadratic utility renders all goods non-essential.

[65] Lee and Pitt (1986) call *H(.)* an unconditional indirect utility function.

[66] Lee and Pitt (1986) refer to these as notional demands: the quantity demanded may be negative.

[67] Eq. (39) generates (40) when $x_i = 0$. Rothbarth (1940-41) and Neary and Roberts (1980) originated the concept of notional or virtual price.

$$0 = \partial H(\rho_1, ., \rho_Q \chi_{Q+!}, .\chi_{N-1}, \rho_N, q, z; \omega)/\partial \rho_i \quad i = Q+1, ., N-1. \tag{40}$$

When combined with the observed normalized prices for the goods that are consumed, the shadow normalized prices, $\chi_i = \chi_i(\rho_1, ., \rho_Q, \rho_N, q, z; \omega)$, have the property that they would lead the consumer to choose to consume exactly zero of the non-consumed goods. The regime where these goods are not consumed occurs when their actual normalized prices exceed these shadow prices:[68].

$$\rho_i \geq \chi_i(\rho_1, ., \rho_Q, \rho_N, q, z; \omega) \qquad i = Q+1, ., N-1. \tag{41}$$

The observed quantities demanded of the goods that are consumed will satisfy Roy's identity at the shadow prices for the unconsumed goods:

$$x_i^* = \frac{\partial H(\rho_1, ., \rho_Q, \chi_{Q+!}(), ., \chi_{N-1}(), \rho_N, q, z; \omega)/\partial \rho_i}{\sum_{j=1}^{N} \rho_j \partial H(\rho_1, ., \rho_Q, \chi_{Q+!}(), ., \chi_{N-1}(), \rho_N, q, z; \omega)/\partial \rho_j} \qquad i = 1, ., Q. \tag{42}$$

The probability of observing the consumption bundle X^* – the probability of observing this general corner solution – is the probability that (41) and (42) both hold.

Eqs. (41)-(42) parallel (37): a set of Q probability equalities in ω combined with a set of *N*-*Q*-1 probability inequalities in ω. Thus, (41)-(42) may be regarded as indirect utility versions of the Kuhn-Tucker conditions, (37). There is an important difference, however. Setting up (37) is a one-step process, while (41)-(42) involves a two-step process, since (40) first has to be solved for the shadow price functions $\chi_{Q+1}(.), ., \chi_{N-1}(.)$. Solving (40) adds complexity. If only one commodity is not consumed, (40) constitutes a single equation which simplifies things. If the right-hand side of (40) contains only linear equations, that also simplifies things. Otherwise, the shadow prices $\chi_j(.)$ obtained by solving (40) are likely to be complicated functions of $(\rho_1, ., \rho_Q, \rho_N, q, z; \omega)$. That would make the right-hand side of (42) an even more complicated function of $(\rho_1, ., \rho_Q, \rho_N, q, z; \omega)$, compounding the complexity of the probability statement that (41)-(42) hold. This has limited the use of the indirect utility Kuhn-Tucker approach.

A special case arises when the indirect Kuhn-Tucker approach is applied to a *bivariate* indirect utility function with scaling or cross-product repackaging making the inside goods perfect substitutes, like $u*(\cdot,\cdot)$ in (8).[69] Making the first good in the bivariate utility non-essential (unlike Hanemann's, 1984 formulation) generates a version of Hanemann's extreme corner solution with the added possibility that *none* of the inside commodities is chosen. This extension of Hanemann's model was developed by Chiang (1991) and Chiang and Lee (1992). With scaling, instead of (18) the modified discrete choice rule becomes: choose the inside good with the lowest quality adjusted price (p_i/ψ_i) as long as that adjusted price is less than the inside good's shadow price; otherwise choose no inside commodity.[70] This formulation is *not* a general corner solution (where more than one of the inside goods is consumed but not all). It is an extreme corner solution where at most one of those goods is chosen. With bivariate utility, estimation is simplified since (40) contains a single equation. As von Haefen (2000) noted, this permits using for the inside goods any demand function that satisfies the integrability conditions in some price-income region.[71]

Otherwise, the indirect Kuhn-Tucker approach is little used for modeling general corner solutions – it has been used mainly with a homothetic translog indirect utility function.[72] This conveniently renders (40) a set of linear equations but it has the implausible consequence that the income elasticities of demand for all *N* commodities are unity.[73] Instead, the literature has generally relied on the direct Kuhn-Tucker conditions, (37), for modeling general corner solutions.

In the context of (37), it is convenient to employ a direct utility function incorporating a translation of the choice variable, as in (4), but with negative translation parameters γ_i.[74] From (5), if the parameter γ_i is negative that pushes the quantity chosen of x_i to become negative at some combination of prices and income; the non-negativity constraint, (3), then forces the choice into a corner solution.[75] This was first suggested by Bockstael et al., (1986, p. 158) who proposed the following version of the Linear Expenditure System (LES)

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[68] When a good is essential, its shadow price is infinite, negating (41).

[69] Or like Hanemann (1984, eq. 3.25), in the case of cross-product repackaging.

[70] The last condition reflects (41).

[71] Von Haefen (2000) called this a "bottom-up" approach. It is actually the approach that Hanemann (1984) used with his semi-log and log-log demand models, (3.16a) and (3.21a). Chiang and Lee (1992) and Chiang (1991) used a bivariate homothetic translog indirect utility function, a problematic choice as explained below, combined with the GEV distribution for the random terms. von Haefen showed that their model could also be used with bivariate LES and AIDS demand models, which Hanemann (eq. 3.22a, 3.23a) had also proposed but without recognizing Chiang and Lee's no-purchase option.

[72] See Lee and Pitt (1986) and Phaneuf (1999); also Chiang (1991) and Chiang and Lee (1992). But, Kao et al. (2001) used (41)-(42) with LES utility.

[73] Pollak and Wales (1992) comment that this makes homothetic translog utility "uninteresting for empirical demand analysis." In the producer context, the indirect Kuhn-Tucker conditions are often employed because Shepherd's Lemma generates an analog of (40) that is linear, hence easily solved for shadow input prices, when combined with the well accepted translog cost function (see, Lee and Pitt, 1987).

[74] In that case, instead of (4) we write utility as a function of $(x_i + \gamma_i)$ and refer to $\gamma_i > 0$ as *non-essentialness parameters* since they prevent x_i from being an essential good.

[75] When γ_i's for two goods are zero, the indifference curves are asymptotic to the axes; when γ_i's are negative, the indifference curves cut the axes, generating a corner solution at certain price ratios.

utility:[76].

$$u = \sum_{i=1}^{N-1} \psi_i(q_i, z; \varepsilon_i) \ln(x_i + \gamma_i) + \ln x_N \tag{43}$$

(43) was used to estimate a general corner solution by Herriges et al., (1999, 2004)), Phaneuf et al. (2000), and Mohn and Hanemann (2006). Independently, Kim et al. (2002) used a translated Constant Elasticity of Substitution (CES) utility to estimate a general corner solution:

$$u = \sum_{i=1}^{N-1} \psi_i(q_i, z; \varepsilon_i)(x_i + \gamma_i)^{\alpha_i} + \psi_N x_N^{\alpha_N}. \tag{44}$$

As variants, von Haefen et al. (2004) used

$$u = \sum_{i=1}^{N-1} \frac{1}{\alpha_i} \varphi_i\left(z, \eta\right) [\psi_i(q_i, \varepsilon_i) x_i + \gamma_i]^{\alpha_i} + \psi_N x_N^{\alpha_N} \tag{45}$$

and von Haefen and Phaneuf (2005) used[77].

$$u = \sum_{i=1}^{N-1} \phi_i(z, \eta) \ln [\psi_i(q_i, \varepsilon_i) x_i + \gamma] + \frac{1}{\alpha_N} \ln x_N^{\alpha_N} \tag{46}$$

In (44) and (45) α_i's are satiation parameters controlling the rate at which marginal utility diminishes as consumption grows, parametrized so that $0 \leq \alpha_i < 1$, to make the utility function quasi-concave.[78] To ensure that the attractiveness indices ψ_i are positive they are parametrized as $\psi_i = e^{\psi_i^*}$. In (44), there are three ways to make a good less apt to be consumed: a higher γ_i means that fewer units of the goods are physically needed to deliver the same consumption impact in terms of $(x_i + \gamma_i)$; a higher α_i makes the consumer tire of the good more quickly as his consumption grows; and a lower ψ_i makes the good inherently less attractive. (45) and (46) satisfy weak complementary; (44) does not satisfy weak complementarity; (43) does only if $\gamma_i = 1$.

Applied to (44), the Kuhn-Tucker conditions (37) become:

$$\begin{aligned} \varepsilon_N - \varepsilon_i &= [\ln(\alpha_N \psi_N x_N^{\alpha_N}) - \ln p_N] - [\ln(\alpha_i \psi_i (x_i^{\alpha_i} + \gamma_i) - \ln p_i] \equiv \theta_i\left(X^*, p\right) \quad \text{when} x_i^* > 0 \\ \varepsilon_N - \varepsilon_i &< [\ln(\alpha_N \psi_N x_N^{\alpha_N}) - \ln p_N] - [\ln(\alpha_i \psi_i (x_i^{\alpha_i} + \gamma_i) - \ln p_i] \equiv \theta_i\left(X^*, p\right) \quad \text{when} x_i^* = 0. \end{aligned} \tag{47}$$

Kim et al. (2002) specified the random terms ε_i as multivariate normal. Let $\nu_i \equiv \varepsilon_1 - \varepsilon_N$and let $f_\upsilon()$, denote their resulting multivariate normal density. The probability that (47) holds viewed as a function of the observed consumption bundle, X^*, is given by:

$$\begin{aligned} &\Pr\left\{x_i = x_i^* > 0 \quad i = 1, ., Q \quad \text{and} \quad x_i = 0 \quad i = Q+1, ., N-1\right\} \\ &= \quad |J| \int_{-\infty}^{\theta_{Q+1}} \cdots \int_{-\infty}^{\theta_{N-1}} f_\nu(\theta_1, ., \theta_Q, \nu_{Q+1}, ., \nu_{N-1}) d\nu_{Q+1} \cdots d\nu_{N-1} \end{aligned} \tag{48}$$

where $|J|$ is the Jacobian associated with the change of variable from $(\nu_1, \ldots, \nu_{N-1})$ to $(x_1^*, ., x_{N-1}^*)$with individual elements

$$J_{ij} = \frac{\partial \theta_i(X^*, p)}{\partial x_j^*}.$$

Kim et al. (2002) applied the model to scanner data on consumer choices among five inside goods, (plain yogurt and four flavored yogurts) plus an outside good (total expenditure on all grocery products). The multivariate normal integral in (48) has no closed form expression and thus required numerical integration which Kim et al. (2002) simulated with the Metropolis-Hastings algorithm. They found it difficult to identify separate γ_i's for each inside good and accordingly fixed $\gamma_i = 1$, $i = 1, \ldots, N$-1. They also found it difficult to estimate separate α_i's for each inside good and therefore restricted the α_i's to be the same across all the inside goods, with a separate α_N for the outside good.[79]

The lack of a closed-form expression for the likelihood function elements (48) complicated Kim et al.'s (2002) task in estimating (44). Bhat (2005) developed a version of (44) that did provide a closed form expression. In (44) he made the ε_i iid extreme value instead of multivariate normal, which generated a closed form expression for the integral in (48). He also formulated another version with a more general stochastic structure, along lines similar to Bernard et al. (1996): the ε_i became a hybrid of iid extreme value plus multivariate normal with heteroscedasticity (different variances) and some correlation. With the latter specification, the probability

[76] They also proposed a version involving translation rather than scaling $u = \sum_{i=1}^{N-1} \kappa_i \ln [x_i + \psi_i(q_i, z; \varepsilon_i)] + \ln x_N$

[77] This was also used by von Haefen (2007, 2008).

[78] (43) is the special case of (44) when $\alpha_i = 0$. If $\alpha_i = 1$ all $i = 1, \ldots, N$-1, (44) generates an extreme corner solution.

[79] Unsurprisingly given the difference in the scales of expenditure, the α-value for the inside yogurt goods was about ten times higher than that for the outside good. Plain yogurt was the flavor most likely to be purchased in isolation. In the fitted model, plain yogurt had an estimated attractiveness coefficient about three to ten times larger than the other flavors.

integral equivalent to (48) required numerical integration. The models were applied to data on weekend time use by individuals where there was a choice of five types of discretionary activity.[80] The best results were obtained with a model having some heteroscedasticity and correlation among the random terms associated with some alternatives. Bhat and Sen (2006) applied a version of the hybrid model to data on household ownership and usage of vehicles, with data on the ownership of five types of vehicles and the annual mileage driven with vehicles of each type.[81] As with the 2005 study, the results indicated the presence of heteroscedasticity and correlation among the unobservables (random terms) associated with some of the choice alternatives.

Bhat (2008) modified (44) by applying a Box-Cox transformation to the translated consumption levels. His model took the form:[82].

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha_i} \psi_i\left(q_i, z\right) e^{\varepsilon_i} \left[\left(\frac{x_i}{\gamma_i} + 1\right)^{\alpha_i} - 1\right] + \frac{1}{\alpha_N} e^{\varepsilon_N} x_N^{\alpha_N}. \tag{49}$$

As with Bhat (2005) and Bhat and Sen (2006), there were several alternative stochastic specifications for the ε_i, including iid extreme value, generalized extreme value (GEV) (producing a nested logit structure), and a hybrid extreme value-multivariate normal formulation. Like Kim et al. (2002), Bhat (2005) found it difficult to estimate the α_i and γ_i terms separately. Therefore, Bhat suggested that the researcher consider three alternative simplified parametrizations of (49): the α-profile, where $\gamma_i = 1$, $i = 1,\ldots,N$-1

$$u = \sum_{i=1}^{N-1} \frac{1}{\alpha_i} \psi_i\left(q_i, z\right) e^{\varepsilon_i} \left[(x_i + 1)^{\alpha_i} - 1\right] + \frac{1}{\alpha_N} e^{\varepsilon_N} x_N^{\alpha_N} \tag{50a}$$

the γ-profile, where $\alpha_i = 0$, $i = 1,\ldots,N$-1[83].

$$u = \sum_{i=1}^{N-1} \gamma_i \psi_i\left(q_i, z\right) e^{\varepsilon_i} \left[\ln\left(\frac{x_i}{\gamma_i} + 1\right) - 1\right] + \frac{1}{\alpha_N} e^{\varepsilon_N} x_N^{\alpha_N} \tag{50b}$$

and the constant α profile, with a single α-value

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha} \psi_i\left(q_i, z\right) e^{\varepsilon_i} \left[\left(\frac{x_i}{\gamma_i} + 1\right)^{\alpha} - 1\right] + \frac{1}{\alpha} e^{\varepsilon_N} x_N^{\alpha}. \tag{50c}$$

In an empirical application, Bhat (2008) found that the constant α parametrization fit best, followed by the γ-profile. The formulations (50a-c) have become the canonical form of Bhat's model in many subsequent empirical exercises. Applications to residential energy demand by Jeong et al. (2011), Yu et al. (2011), Yu and Zhang (2015), Frontuto (2019), and Iraganaboina and Eluru (2021) all found that the γ-profile fit their data best.

Bhat's primary motivation for generalizing the utility function from (44) to (49) was weak complementarity, (6), satisfied by (49) but not (44).[84] Whether or not it is reasonable to impose weak complementarity is a judgment by the modeler. It may not be reasonable theoretically in some cases.[85] Empirically, there may be little reason to impose weak complementarity in the energy demand context because almost none of the energy demand applications of Bhat's model (49) have actually incorporated attribute variables corresponding to q; instead, they used household/building characteristics corresponding to z.[86] Since z is not specific to any choice alternative, there is no reason why it should satisfy weak complementarity.[87] Without a need for weak complementarity, little seems gained from the additional complexity of (49) compared to Kim et al.'s (44). For example, a γ-profile version of (44) could be

$$u = \sum_{i=1}^{N-1} \psi_i(q_i, z) e^{\varepsilon_i} \ln(x_i + \gamma_i) + \psi_N e^{\varepsilon_N} x_N^{\alpha_N} \tag{51}$$

Not having γ_i appear twice in the summation term of (51) versus (50b) may simplify estimation, at the cost of violating weak complementarity.

[80] Nobody participated in all five activities, 61% participated in only one activity and the rest participated in between two and four activities. There was no outside good.

[81] Again, there was no outside good.

[82] The outside good, x_N, has no translation parameter, γ_N, and – unlike Kim et al. (2002) –no attractiveness index, ψ_N.

[83] Bhat (2008) refers to γ_i as satiation parameters. A more accurate term is non-essentialness parameters. Satiation is governed more directly by the α_i.

[84] Along with (10), (45) and (46), (49) satisfies weak complementarity through scaling. As noted above, it is also possible to satisfy weak complementarity through cross-product repackaging. Larson (1991) and von Haefen (2007) identify a third way of imposing weak complementarity called generalized translating. In fact, (49) can also be seen as generalized translating. von Haefen (2007) shows that generalized translating can usefully be applied in the bivariate utility setting that leads to an extreme corner solution, but concedes (p.22) that, when it is not also scaling or cross-product repackaging, generalized translating is unlikely to be useful for a general corner solution.

[85] For example, when the consumer's choice depends not on the *absolute* level of attributes, q, but rather their levels *relative* to that of another choice alternative (Quandt and Baumol, 1966). In that case, the attributes of the other alternative affect the utility assessment of every alternative even when it itself is not consumed. That type of formulation has not yet been employed in the DCC literature.

[86] The exception is Frontuto's (2019) demand function for gas, which included a variable that might be subjected to weak complementarity.

[87] There is no reason to assume that $x_j = 0 \rightarrow \partial u / \partial z = 0$.

Utility functions (43) – (46) and (49) – (51) are all additive: the marginal utility of alternative is unaffected by the consumption of any other good. This rules out complementarity or substitutability among alternative commodities with respect to the deterministic component of the utility function.[88] Vásquez-Lavín and Hanemann (2008) proposed a non-additive version of Bhat's utility function (49) taking the form:[89].

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha_i} \psi_i e^{\varepsilon_i} \left[\left(\frac{x_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right] + \frac{1}{2} \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \theta_{ij} \left[\left(\frac{x_i}{\gamma_i} + 1 \right)^{\alpha_i} - 1 \right] \left[\left(\frac{x_j}{\gamma_j} + 1 \right)^{\alpha_j} - 1 \right] + \frac{1}{\alpha_N} x_N^{\alpha_N}. \tag{52}$$

Subsequently, Bhat et al. (2006) suggested dropping the diagonal terms, θ_{ii}. These added to the quasiconcavity of what was already a quasiconcave function; dropping them simplified the first-order conditions and facilitated estimation. The γ-profile version of (52) with the diagonal elements of θ omitted is:

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha_i} \psi_i e^{\varepsilon_i} \ln \left(\frac{x_i}{\gamma_i} + 1 \right) + \frac{1}{2} \sum_{i=1}^{N-1} \sum_{j \neq i} \theta_{ij} \ln \left(\frac{x_i}{\gamma_i} + 1 \right) \ln \left(\frac{x_j}{\gamma_j} + 1 \right) + \frac{1}{\alpha_N} x_N^{\alpha_N}. \tag{53}$$

Vásquez-Lavín and Hanemann (2008) applied (52) to consumer choices among quality-differentiated commodities. Neither (52) nor (53) has yet been applied to energy data, but Yu and Zhang (2015) used a somewhat similar non-additive model for household end uses of energy in Beijing, which they compared with the versions of Bhat's model in (50a,b,c). Yu and Zhang's model, called the Resource Allocation Model – Multi-Linear Function (RAM-MLF), took the form:

$$u = \sum_i w_i \psi_i e^{\varepsilon_i} \ln(x_i + 1) + \sum_i \sum_{j>i} \lambda w_i w_j \psi_i e^{\varepsilon_i} \psi_j e^{\varepsilon_j} \ln(x_i + 1) \ln(x_j + 1) \tag{54}$$

where w_i are a set of weights to be estimated and ψ_i are function of household and home characteristics, z. In this formulation, all the commodities are either substitutes or complements depending on λ – they are substitutes if $\lambda < 0$ (which turned out to be the case) or complements if $\lambda > 0$. Yu and Zhang (2011) found that the γ-profile fit the data best among the Bhat formulations, but their non-additive model (54) fit even better.[90]

It is possible to formulate a utility-theoretic model combining elements of both an extreme corner solution and a general corner solution. For example, suppose there are two groups of inside goods. Inside goods in group A = $\{j | j = 1, \ldots, Q\}$ are perfect substitutes, so that a consumer chooses at most one of them, while inside goods in group B = $\{j | j = Q+1, \ldots, N\text{-}1\}$ are not perfect substitutes and therefore consumers generally choose more than one of them but not all. With x_N the outside good and using scaling to generate perfect substitutes, by analogy with (8) a utility function generating this outcome has the form[91].

$$u = u\left(\sum_{i=1}^{Q} \psi_i x_i, \psi_{Q+1} x_{Q+1}, ., \psi_{N-1} x_{N-1}, x_N \right) \tag{55}$$

To generalize (49) into a combination of perfect and imperfect substitutes, Bhat et al. (2006) proposed the following:

$$u = \max_{i \in A} \{\psi_i\} [x_A + 1]^{\alpha_A} + \sum_{j=Q+1}^{N-1} \psi_j (x_j + 1)^{\alpha_j} + \psi_N x_N^{\alpha_N}. \tag{56}$$

The notion appears to be that the consumer first chooses a discrete alternative in group A and then, having chosen that alternative, chooses the quantity to consume (x_A) along with (x_{Q+1},.,x_N) to satisfy the budget constraint. This sequence seems questionable given that alternatives in group A have different prices: the heating fuel choice should involve a tradeoff between a fuel's price and its quality, ψ_i.[92] A formulation satisfying (55) is:

$$u = \left(\left[\max_{i \in A} \sum_{i \in A} (\psi_i x_i) \right] + 1 \right)^{\alpha_A} + \sum_{j=Q+1}^{N-1} \psi_j (x_j + 1)^{\alpha_j} + \psi_N x_N^{\alpha_N}. \tag{57}$$

In (56), the discrete alternative chosen in group A has the highest ψ_i; in (57) the discrete alternative chosen has the lowest (p_i/ψ_i).

[88] Bhat et al. (2006) note that substitutability among alternatives can be introduced through the ε_i, in the form of a nested logit structure, as in Bhat et al. (2009).

[89] To simplify estimation, they dropped the stochastic term from the outside good. (52) includes many of the utility functions used in this literature. If $\alpha_i = 0$ all i, it becomes the translog direct utility; if $\alpha_i = 1$ all I it is quadratic utility.

[90] The energy end uses considered were refrigerator, fan, air conditioning, shower, washer, TV, personal computer, microwave oven and car. It is not surprising that, overall, some degree of substitutability was found.

[91] This is for the case where a single group of goods is viewed as perfect substitutes.

[92] In Frontuto's (2019) application of (56), discussed below, group A is fuel used for space heating with the alternatives being oil, gas, LPG and wood. These have different prices and different thermal efficiencies.

7. Theory meets data – general corner solutions

As before, the choice is to model household demand (expenditure) for fuels or for end uses. Of papers applying Bhat's model to residential energy demand,[93] while the other paper modeled expenditure on end uses, those by Iraganaboina and Eluru (2021) and Pinjari and Bhat (2021) modeled expenditure on fuels independent of end uses. Both used data from the US Residential Energy Consumption Survey (RECS).[94] In 2005, for example, 28.7% of the households used only electricity while 58.7% used a mix of electricity and gas, and 5.8% used electricity and oil, etc. By ignoring end uses, these papers questionably ascribe differences in fuel use to fuel preferences: if one household uses electricity alone while another uses electricity and gas combined, the latter must have a stronger taste for variety in its fuels.

Pinjari and Bhat (2021) made the household's expenditure on non-fuel items the outside good, so the household was trading off its fuels consumption against non-fuel consumption subject to a budget constraint involving the price per British Thermal Units (BTU) of each of the four fuels.[95] In Iraganaboina and Eluru (2021) there are three other fuels plus, as the outside good, electricity (which every home uses), and the household allocates a pre-set (but unexplained) total BTU consumption among those fuels. [96] This amounts to estimating a weakly separable sub-utility function for fuels.

Authors modeling the demand for end uses of energy had to deal with the fact that their data broke down energy use by fuel type but *not by end use*. Unless a fuel could safely be presumed to be employed for only one end use, it was necessary to impute the amount of fuel consumed by alternative end uses. The imputation was done in two ways.

One method of imputation, used by Dubin and McFadden (1984), employed engineering data. Thus, Yu et al. (2011) had data from a survey of Beijing residents covering their energy consumption and expenditures plus their ownership and weekly usage of appliances and vehicles. Yu et al. then imputed each household's annual energy expenditure for each major appliance and vehicle based on its reported usage combined with engineering data on unit energy consumption.

The other imputation used regression. Jeong et al. (2011) had household survey data covering appliance ownership and usage plus electricity and gas consumption. They focused on energy consumption by three appliances used for space heating: electric heaters, an electric heating bed, and a gas boiler. Gas was also being used for cooking. The authors had other data on the typical monthly ratio of gas usage for cooking and space heating which generated an estimate of usage for the gas boiler. Electricity usage for heaters and for a heating bed, as opposed to other uses, was obtained using a statistical model known as conditional demand analysis (CDA) originated by Parti and Parti (1980) and refined by Larsen and Nesbakken (2004). The particular CDA regression equation used by Jeong et al. (2011) for electricity end uses was:[97].

$$x_h = \beta_h + \sum_{k=1}^{K} \varphi_k N_{kh} + \sum_{k=1}^{K} \sum_{l=1}^{L} \rho_{lh}(z_{lh} - \bar{z}_{lh}) N_{kh} + \sum_{k=1}^{K} \sum_{l=1}^{M} \rho_{mh} \ln(z_{lh} - \bar{z}_{lh}) N_{kh} + \overline{\omega}_h \tag{58}$$

where x_h is the total amount of electricity consumed by household h, β_h is a household-specific fixed effect, N_{kh} is the number of household appliances of type k (= electric heater, electric heating bed, or non-heating electric appliances), z_{lh} is the i^{th} characteristic of household h; and$\bar{z}_{lk}$is the average value of attribute l across all households owning appliance k.[98] With Eq. (58) estimated, the predicted usage of electricity by household h for end use k (= electric heaters, electric heating bed) was given by

$$\widehat{x}_{kh} = \widehat{\varphi}_k N_{kh} + \sum_{l=1}^{L} \widehat{\rho}_{lh}(z_{lh} - \bar{z}_{lk}) N_{kh} + \sum_{l=1}^{L} \widehat{\sigma}_{lh} \ln(z_{lh} - \bar{z}_{lk}) N_{kh} \tag{59}$$

where "^" denotes the estimated value of the regression coefficient. This imputes to household h the average level of predicted electricity usage across all households who own that appliance and have the same household characteristics z_l.

Frontuto (2019) used the Italian household consumption survey, which gave data on expenditure for goods and services, including expenditure on fuels. The fuels covered were electricity, gas, oil, gasoline, diesel, wood and LPG. Gasoline and diesel were presumed to be used by a household only for private transportation; oil, wood and LPG were presumed to be used only for heating space and water. But electricity and gas were used for multiple purposes. Without providing details, Frontuto (2019) conducted a CDA analysis like

[93] See Table 1.

[94] Iraganaboina and Eluru (2021) used data from the 2015 RECS survey; Pinjari and Bhat (2021) and Bhat (2008) used data from the 2005 RECS survey.

[95] Pinjari and Bhat (2021) noted that utility customers often face *increasing block* prices for fuel. They sought to finesse the endogeneity of fuel price by using aggregate unit price values from RECS data.

[96] Iraganaboina and Eluru (2021) set the γ parameter for electricity to zero, as the outside good. Pinjari and Bhat (2021) set the γ parameters for both electricity and for the outside good to zero. This reflects the fact that every household consumes some electricity. Another approach would have been to set up electricity with a negative γ parameter reflecting the fact that is a subsistence good.

[97] Dubin and McFadden (1984) considered CDA, but rejected it because the random error term, ω_h, was likely to include unobserved household characteristics that were correlated with appliance ownership and intensity of use.

[98] In Jeong et al.'s (2011) application, N_{hk} was zero or one for gas boilers. The z variables used in the second term on the right-hand side of (58) were the deviation from the average household's gas price, the deviation from the average household's electricity price and the deviation from the average household frequency or duration of usage. The z variables used in the third term of (58) were the natural logarithm of the deviation from the average household monthly living expenditure and the natural logarithm of the deviation from the average household's electricity price.

Jeong et al. (2011) to allocate electricity and gas expenditures to space and water heating.

Whatever the method used to fuel consumption among specific end uses, it inevitably omits some of the behavioral variation found among households. It yields a noisy estimate of the household's actual fuel usage for that end use. A household's actual level of energy consumption for end use k might be represented as:[99].

$$x_{kh} = \widehat{x}_{kh} + \zeta_{kh}. \tag{60}$$

The measurement error ζ_{kh} could bias the estimation of, say, (50a,b,c). One might consider substituting (60) into the Kuhn-Tucker conditions (47) to generate a modified version of Bhat's likelihood function (48).[100]

Given their allocation of fuel expenditures to end uses, Yu et al. (2011), Yu and Zhang (2015) and Jeong et al. (2011) used Bhat's (50a,b,c) model to estimate a system of energy demands by type of appliance/end use. Yu et al. (2011) and Yu and Zhang (2015) made non-energy expenditure the composite outside good.[101] Jeong et al. (2011) had no outside good.[102]

Frontuto (2019) analyzed household expenditures on transportation, on space and water heating, and on electricity for non-heating purposes, with non-energy expenditure as the outside good. Non-energy expenditure and non-heating expenditure on electricity were treated as essential goods with a γ-value of zero.[103] Frontuto employed Bhat's model (56). Expenditures on transportation and heating each had internal choices that were perfect substitutes, leading to an extreme corner solution within the expenditure category.[104] Transportation involved a choice between expenditure on gasoline, on diesel, on a combination of both (when households owned both types of vehicle), and on non-fuel transportation (bicycles and public transit). Rather than treating these as perfect substitutes, as in (8), it might be better to view them as mutually exclusive alternatives, as in (11).[105]

Papers in this literature found that Bhat's γ-profile formulation fit best. In that formulation, the attractiveness indices, ψ_i, form the basis for parametrizing characteristics, z_{lh}, as determinants of preferences, making the attractiveness of choice alternative i for household h:

$$\psi_{ih} = \exp\left(\mu_i + \sum_{l=1}^{L} z_{lh}\beta_{li}\right) \tag{61}$$

where μ_i measures the baseline attractiveness of the choice alternative absent household characteristics. As characteristics, papers include household size, housing unit characteristics, house location, and climate variables such a heating and cooling degree days. Other z variables sometimes raise issues.

Frontuto included fuel prices among his z's. Prices are not conventionally considered arguments of a *direct* utility function. Pinjari and Bhat (2021), Yu et al. (2011), Yu and Zhang (2015) and Jeong et al. (2011) included household income among their z's, which also is not conventionally an argument of the direct utility function.[106] A better approach would be to coarsen the relationship between income and preferences, for example through a dummy variable for high versus low income, as done by Jeong et al. (2011). Jeong et al. also divided their data into high- and low-income groups, estimating separate utility models for each group. Similarly, for large versus small dwellings, and high versus low heating degree day areas. Each time, they obtained notably different estimates of the coefficient vector in (61), indicating some fundamental interactions among the z_h's.

Two features stand out in the existing literature: narrow reliance on the attractiveness indices, ψ_i, to represent preference heterogeneity and lack of consideration for heterogeneity in household choice sets.

The ψ_i are the vehicle for introducing random preference elements, ε_i, and for parametrizing preferences as functions of household

[99] If there is some degree of correlation between the error term in (58) and the explanatory variables, as Dubin and McFadden (1984) suggested, the resulting bias would contribute to the error ζ_{kh} in (60).

[100] This could result in something like the utility function mentioned in footnote 69 above.

[101] For Yu et al. (2011), the demands were for energy used by refrigerators, air conditioning, fans, clothes washer, electrical shower, gas shower, and cars. For Yu and Zhang (2015), using a later version of the same survey analyzed in a similar manner, the demands were for refrigerators, fans, air conditioner, gas shower, clothes washer, TV, PC, microwave oven and cars.

[102] For Jeong et al. (2011), the demands were for energy used by electric heaters, electric heating beds, and gas boilers.

[103] Expenditures on heating and transportation were modeled as non-essential.

[104] The heating fuels treated as perfect substitutes – oil, gas, LPG and wood – are not treated as perfect substitutes by other researchers, who find homes using a mixture of gas and wood, or oil and wood, for space heating. Electricity does not appear as a choice alternative for heating in Frontuto's data.

[105] There are four types of households: some don't drive; some drive only gasoline-fuel cars; some drive only diesel-fuel cars; some drive both types of car. There is a discrete choice of household type and then, conditional on that choice, a choice of transportation fuel expenditure.

[106] The notion that households with different incomes have different preferences is not unreasonable. Yet it seems undesirable to represent direct utility as a continuous function of household income.

characteristics, z. The α_i have not had played a role in consumer heterogeneity since the α-profile was often rejected. The γ_i have also not been made functions of household characteristics.[107] This is surprising, since the γ_i are primary determinants of whether a commodity is purchased or not.[108] Jeong et al.'s (2011) findings suggest that the γ_i should receive more attention. Jeong et al. divided their data and estimated separate utility functions for subgroups of households; they found different subgroups had different sized γ_i, leading them to select systematically different corner solutions. That heterogeneity could be captured by making γ_i functions of z.

Another approach to preference heterogeneity is to latent class modeling.[109] Some lessons may also be learned from recent developments in the econometric modeling of discrete choices which use mixture models to represent a combination of both separate preference classes and preference heterogeneity within classes.[110]

The literature summarized here assumes that all households have their choice of fuels and face the same choice set. Renters do not have a fuel choice and should usually be excluded. Among fuels, gas is often available only in some areas; the same may be true with other fuels. Consequently, it would be appropriate to have individualized choice sets, with gas (or whatever) not a choice option where it is unavailable. [111]

Finally, if end uses are the primitives of consumer preferences, household fuel choices might be conceived as an example of household production in the manner of Becker (1965) and Muth (1966), with production functions using fuel inputs to produce end uses and household preferences over end uses – a formulation not yet considered in the literature.

8. Welfare evaluation

Once estimated, a model of residential energy demand can be used: (a) to analyze or predict household behavior, for example by estimating price elasticities of demand for fuels; and (b) to perform welfare analysis, for example calculating the compensating variation measure for a fuel price increase or for quantity rationing. For (a), any statistical model of demand could be used – the model does not have to be consistent with utility maximization. For (b), inconsistency with utility theory ends the link from observed household behavior to inferred household preference and welfare.

Demand prediction is straightforward with an extreme corner solution because the likelihood function being maximized is the formula for the probability of the quantity demanded. Not so with a general corner solution: as noted, the formula for the demand function is not part of the likelihood function and must be derived through a separate calculation.

In both cases, the random terms ω in $u(x,q,z;\omega)$ in (37) cause problems. These are known to the consumer but are random for the econometric investigator. At best the investigator recovers the consumer's demand function up to a probability distribution. To predict the consumer's behavior, the investigator uses the expected value of the consumer's unconditional demand function. Because of the nonlinearity of the utility function formulation

$$E\{\text{argmax}_x u(x,q,z;\omega)\} \quad \neq \quad \text{argmax}_x E\{u(x,q,z;\omega\} \tag{62}$$

The left-hand side of (62) is conceptually the correct quantity; the right-hand side might be seen as a representative consumer, but the non-linearity renders the aggregation suspect. Bhat (2005), Bhat and Sen (2006) and Bhat et al. (2006) used the right-hand side of (62), while Saxena et al., 2022 used the left-hand side. Similarly, with price and income elasticities/derivatives of demand:[112].

$$E\left\{\frac{\partial \text{argmax}_x u(x,q,z;\omega)}{\partial p_i}\right\} \quad \neq \quad \frac{\partial E\{\text{argmax}_x u(x,q,z;\omega\}}{\partial p_i} \tag{63}$$

Dubin and McFadden (1984) and Bernard et al. (1996) and most other researchers used the right-hand side of (63); the left-hand side is conceptually correct.

With an extreme corner solution model, the multivariate distribution of ω., $f_w()$, *is* the key to calculating the left-hand sides of (62) and (63). Having estimated the model, including $f_w()$, the procedure is to take, say, 200 draws from the estimate of $f_w()$ and, using the known value of ω from each draw separately, solve the utility maximization on the left-hand side of (62), or calculate the derivative on the left-hand side of (63). Repeating this process with each draw yields an empirical distribution of the unconditional ordinary demand

[107] This may be due to the complicated way in which γ_i enter the utility function in Bhat's formulation in (49) as compared to (43) – (45) and (51).

[108] The larger its γ-value, the less likely a commodity is to be purchased.

[109] See Kuriyama et al. (2010) for an example.

[110] See, for example, Kabaya and Kuriyama (2021), Keane and Wasi (2013) and Wasi and Carson (2013). The latter show that, in the presence of preference heterogeneity, using the right-hand side of (61) for calculations instead of the left-hand side produces misleading results.

[111] Pinjari and Bhat (2021) handled gas unavailability as a dummy variable in z, making it a preference shifter. Individual choice sets make it a supply factor. This was done by Newel and Pizer (2008) who had separate choice sets for building with a gas connection versus those with no gas connection. Frontuto (2019) included in z the density of the gas distribution network. That is a legitimate preference shifter – indeed it could be seen as a q_i-variable possibly requiring weak complementarity.

[112] A referee noted that the left-hand side is the average partial effect; the right-hand side is loosely but not exactly the partial effect at the average.

functions or the price/income derivative, from which the mean is then computed.

With a general corner solution, there is an additional complication – besides the random terms ω, the consumer may choose any of multiple subsets of goods to consume. Therefore, in addition to dealing with multiple draws from the distribution of $f_w()$ one must allow for the different corner solutions that might arise. This requires more extended sampling over alternative choice outcomes in addition to having sampled over multiple possible draws from $f_w()$.

Welfare evaluation is conducted using the unconditional indirect utility function:

$$v(p, q, z, y; \omega) = \max_x u(x, q, z; \omega) \tag{64}$$

Suppose, for simplicity, that there is a change in prices and/or commodity attributes from (p^0,q^0) to (p^1,q^1) that brings about an improvement in the consumer's welfare. The consumer's willingness to pay for this change, C, satisfies

$$v(p^1, q^1, z, y - C; \omega) = v(p^0, q^0, z, y - C; \omega) \tag{65}$$

Note that C depends on ω and is therefore random for the outside observer

$$C = C(p^1, p^0, q^1, q^0, z, y; \omega)$$

The econometrician would evaluate the expected value, $E\{C(p^1,p^0,q^1,q^0,z,y;\omega)\}$.

With an extreme corner solution model, the process is similar to that for calculating demand. Take draws from the estimate of $f_w()$ and, using the known value of ω from each draw separately, solve the utility maximization in (62) using (p^0,q^0), and calculate $v(p^0,q^0,z,y;\omega)$. Using the same draw, solve the utility maximization in (62) using (p^1,q^1), and calculate $v(p^1,q^1,z,y;\omega)$. Using some appropriate routine such as numerical bisection, determine the value of C that satisfies (65). Repeat over all draws of ω, and average the resulting estimates of C.

In the case of a general corner solution, there is the additional complication of multiple potential corner solutions, each involving a different set of goods consumed. Therefore, one samples over both $f_w()$. and the alternative choice outcomes. While not yet applied to energy demand, this approach is often used on the demand for quality differentiated recreation sites.[113]

When a discrete-continuous choice model has been estimated that is not consistent with the maximization of a utility function, welfare evaluation is challenging. An example is when two distinct indirect utility functions are invoked, one underlying the discrete choice, (35), and another derivable from the equation for the continuous choice, (36), as with Bernard et al. (1996) and others. Which one should be used for welfare evaluation? Or, should both somehow be used?

Davis and Kilian (2011) faced this question when calculating gas users' lost consumer's surplus due to price regulation and rationing of natural gas. They calculated gas user's individual cut-off prices for natural gas using (35), the conditional indirect utility functions underlying the discrete choice of natural gas versus other fuels. Letp_h^* be the cut-off gas price calculated from (35) at which the h^{th} household becomes indifferent between using gas versus the next best fuel. Let p_h be the household's current price of gas by. Denote gas consumption predicted at the current price using the fitted demand Eq. (36) by $\widehat{x}_h$. Davis and Kilian (2011) measured the household's current consumer's surplus as[114].

$$(p_h^* - p_h)\widehat{x}_h \tag{66}$$

This uses both (35) and (36) but combines them in an odd manner. It assumes an inelastic demand curve for natural gas, contrary to what the estimated demand function (36) shows.[115] Moreover, the estimated demand function (36) itself generates a cut-off price – the price at which gas demand falls to zero – which is different from p_h^*computed from (35). There is no rationale for choosing one cut-off price over the other, or for assuming a fixed consumption of gas.[116]

9. Conclusions

In a world of differentiated commodities, corner solutions, where some available commodities are systematically not consumed, are a ubiquitous phenomenon. In that context, zero consumption is a distinctive action, a qualitative choice, not just another quantitative choice. Logically, the qualitative and quantitative choices should be connected – they reflect an underlying utility maximization. Recognizing this, theory-consistent empirical models of consumer choice were developed that are widely applied to various types of consumption data, including on residential energy demand. Two distinctively different types of corner solution emerged – extreme

[113] For further details, see the sequence of papers that starts with Bockstael et al. (1986), and continues with Phaneuf et al. (2000), von Haefen et al. (2004), von Haefen and Phaneuf (2005), von Haefen (2007) and Lloyd-Smith (2018).

[114] Their Eq. (5).

[115] They find a decreasing price sensitivity over time, with the point estimate of price elasticity ranging from − 0.34 in 1980 to − 0.10 in 2000.

[116] If the household's fuel choice, underlying (35), occurred in the past while its fuel usage, underlying (36), is current, ignoring (35) and using (36) for welfare evaluation might be more appropriate.

Table 1
Discrete-continuous models of residential energy demand.

Paper	Type of corner solution[a]	Discrete choice	Continuous choice	Utility-theoretic[b]	Estimation method	Outside good[b]
Dubin and McFadden (1984)	EX	Electricity v gas for heating space/ water	Household demand for electricity	Y	2-STEP	Y
Bernard et al. (1996)	EX	Fuels for space/water heating: gas/ gas, gas/electricity, dual energy/oil, dual energy/electricity, oil/oil, oil/ electricity, wood/electricity, wood-electricity/electricity	Household demand for electricity	N	2-STEP	Y
Nesbakken (1999)	EX	Fuel for home heating: electricity, electricity & wood, electricity & oil, electricity & wood & oil	Total household energy demand	Y	FIML	Y
Vaage (2000)	EX	Fuel for home heating: electricity, electricity & wood, electricity & oil, electricity & wood & oil	Total household energy demand	Y	FIML	Y
Nesbakken (2001)	EX	Fuel for home heating: electricity, electricity & wood, electricity & oil, electrcity & wood & oil	Total household energy demand	Y	FIML	Y
Liao and Chang (2002)	EX	Electricity, gas, oil for space heating; Electricity, gas, water for water heating	Fuel demand by fuel and end use	N	2-STEP	
Mansur et al. (2008)	EX	Residential fuel choice: electricity, electricity & gas, electricity & oil	Household demands for electricity, gas, oil	N	2-STEP	
Newell and Pizer (2008)	EX	Commercial building fuel choice for each of 5 end uses: electricity, gas, oil, district heating, electricity & gas, electricity & oil, gas & oil, electricity & district heating	Demands by end use for electricity, gas, oil, district heating	N	2-STEP	
Davis and Kilian (2011)	EX	Residential fuel choice for space heating: electricity, gas, oil	Household demand for gas	N	2-STEP	
Jeong et al. (2011)	GEN	Method of space heating: electric heaters, electric heating bed, gas	Household energy demand for space heating	Y	FIML	N
Yu et al. (2011)	GEN	Household energy usage for refrigerator AC, fan, clothes washer, electrical shower, gas shower, cars	Household energy usage for refrigerator AC, fan, clothes washer, electrical shower, gas shower, cars	Y	FIML	Y
Couture et al. (2012)	EX	Not use wood; wood primary for heating; electricity primary, wood backup; gas primary, wood backup; oil primary, wood backup	Demands for wood when used alone, when used as backup for each individual primary fuel	N	2-STEP	
Yu and Zhang (2015)	GEN	Household energy usage for refrigerator AC, fan, clothes washer, gas shower, TV, PC, microwave, cars	Household energy usage for refrigerator AC, fan, clothes washer, gas shower, TV, PC, microwave, cars	Y	FIML	Y
Risch and Salmon (2017)	EX	For homes: electricity, gas, oil For flats: electricity, gas, collective heating	Total residential fuel use (all fuels, all end uses) per square meter	N	2-STEP	
Frontuto (2019)	GEN-EX	Household expenditure on energy for heating space & water, on electricity for all other end uses, on fuels for private transportation	Household expenditure on energy for heating space & water, on electricity for all other end uses, on fuels for private transportation	Y	FIML	Y
Iraganaboina and Eluru (2021)	GEN	Choice of fuels for household energy: electricity, oil, gas, LPG	Household demand for electricity, oil, gas, LPG	Y	FIML	N
Pinjari and Bhat (2021)	GEN	Choice of fuels for household energy: electricity, oil, gas, LPG	Household demand for electricity, oil, gas, LPG	Y	FIML	Y

Notes.
[a] EX = extreme; GEN = general; EX + GEN = general plus extreme
[b] Y = Yes; N = No

corner solutions, where at most one of a set of related commodities is consumed, and general corner solutions, where more than one but not all are consumed. Two alternative pathways generate an extreme corner solution – choice alternatives are either mutually exclusive or considered perfect substitutes. The only requirement for a general corner solution is preferences allowing for non-essential commodities. Both types of corner solution are used for residential energy demand.

The type of data available always shapes not just empirical estimation of demand functions but, also, their conceptualization. Residential energy data can have features that complicate the conceptualization of a corner solution. (i) Households sometimes use multiple fuels for the same end use (e.g., using gas and electricity for space heating, or wood and electricity). Electric heaters can be used in any room, and one can have a fireplace in a house with central heating. Hence, the fuel alternatives seem neither mutually exclusive nor perfect substitutes, invalidating the conceptualization of an extreme corner solution, yet such models are employed (Table 1). (ii) The discrete choice is between fuels for a particular end use but the fuel consumption data includes other end uses as well. That complicates aligning the continuous choice with the discrete choice in a theoretically consistent manner.

As between extreme and general corner solutions, there is a paradox: extreme corner solutions permit a wide range of (bivariate) utility-theoretic formulations and are less complicated to estimate; general corner solutions work with a relatively limited range of utility formulations and are very complicated to estimate. Yet, utility-theoretic formulations are still used for general corner solutions, while being abandoned for extreme corner solutions. Extreme corner solutions now use a set-up like (35)-(36), with ad hoc formulations of the right-hand sides.[117] The only linkage between discrete and continuous choices arises through unobservables believed to influence the discrete and continuous choices.

Two factors explain the abandonment of utility theory in residential energy demand, one specific to residential energy. Some energy equipment – especially space/water heating – is long lived and embedded in the physical structure of the building: changing a heating system is more complicated than changing a refrigerator. That clouds the link between decisions on appliance ownership and utilization. The heating system was perhaps installed long ago and is exogenous to current decisions on usage. If so, it becomes difficult to determine the buyer's expectation of fuel (operating) cost at the time the appliance was purchased. Likewise, it is difficult to determine now the capital (purchase) cost back then, to amortize it and to include that alongside the operating cost.[118] The time lapse between appliance purchase and observed utilization also casts doubt on the justification for a unified theoretical model: if appliance ownership and utilization decisions occur at different times, why are they generated by the same utility function?

The second factor is the general trend in economics to reject structural economic models in favor of non-parametric and reduced form models.[119] It is asserted that structural models rely on assumptions about parametric functional form that have no specific foundation in economic theory and are essentially arbitrary; and that structural models are vulnerable to bias from omitted explanatory variables and from endogenous included variables. Even conceding these points, two things of value result from the structure in a structural model if it can reliably be estimated: (1) the ability to simulate counterfactual scenarios such as price/attribute combinations not observed in the data, and (2) the ability to infer preferences and use those to calculate welfare measures for changes in fuel prices, attributes or quantities associated with energy choices.

The latter is illustrated by the experience with Davis and Kilian's (2011) calculation of the welfare cost of natural gas shortages. Instead of a unified choice model, they deployed two separate models for the discrete and continuous choice of gas heating. Each model could be used to estimate the shutoff price at which gas would not be chosen/used by the household. Each model could generate a gas demand function (or an approximation thereto). As noted, Davis and Kilian (2011) combined information from the two separate models in a dubious manner. But, if the lapse of time between appliance purchase and utilization is a concern, why treat both models as equally valid – why not focus on the current utilization decision alone (the continuous choice) as the relevant basis for welfare estimation?

Given the significance of the linkage between appliance ownership and utilization decisions for household energy demand, it is unfortunate that the conventional assumption of a unified decision process has not actually been tested in the literature. If it were found to be rejected, that could imply that residential energy demand should be modeled separately for households that have recently made a major appliance purchase, and therefore have faced both ownership and usage choices, versus other households, who face only utilization decisions. In current research we are testing this with data on home heating in Spain. Our findings will be the subject of a separate paper, intended as a complement to this paper.

Conflict of interest statement

The authors whose names are listed immediately below certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

[117] It is hard to imagine an atheoretic formulation for a general corner solution like (35)-(36).

[118] For this reason, Dubin and McFadden (1984), who had energy usage data for 1975, went to great lengths to determine the original capital cost of the heating equipment. Bernard et al. (1996), who had data on residential energy usage in 1989, limited their analysis to homes built between 1986 and 1989. Nesbakken (2001), who had energy usage data for 1990, struggled with the fact the heating equipment in the homes surveyed was installed between 1971 and 1990. The applications of Baht's model in Table 1 all omit the fixed cost of ownership. Thus, they model utilization, not ownership.

[119] For a discussion, see the symposium on this topic in the *Journal of Economic Perspectives*, Spring 2010.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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