Information Release in Second–Price Auctions☆

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Abstract

This paper studies the incentives faced by competing auctioneers who can release information to prospective bidders before bidders choose trading partners. I provide sufficient conditions that ensure the existence of a unique equilibrium in which both sellers release all available information. Contrary to previous findings in the literature, the existence of this equilibrium holds true even if there are only two bidders in the market. Thus, the findings of this paper provides support to the idea that competition among sellers improves informational efficiency relatively to monopoly.

Keywords: Competing Auctions, Information Structures, Private Provision of Information.

1. Introduction

The literature on auction theory has devoted considerable effort to investigating whether it is profitable for an auctioneer to release information about the auctioned item prior to the auction. Most of this effort has been focused on environments with a single auctioneer, leaving almost unexplored

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the question about the seller's incentives to release information in environments where several auctioneers compete for the same pool of bidders. The aim of this paper is to investigate this question and to examine the relation between competition and the seller's incentives to release information in competing auctions when this information affects the decision of bidders about which auction to visit.

In practice, there are many situations in which sellers can control the amount of information about the characteristics of their objects that is made available to potential buyers. For instance, internet auctions, where thousands of independent auctions are run every day, provide sellers with the opportunity to add information and pictures describing some characteristics of their goods, enabling potential bidders to improve their estimates about the items. In those cases where bidders' tastes are different –for instance, they care about some particular attribute of the good– the information that a seller releases will cause some bidders to increase their estimates and others to decrease them, depending on how well the characteristics of the good match their preferences[1]. In environments with several competing auctioneers this is important because bidders who find out that a seller's object is a poor match to their preferences may choose to bid in an alternative auction. This suggests that releasing information in such environments generates a trade–off between price and market share that differs from the one between efficiency and informational rents that arises in environments with a single auctioneer.

In order to examine the nature of this trade–off I develop a simple two stage game where two auctioneers simultaneously choose whether or not to release information in the first stage, and buyers simultaneously decide on which auction to visit in the second one. Prior to any interaction with the sellers, buyers are uninformed about their valuations and think of these two goods as being perfect substitutes. Sellers can release information about the characteristics of their goods before bidders choose on which auction to bid. For simplicity, I assume that the decision to provide information is binary: sellers can choose between granting full access to information about the product to every potential buyer, or preventing any access to it. In the

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[1]Alternatively, bidders may react symmetrically to the information released by the seller if, for instance, valuations are affiliated. In such cases, the linkage principle of Milgrom & Weber (1982) states that releasing information raises the sellers' profits if the seller commits to publicly revealing this information.

case of a seller releasing information, all bidders learn their true value of this seller's good; otherwise, bidders remain uninformed. To avoid the possibility that sellers use information to stochastically bias bidders' valuations, information only affects the dispersion of bidders' posterior valuations but not their mean. Thus, sellers can use information to generate horizontal but not vertical differentiation among the bidders. Furthermore, as the paper focuses on private value environments, bidders' valuations are assumed to be identically and independently distributed both across bidders and across sellers.

The main contribution of this paper is to show that competition between auctioneers may be sufficient to improve informational efficiency relative to monopoly. In particular, I provide sufficient conditions in terms of the distribution of bidders' valuations such that the sellers' decision to release information becomes a dominant strategy and hence, the unique equilibrium of the game is one in which both sellers release all available information. Intuitively, information serves as a screening tool that sellers can use to attract those bidders whose preferences represent a better match to the characteristics of their goods. For a seller who expects his competitor not to release information, having a distribution of bidders' valuations with a mean lower than its median suffices for this screening to be done without reducing traffic. Thus, a seller who expects his competitor not to release information is better off releasing information because he thus attracts sufficient bidders with relatively high valuations such that the number of visitors does not fall relative to the number of visitors he would receive if he did not release information. Likewise, a seller who expects his competitor to release information is also better off releasing information provided that the distribution of bidders' valuations is either convex or it has a mean lower than its median, and a log–concave density satisfying an additional requirement on its slope evaluated at one. The reason for these extra requirements is to ensure that those bidders who value the seller's good the most have sufficiently high valuations for the expected price to be greater than the average valuation, which is the price the seller would receive by not releasing information. Thus, when the distribution of bidders' valuations satisfies either of these two conditions, the seller expects to see, on average, bidders with higher valuations relative to the valuation of the average bidder and hence, releasing information will not only attract bidders with high willingness to pay but it will also ensure that they come in a number that is never lower than the number of visitors this seller would have received had he not released information.

The previous result is obtained using a highly stylized model in which information structures are binary and sellers cannot compete using reserve prices. In order to better assess the role played by these assumptions in the derivation of this result, I develop two extensions of the benchmark model. The first extension considers the case in which sellers can choose the probability with which bidders learn the truth about their valuations. I show that the probability with which any seller chooses an uninformative structure in equilibrium is nil provided that the number of bidders is sufficiently large. This suggests that the main result of the paper showing that competition between auctioneers improves informational efficiency relative to monopoly is not a simple artifact of the binary information structure assumption. The second extension introduces competition in reserve prices. Contrary to previous results in the literature, I show that releasing information need not induce more restrictive reserve prices as the number of bidders increases. In particular, I show the existence of an equilibrium in which sellers release all information and post reserve prices equal to production costs provided that the number of bidders is sufficiently large. Thus, this result suggests that competition may lead to equilibria in which sellers release information but cannot appropriate of the surplus that this information generates.

The rest of the paper is organized as follows. Section 2 briefly reviews the related literature, section 3 outlines the fundamentals of the model, section 4 presents the equilibrium analysis and the main results of the paper, and section 5 presents two alternative settings in which sellers can choose intermediate degrees of informativeness and they can also post reserve prices. The paper concludes with some final comments and conclusions. All proofs are presented in the appendix.

2. Related Literature

As mentioned in the introduction, the auction literature has almost exclusively focused on the problem of information disclosure in monopolistic environments (e.g. Milgrom & Weber (1982); Ganuza (2004); Bergemann & Pesendorfer (2007); Vagstad (2007); Board (2009); Ganuza & Penalva (2010, 2014)). In such environments, this literature has shown that the seller's incentives to release information depend on a trade–off between efficiency and

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bidders' informational rents[2]. Thus, depending on whether or not the gains from efficiency are sufficient to compensate for the higher costs of additional informational rents, there might be cases in which the auctioneer finds optimal not to release information. Board (2009) finds that the cost of higher informational rents overcomes the gains from efficiency when a seller running a second–price private value auction faces only two bidders. Nonetheless, this literature has also shown that this need not be true if there are more than two bidders or if the seller can charge for the extra information. As shown by Board (2009) and Ganuza & Penalva (2010), releasing information tends to benefit the seller if there are enough bidders in the market. Moreover, even though the auctioneer has incentives to release less information than the efficient level, this inefficiency vanishes as the number of buyers increases (Ganuza, 2004; Ganuza & Penalva, 2010). When information can be sold, Eső & Szentes (2007) (see also Gershkov (2009)) have shown that the optimal information disclosure policy for the seller is to reveal all available information and to charge for this information in order to extract all the resulting gains in efficiency.

Forand (2013) is, to the best of my knowledge, the only work that examines the problem of endogenous information structures in competitive environments. Forand considers a model in which two sellers direct the search of two bidders through commitments to providing information. His main conclusion is that competition tends to improve informational efficiency relative to monopoly, a result that is also found in this paper. When competition in mechanisms is restricted to second price auctions, Forand shows the existence of a continuum of equilibria with both auctioneers promising to offer the same amount of information to bidders. Since in Forand's setup a monopolist would choose not to release information, this multiplicity of equilibria

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[2]Milgrom & Weber (1982) assume that bidders' valuations are affiliated while all other papers –including this, study private value environments. Bergemann & Pesendorfer (2007) study the choice of an information structure as part of the design of the mechanism, and show that the optimal information structures can be represented by finite partitions that are asymmetric across bidders. In all the remaining papers, as well as in this, sellers are restricted to treating bidders symmetrically with respect to information provision. Vagstad (2007) studies the problem of information provision in a model where potential bidders must pay an entry cost before the auctioneer can release information. He shows that early information induces screening of high–valuation bidders (an effect that is also present in this paper) but information may reduce equilibrium entry, making the overall effect of information on profits ambiguous.

makes it difficult to evaluate the extent to which competition affects informational efficiency. In the present paper, there is no such multiplicity, which allows me to make a better assessment about the impact of competition on informational efficiency when sellers compete in auctions.

This paper also relates to, but it is separated from, the strand of literature that studies the incentives to provide information in environments where sellers compete through prices. Damiano & Li (2007) consider a model with two sellers, one buyer and binary information structures, and show that information is used by sellers to soften price competition. Ivanov (2013) extends Damiano & Li (2007)'s setup to an environment with an arbitrary number of sellers and a continuum of types but a single buyer. He shows the existence of a critical number of sellers such that in the unique symmetric equilibrium all sellers provide information and charge prices equal to their marginal costs. Similar to Ivanov (2013), I show that competition in reserve prices may not soften price competition when there is a large number of bidders because increasing reserve prices costs the auctioneer a lower number of visitors from both low and high–valuation types, an effect that is absent in models where sellers compete using prices.

3. The Benchmark Model

There are two risk-neutral sellers (seller 1 and seller 2) and $n \geq 2$ risk neutral buyers. Each seller has one unit of an indivisible good and each buyer wants to buy exactly one unit of this good. Trade takes place using second-price sealed bid auctions. Bidder i's true valuation of seller j's good is described by a random variable V_j^i. For each good j, $j = 1, 2$, the random variable V_j^i is independently distributed on $[0, 1]$ according to a distribution function $F(\cdot)$ with mean μ and positive density f. Furthermore, for every bidder i, $i = 1, \ldots, n$, the random variables V_1^i and V_2^i are independently distributed, i.e., bidders' valuations are independently and identically distributed both across goods and across sellers. The payoff of a bidder who wins seller j's object when his realized valuation is v_j^i and makes a payment of t^i, is $v_j^i - t^i$.

Initially, bidders are uninformed about their valuations of the two goods, which are ex ante identical. However, each seller generates private random signals correlated with bidders' true valuations of his object. Although the sellers control how informative their signals are, they cannot observe the private signals received by each bidder. For simplicity, I consider the class of

binary truth-or-noise information structures[3] where bidders either perfectly learn the truth about their valuation of the object or remain uninformed about them. Formally, let (s_1^i, s_2^i) be a realization of a vector of independent and identically distributed random signals (S_1^i, S_2^i), with each random variable following the common distribution function F. For each bidder i, the realized signal s_j^i is informative only about this bidder's valuation of seller j's good, and this bidder i is the only one who observes the realization of the vector of random signals (S_1^i, S_2^i). When seller 1 (resp. seller 2) decides to release information, a bidder who has observed the pair (s_1^i, s_2^i) learns the truth about his valuation of seller 1's good (resp. seller 2's good), i.e., $v_1^i = s_1^i$ (resp. $v_2^i = s_2^i$) with probability one. Alternatively, if seller 1 (resp. seller 2) decides not to release information, then the bidder treats the signal coming from seller 1 (resp. seller 2) as an indistinguishable draw, independently distributed from V_1^i (resp. V_2^i), coming from the distribution $F(\cdot)$. That is, a seller who releases information lets every bidder privately learn their true valuation of the object whereas a seller who does not release information leaves the bidders only with the public pool of information contained in the prior distribution of the valuations whenever he decides not to release information. In what follows, superscripts are omitted whenever they are clear from the context.

The game begins with sellers simultaneously deciding on whether or not to release information. With the exception of section 5.2 where I study an extension of the model in which sellers also post reserve prices, it is assumed throughout the paper that both objects are always sold. That is, unless otherwise noted, releasing information is the only decision that sellers have to make. Upon observing the sellers' decisions, each buyer independently and privately draws a signal from each seller and chooses one and only one seller as trading partner. Since the realization of the signals are observed only by the bidders, their estimates about the valuations of the goods remain bidders' private information. After bidders have chosen on which auction to bid, each seller collects the bids and awards the good to the bidder who submitted the highest bid at a price equal to the second highest bid. To simplify the analysis I assume that once in an auction bidders bid truthfully any estimate

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[3]The truth–or–noise information technology was introduced by Lewis & Sappington (1994) in the context of a monopolist who must decide on how much information to let potential buyers learn about his product.

that they have[4]. The behavior of any non participant is treated as equivalent to the behavior of a non serious bidder who participates in auction 1 with probability one.

This paper focuses on equilibria in which bidders use symmetric participation rules. A participation rule is symmetric if two bidders with the same vector of signals visit the sellers with the same probabilities. This is not the only plausible continuation equilibria of the game. There are asymmetric continuation equilibria that can be used to support equilibria in which only one seller releases information. I will defer to section 6 a more thoughtful discussion about this issue as well as the existence of some alternative continuation equilibria.

4. Equilibrium Analysis

This section provides a characterization of the equilibrium set of the game. Consistent with subgame perfection, the equilibria of the bidders' participation game taking sellers' decisions as given is characterized first. As a consequence of the binary nature of sellers' decisions, this characterization is organized around three classes of subgames that arise depending on whether both, one or none of the sellers reveals information. Then, this equilibrium participation rule is used to characterize the equilibrium set of the resulting normal form game played between the sellers.

4.1. Bidders' Participation Game

Without loss of generality, take the perspective of bidder 1. Suppose that this bidder decides to participate in auction j, $j = 1, 2$. From McAfee (1993), the probability that bidder 1 wins item j must be written as the sum of the probability that no one else visits the auction plus the probability that any other participant who comes to the auction does so with a valuation lower than bidder 1's valuation of item j. Since bidding is truthful and valuations are independently distributed both across bidders and across sellers, this probability must be the same for any two bidders whose signals about seller j's good are the same and hence, bidder 1's payoff if bidding in auction j must be a function of her signal about seller j's good and not on her signal

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[4]Truthful bidding is a common assumption in the literature of competing auction models (e.g. Peters & Severinov (1997) or Virag (2010)) and it is made in order to focus on the analysis of the bidders' participation problem where they have to choose trading partners.

about the other seller's good. Let $\mathcal{U}_j(s_j; \pi)$ be bidder 1's payoff when this bidder observes the signal s_j about seller j's good, and expects the other bidders to use the array of participation rules $\pi := \{\pi^k\}_{k \neq 1}$. A necessary and sufficient condition for the participation rule π' to be bidder 1's best response to π is:

$$\pi'(s_1, s_2) = \begin{cases} 0 & \text{if } \mathcal{U}_1(s_1; \pi) < \mathcal{U}_2(s_2; \pi) \\ 1 & \text{if } \mathcal{U}_1(s_1; \pi) > \mathcal{U}_2(s_2; \pi) \\ \in [0, 1] & \text{if } \mathcal{U}_1(s_1; \pi) = \mathcal{U}_2(s_2; \pi) \end{cases}$$

Consider any subgame in which no seller releases information. As releasing no information implies that all bidders share (and hence bid) the same estimate of both seller's good, winning either auction yields a positive payoff if and only if the winner is the unique participant in the auction. Moreover, as bidders do not coordinate on the selection of trading partners, bidder 1's best response to π must be to visit either seller with equal probability. It follows that the unique (symmetric) equilibrium of this subgame must be one in which every bidder visits each seller with probability equal to a half.

Next, consider any subgame in which only one seller releases information. For simplicity, let seller 1 be the seller who releases information. Since seller 2 has not released information, every bidder's estimate about this seller's good must be equal to μ and hence, bidder 1's payoff if bidding in auction 2 must be independent of bidder 1's signals. Alternatively, bidder 1's payoff if bidding in auction 1 must increase with the signal coming from seller 1 because a bidder with a higher signal about seller 1's good should trade more often with this seller than a bidder with a lower signal, as the first type is less likely to face high valuation competitors than the second type. Consequently, bidder 1's best response to π must take the form of a cutoff strategy, i.e., a strategy characterized by a cutoff value with the property that types whose signals from seller 1 are higher than this cutoff visit seller 1 for sure, otherwise they visit seller 2 for sure.

It is not difficult to show that this class of subgames always possesses an equilibrium in which every bidder uses the same cutoff value $s^* \in (0, 1)$. However, this is not the only plausible equilibrium. Provided that the distribution of bidders' valuations is convex then it is possible to show the existence of an equilibrium in which bidders use different cutoffs, similar to what oc-

curs in single–seller private value auctions with costly participation[5]. The intuition behind this result can be illustrated using the simple two–bidders case in which bidder 1 and bidder 2 use cutoffs s^1 and s^2 respectively, with $s^1 \leq s^2$. By construction, a type of bidder 1 who observes a signal about seller 1's good equal to s^1 must be indifferent between bidding in auction 1 or auction 2, which yields the following indifference condition for this type of bidder,

$$s^1 F(s^2) = \mu(1 - F(s^2))$$

The left–hand side of this expression gives the payoff that this type expects if bidding in auction 1 where this type wins the object if and only if bidder 2's signal about seller 1's good is below s^2. The right–hand side gives the payoff that this type expects if bidding in auction 2. Applying a similar reasoning for a type of bidder 2 with signal s^2, yields,

$$s^2 F(s^1) + \int_{s^1}^{s^2} (s_2 - t) f(t) dt = \mu(1 - F(s^1))$$

As expected, these expressions always admit a symmetric solution, i.e., a solution in which cutoff values are equal. To have an equilibrium in which $s^1 < s^2$, the payoff of type s^2 of bidder 2 when bidding in auction 1 must be greater than the payoff of type s^1 of bidder 1 when bidding in this same auction, i.e.,

$$(s^2 - s^1) F(s^2) - \int_{s^1}^{s^2} t f(t) dt > 0 \tag{1}$$

because $s^1 < s^2$ implies that bidder 1 must visit auction 2 less often than bidder 2 does. As conditional on winning, the winner obtains seller 2's good for free, and both bidders have the same valuation of this good (equal to

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[5]This resembles Proposition 2 in Tan & Yilankaya (2006) with the difference that instead of concavity, I require convexity to establish uniqueness. The difference arises because in the current model the bidders' decision to bid in some auction depends on the value of an option that is endogenous to the model –to enter the other auction– while this option is exogenously given in Tan & Yilankaya (2006)'s game.

μ), type s^2 of bidder 2 must expect a relatively larger payoff than that of type s^1 if they both bid in auction 2. Given the equality of payoffs that characterizes type s^2, this implies that this type must also expect a higher payoff in auction 1 relative to the payoff expected by type s^1 of bidder 1, which is exactly what expression (1) reflects. When $F(\cdot)$ is skewed towards the high end of the support, the expected payment made by type s^2 in auction 1 is large relative to the difference of the gross expected payoffs $(s^2 - s^1)F(s^2)$ because, conditional on him visiting, bidder 1 comes with high valuations. Hence, the convexity of $F(\cdot)$ is sufficient for this payment to be large enough to make expression (1) negative, violating expression (1) and ruling out the existence of asymmetric equilibria for this class of subgames.

Finally, consider any subgame in which both sellers release information. It is not difficult to show that apart from being a function of just the signal coming from the seller in which the bidder chooses to bid, a bidder's payoff is a continuous and increasing function of this seller's signal. Therefore, the existence of a bidder with signals (s_1, s_2) who is willing to visit seller 1 with positive probability suggests that any other bidder whose signals (s_1', s_2) are such that $s_1' > s_1$ should visit seller 1 with probability one, as her payoff in auction 1 must be strictly greater than her payoff in auction 2. Formally, for every value s_1 there must exist a number $\rho(s_1)$ such that $\mathcal{U}_1(s_1; \pi) \geq \mathcal{U}_2(\rho(s_1); \pi)$ whenever $s_1 \geq \rho(s_1)$. This gives a characterization of bidder 1's best response in terms of a cutoff strategy defined by a continuous and nondecreasing function ρ such that $\pi'(s_1, s_2) = 1$ if and only if $s_1 \geq \rho(s_1)$, and $\pi'(s_1, s_2) = 0$ if and only if $s_1 < \rho(s_1)$. This is important for at least two reasons. First, it shows that no matter what π may look like, every best response to it must take the form of a pure strategy in which the bidder also uses a continuous and nondecreasing function to decide on which auction to bid. Second, bidder's best response must be a mapping from the set of continuous and nondecreasing functions into itself and hence, questions about existence and uniqueness of a continuation equilibrium in subgames following histories in which both sellers release information become questions about existence and uniqueness of a fixed point of this mapping.

The characterization of bidder 1's best response in terms of cutoff functions holds regardless of whether or not the sellers post reserve prices (see section 5 for an extension of the model in which sellers compete in reserve prices). However, normalizing reserve prices to be equal simplifies the analysis because the (symmetric) cutoff function takes the simple form of the identity function. Intuitively, if bidder 1 with signals (s_1, s_2) expects the

other bidders to bid in auction 1 whenever their signals about seller 1's good are greater than their signals about seller 2's (i.e., bidder 1 expects the other bidders to use the cutoff function $\rho^*(s_1) = s_1$), then bidder 1's payoff if bidding in auction 1 is,

$$\mathcal{U}_1(s_1; \rho^*) = \int_0^{s_1} \left(\frac{1}{2} + \frac{F^2(t)}{2} \right)^{n-1} dt \tag{2}$$

while her payoff if bidding in auction 2 is,

$$\mathcal{U}_2(s_2; \rho^*) = \int_0^{s_2} \left(\frac{1}{2} + \frac{F^2(t)}{2} \right)^{n-1} dt \tag{3}$$

because the probability of winning seller 1's (resp. seller 2) object is equal to the probability that any other bidder visits seller 2 (resp. seller 1) plus the probability that conditional on visiting this seller the other bidder does so with a signal about seller 1's (resp. seller 2's) good lower than s_1 (resp. s_2). One can readily see that $\mathcal{U}_1(s_1; \rho^*) \geq \mathcal{U}_2(s_2; \rho^*)$ if and only if $s_1 \geq s_2$, from where it follows that bidder 1's best response is to also use the identity function ρ^*.

The following Proposition summarizes our discussion so far.

Proposition 4.1. *The participation rule $\pi^* : [0,1]^2 \longrightarrow [0,1]$ is the unique symmetric equilibrium of the continuation game in which bidders select trading partners if and only if:*

1. *In any subgame where no seller releases information, $\pi^*(s_1, s_2) = {}^1\!/_2$ for all $(s_1, s_2) \in [0,1]^2$.*
2. *In subgames where only one seller releases information, there exists a value $s^* \in (0,1)$ such that bidders visit the seller who releases information with probability one if and only if their signals are greater or equal than s^*, and visit the other seller if and only if their signals are lower than s^*. The value s^* is given by the unique solution to:*

$$s^*[F(s^*)]^{n-1} - \mu[(1 - F(s^*)]^{n-1} = 0 \tag{4}$$

3. *In subgames where both sellers release information, every bidder with signals (s_1, s_2) visits seller 1 with probability one iff $s_1 \geq s_2$, and visits seller 2 with probability one iff $s_1 < s_2$, $(s_1, s_2) \in [0,1]^2$.*

4.2. The Sellers' Game

The continuation equilibrium described in Proposition 4.1 induces a 2×2 normal form game played by the sellers who choose whether or not to release information and whose payoffs depend on the probability of receiving a given number of visitors times the price that these visitors are expected to pay. Thus, to evaluate the sellers' incentives to release information it is necessary to determine how the seller's decisions affect the probability with which bidders visit the auctions and the distribution of bidders' valuations conditional on participation.

Consider the case in which both sellers release information. From Proposition 4.1, the equilibrium participation rule tells bidders to visit seller 1 for sure whenever their signals about seller 1's good are greater than their signals about seller 2's object. It is not difficult to see that this rule induces a probability of visiting seller 1 equal to a half, and a distribution of valuations conditional on participation equal to $F^2(\cdot)$. Let $V(1,1)$ be seller 1's payoff when both sellers release information. Then,

$$V(1,1) = \sum_{k=2}^{n} \binom{n}{k} \left(\frac{1}{2}\right)^n \mu_k \tag{5}$$

where,

$$\mu_k = \left[2k(k-1) \int_0^1 t \left[F^2(t)\right]^{k-2} (1 - F^2(t)) F(t) f(t) dt \right] \tag{6}$$

is the expected value of the second highest valuation when seller 1 is matched with exactly k bidders, $k = 1, \ldots, n$.

Alternatively, if only seller 2 released information, the probability of visiting seller 1 would be equal to $F(s^*)$ because the equilibrium participation rule indicates that bidders visit seller 1 with probability one if and only if bidders' signals about seller 2's good are below the cutoff value s^*. Moreover, the fact that seller 1 is not releasing information implies that every participant in the auction submits a bid equal to μ and hence, seller 1's expected price when only seller 2 releases information must be equal to $\max\{0, \mu\}$. Therefore, seller 1's payoff, $V(0,1)$, when only seller 2 releases information is,

$$V(0,1) = \sum_{k=2}^{n} \binom{n}{k} [F(s^*)]^k [1 - F(s^*)]^{n-k} \mu \tag{7}$$

Inspection of expressions (5), (6), and (7), suggests that $F(s^*) \leq \frac{1}{2}$ and $\mu \leq \mu_k$, $k = 2, \cdots, n$ suffice for $V(1,1)$ to be greater than $V(0,1)$ and hence, for seller 1's best response to be to release information when this seller conjectures that seller 2 will also release information. The first of these conditions ensures that expected traffic in auction 1 is higher if seller 1 releases information, and it is more likely to hold when the distribution of bidders' valuations concentrates relatively more values on its right tail. Intuitively, left–skewed distributions generally have means lower than their medians[6] and therefore, a cutoff above the median would imply a higher valuation and a higher probability of trading with seller 1 relative to the valuation and the probability of trading with seller 2 for a bidder with signal s_2 equal to s^*, contradicting the fact that this bidder is indifferent between the sellers. The next lemma provides a necessary and sufficient condition relating the mean and the median of $F(\cdot)$ that formalizes the above intuition.

Lemma 4.1. *Let s^* be the unique solution to Eq. (4) and m be the median of $F(\cdot)$. Then, $F(s^*) \leq \frac{1}{2}$ if and only if $\mu \leq m$, and $F(s^*) > \frac{1}{2}$ if and only if $\mu > m$.*

Most of the commonly used distribution functions in the auction theory literature are log–concave, which are a class of strongly unimodal distributions[7]. When $F(\cdot)$ is unimodal, the condition in lemma 4.1 can be reformulated in terms of $F(\cdot)$ and the density $f(\cdot)$ using corollary 1 in Basu & DasGupta (1997). According to this corollary, the mean of a unimodal random variable X is lower than its median if the density $f(\cdot)$ and c.d.f. $F(\cdot)$ of X satisfy $f(F^{-1}(t)) \leq f(F^{-1}(1-t))$ for all $0 < t < \frac{1}{2}$. Thus, if $F(\cdot)$ is a log–concave c.d.f. then $f(F^{-1}(s)) \leq f(F^{-1}(1-s))$ for all $0 < s < \frac{1}{2}$ suffices for traffic to increase with seller 1's decision to release information.

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[6]For unimodal distributions, Theorem 1 in Dharmadhikari & Joag-dev (1983) shows that the mean of the distribution must lie below its median whenever the probability of drawing a signal above the median is at least as high as the probability of drawing a signal below it.

[7]For a complete characterization of log–concave distribution functions, see An (1998).

As previously noted, seller 1's decision to release information induces a traffic and a price effect, which can be estimated by comparing $F(s^*)$ with $^{1}/_{2}$, and μ_k with μ. Lemma 4.1 provides a simple condition that can be used to estimate the traffic effect. As for the price effect, the fact that μ and μ_k are expected values of random variables with different but related cumulative distribution allows to focus on the relationship between $F(\cdot)$ and the distribution of the second highest valuation conditional on participation, $F^2(\cdot)$. Furthermore, it can be shown that the family of distribution functions of the second highest valuation can be ordered in the first–order stochastic sense according to the number of bidders k, with the distribution of the second highest valuation when seller 1 is matched with $k+1$ bidders dominating the distribution when this sellers is matched with k bidders, $k = 2, \ldots, n$[8]. Thus, an estimate of the price effect can be obtained by comparing the expected value of the second highest valuation when $n = 2$ with μ.

Lemma 4.2. *Suppose that, in addition to being strictly positive, the density function $f(\cdot)$ satisfies either of the following two conditions: (i) $f'(s) \geq 0$ for all $s \in [0,1]$; or (ii) $f'(1) \geq 0$ and $\frac{d}{ds}\left(\frac{f'(s)}{f(s)}\right) \leq 0$, $s \in [0,1]$. Let μ_k be the expected value of the second highest valuation when seller 1 is matched with k bidders in a subgame in which both sellers release information. Then, $\mu_k \geq \mu$ for all $k = 2, \cdots, n$.*

The intuition behind this result is as follows. By releasing information, seller 1 induces a continuation equilibrium that leads bidders whose signals about seller 1's good are higher than their signals about seller 2's good to participate in seller 1's auction for sure. This means that bidders who end up bidding in auction 1 are those with a better match between their personal tastes and the characteristics of the seller's good. However, it might be the case that even those bidders who value seller 1's good the most have relatively low valuations. The conditions in the lemma ensures that this does not occur and hence, that seller 1 always expects to see, on average, bidders with higher valuations relative to the valuation of the average bidder in unconditional terms.

The following Theorem formalizes the discussion so far and presents the main result of the paper.

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[8]Recall that whenever X first–order stochastically dominates Y then $\mathbb{E}[X] \geq \mathbb{E}[Y]$. See Shaked & Shantikumar (1994).

Theorem 4.1. *Consider the normal form game induced by the continuation equilibrium given by Proposition 4.1 in which sellers simultaneously choose whether or not to release information. Then, $F(\cdot)$ convex is sufficient for the existence of a unique symmetric equilibrium in which both sellers release information. Alternatively, having a distribution function with a mean lower than its median, and whose density function satisfies $f'(1) \geq 0$ and $\frac{d}{ds}\left(\frac{f'(s)}{f(s)}\right) \leq 0$, $s \in [0,1]$, is also sufficient for the existence of a unique symmetric equilibrium in which both sellers release information.*

Notice that Theorem 4.1 guarantees not only existence but also uniqueness of an equilibrium in which both sellers release information. Uniqueness follows from the observation that, by releasing information against a competitor who is expected not to release information, a seller induces a continuation equilibrium in which bidders select this seller's auction based on a strictly positive cutoff value s^* determined by Eq. (4). Thus, the seller's decision to release information induces screening of high-valuation types as only bidders with valuations above s^* visit the auction. When the mean of $F(\cdot)$ is lower than its median, this screening favours the release of information because participants in the auction of the seller who releases information come with valuations that are never lower than μ, and visit this seller's auction with a probability that is never lower than a half. Consequently, the seller expects a payoff that must be higher than the payoff that he would obtain by not releasing information, making the release of information a dominant strategy for the sellers. Theorem 4.1 also suggests that some of the findings in the literature (Board, 2009; Ganuza & Penalva, 2010) showing that providing information is suboptimal for a monopolist who faces two bidders is not robust to the introduction of competition between auctioneers. The reason is that the interaction between information and competition affects not only bidders' willingness to pay but also their participation decisions, giving information the potential to alter both the number and the types of bidders who visit the auctions.

As previously discussed, corollary 1 in Basu & DasGupta (1997) allows the reformulation of the condition requiring the mean of $F(\cdot)$ to be lower than its median directly in terms of the density function $f(\cdot)$ for unimodal distribution functions. The following corollary follows from Theorem 4.1 and this observation.

Corollary 4.1. *Suppose that in addition to condition (ii) in lemma 4.2, the density function satisfies $f(F^{-1}(s)) \leq f(F^{-1}(1-s))$ for all $0 < s <$*

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$^1/_2$. *Then, the game possesses a unique symmetric equilibrium in which both sellers release information.*

Although interesting, Theorem 4.1 (and Corollary 4.1) only provides sufficient conditions for the existence of an equilibrium in which both sellers release information. When this sufficient condition does not hold, it is possible to construct games that only have an equilibrium in which sellers do not release information. To better illustrate this possibility, consider the two–bidders case with a distribution of bidders' valuations given by the truncation of the exponential distribution function with parameter $\lambda = 2$, $F(x) := \frac{G(x)-G(0)}{G(1)-G(0)}$, where $G(x) = 1 - \exp^{-2x}$. Given the strict concavity of $F(\cdot)$, $\mu > m$ and $f'(1) < 0$. Simple calculations[9] shows that seller 1's payoff when both sellers do not release information is equal to $V(0,0) = 0.2090$ whereas his payoff had he released information is equal to $V(1,0) = 0.0750$. Hence, not releasing information is a best response to not releasing information and thus, not releasing information must be an equilibrium of the game. Moreover, as this seller's profit when both sellers release information, $V(1,1)$, is strictly lower than his profits when he does not release information, $V(0,1)$, this equilibrium must be unique[10].

The existence of equilibria in which no seller releases information originates from a screening effect that is too weak to overcome the negative traffic effect that results from seller 1's decision of releasing information[11]. Nevertheless, intuition suggests that this traffic effect should become less important the greater the number of bidders because a greater number of bidders means a greater pool of participants in both auctions. Moreover, the price that the seller expects to receive when releasing information should increase with n (it is the expected value of a second order statistics), making the price effect more important than the traffic effect the greater the number of bidders. To gain a better understanding about the role played by the number of bidders, consider the difference between seller 1's payoffs when this seller releases and

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[9] All calculations have been performed using the open code software wxMaxima v11.08.0 (http://maxima.sourceforge.net/).

[10] Seller 1's payoff when both sellers release information is $V(1,1) = 0.1902$ whereas seller 1's profit if he is the unique seller not releasing information is $V(0,1) = 0.3672$.

[11] Although the game always has an equilibrium, this equilibrium may involve the use of mixed strategies. Such situation may occur when the price effect is not strong enough relative to the traffic effect to support an equilibrium with information provision, but not too weak to induce sellers not to release information.

when he does not release information, for the two alternative strategies that seller 2 may adopt. First, consider a seller 2 who is expected not to release information and denote by $\Delta V^{NI}(n) = V(1,0) - V(0,0)$ the difference between seller 1's payoff when he releases information and his payoff when he does not release information, conditional on having a total of n bidders in the market,

$$\Delta V^{NI}(n) :=$$

$$\sum_{k=2}^{n} \binom{n}{k} [1 - F(s_n^*)]^k [F(s_n^*)]^{n-k} \hat{\mu}_k - \left(1 - \left(\frac{1}{2}\right)^n - n\left(\frac{1}{2}\right)^n\right)\mu$$

where s_n^* is the cutoff value that solves Eq. (4) when the number of bidders is equal to n, and $\hat{\mu}_k$ is the expected price when seller 1 is matched with $k = 2, \ldots, n$ bidders, and the distribution of bidders' valuations conditional on participation is $G(s) = \frac{F(s) - F(s_n^*)}{1 - F(s_n^*)}$. Likewise, let $\Delta V^I(n) = V(1,1) - V(0,1)$ be the difference between seller 1's payoff when he releases information and his payoff when he does not release information, given that seller 2 releases information and there are n bidders in the market,

$$\Delta V^I(n) :=$$

$$\sum_{k=2}^{n} \binom{n}{k} \left(\frac{1}{2}\right)^n \mu_k - \left(1 - (1 - F(s_n^*))^n - nF(s_n^*)(1 - F(s_n^*))^{n-1}\right)\mu$$

because bidders visit seller 1 when this seller does not release information if and only if their signals about seller 2's good are below the cutoff value.

A reasonable conjecture is that there exists a threshold number of bidders n^* such that $\Delta V^{NI}(n) > 0$ and $\Delta V^I(n) > 0$ whenever $n \geq n^*$. Intuitively, this should hold true provided that the cutoff value does not increase with n as otherwise one would have to compare second–order statistics coming from distributions with increasing truncation points[12]. If $s_n^* \geq s_{n+1}^*$ then $\hat{\mu}_k$

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[12] Menezes & Monteiro (2000) analyze a model with a single auctioneer in which bidders' participation is endogenous but the cost of participating is given exogenously. They show that seller's payoff may increase or decrease as n grows large because the truncation point increases with the number of potential bidders.

should be larger than μ for some sufficiently large value of k because the expected value of this second order statistic would be an increasing function of k (from lemma 4.2 $\hat{\mu} \geq \mu$ for all $k = 2, \ldots, n$). Moreover, as s_n^* does not increase with n, the probability with which bidders visit seller 1 when this seller releases information and seller 2 does not, increases with n while the visiting probability when seller 1 does not release information but seller 2 does, cannot increase with n. Thus, for a sufficiently large number of bidders the price effect should dominate the traffic effect regardless of the decision adopted by seller 2, suggesting that the existence of equilibria in which no seller releases information can only happen when the number of bidders is small.

Proposition 4.2. *Consider the normal form game induced by the continuation equilibrium given by Proposition 4.1 in which sellers simultaneously choose whether or not to release information. Then, there exists a threshold number of bidders n^* such that this game has a unique equilibrium in which both sellers release information provided that the number of bidders is greater than n^*.*

A related question to that addressed by Proposition 4.2 is how these differences in payoffs compare with the difference in payoffs that a monopolist would obtain in this market. Intuitively, a competing seller should have weaker incentives to release information compared to those of a monopolist. A competing seller who faces a competitor who releases information trades with bidders more often than the monopolist does and hence, gives away informational rents more often than the monopolist. As the profit of this competing seller when he does not release information tends to μ as n increases, one should expect that for a sufficiently large value of n, the difference in payoffs for the monopolist overcomes the difference in payoffs for the competing seller. Nevertheless, the same need not be true when the competing seller faces a competitor who does not release information. This stems from the fact that a competing seller who faces a competitor who does not release information has to give away a strictly positive participation rent to the bidder whose valuation is equal to the cutoff value, and this rent may grow at a faster or a slower rate than the positive difference in payoffs that results from comparing the seller's and the monopolist's payoffs when both do not release information. For instance, if bidders' valuations are assumed to be uniformly distributed on $[0, 1]$, then the monopolist's difference in payoffs

from releasing and not releasing information is higher than the corresponding difference in payoffs of a competing seller who faces a competitor who releases information provided that there are four or more bidders in the market[13]. Nevertheless, a complete different conclusion arises in the case of a competing seller facing a competitor who does not release information. In this case, the monopolist's difference in payoffs is strictly lower than the corresponding competing seller's difference regardless of the number of bidders in the market.

5. Extensions

Theorem 4.1 is interesting because it provides some support to the idea that competition between sellers increases informational efficiency relative to the monopoly. However, it relies on two simplifying assumptions that make the benchmark model highly stylized. In the first place, sellers are restricted to only choosing between granting full access to information about their products and preventing any access to it. A natural question is whether this finding would persist even if intermediate degrees of informativeness are considered. In the second place, sellers are not allowed to post reserve prices. Yet, competing auction models, as well as standard monopolistic models, usually involve the sellers' decisions about reserve prices. In this section I discuss some of the implications of relaxing these two assumptions and explore the extent to which our previous findings persist in these more general environments.

5.1. Information Structures

Consider an extended version of the benchmark model in which the sellers can choose the probability with which a bidder's private signal matches this bidder's true valuation about the seller's good. Formally, let each seller choose an information structure indexed by p such that every bidder i whose signal about this seller's good is equal to s_i learns that $v^i = s^i$ with probability p, and that s_i is pure noise independently drawn from the common prob-

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[13]It is possible to show that the monopolist's difference in payoffs is strictly higher than the competing seller's one when this seller faces a competitor who does release information provided that there is a sufficiently large number of bidders in the market.

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ability distribution F with complementary probability $(1-p)^{14}$. This simple generalization of the binary truth–or–noise information technology used in the benchmark model induces a distribution of valuations conditional on p, $G(\cdot; p)$, given by,

$$G(\omega; p) = \begin{cases} 0 & \text{if } \omega < \underline{\omega}_j \\ F\left(\frac{\omega - (1-p)\mu}{p}\right) & \text{if } \underline{\omega}_j \leq \omega \leq \overline{\omega}_j \\ 1 & \text{if } \omega > \overline{\omega}_j \end{cases} \tag{8}$$

with $[\underline{\omega}, \overline{\omega}] = [(1-p)\mu; p+(1-p)\mu]$ the support of bidders' posterior valuations. As can readily be seen, the family of distribution functions $\{G(\cdot; p)\}_p$ can be ordered according to their degrees of informativeness p using second–order stochastic dominance as the ordering criterion with $G(\cdot; p')$ dominating $G(\cdot; p)$ in the second–order stochastic sense whenever $p' > p$. Thus, this information technology captures in a simple way the idea that releasing information affects the dispersion but not the mean of bidders' posterior valuations[15].

As expected, the equilibria in subgames following histories in which both sellers choose uninformative information structures and in subgames following histories in which only one of them chooses an informative structure, are straightforward extensions of parts (1) and (2) of Proposition 4.1 (after appropriate substitutions of $F(\cdot)$ by $G(\cdot; p)$, and the support $[0, 1]$ by $[\underline{\omega}, \overline{\omega}]$). Nevertheless, the same reasoning does not apply when characterizing the

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[14]In order to avoid issues about equilibrium existence, I assume that p belongs to a finite subset $\mathcal{P}$ of the $[0, 1]$ interval that includes at least one element different from zero and one.

[15]Let $p' > p$ and define $\alpha(t) = \int_0^t \{G(\omega; p') - G(\omega; p)\} \, d\omega$. Then, $\alpha(0) = \alpha(1) = 0$, $\frac{d}{dt}\alpha(t) = 0$ at $t = \mu$, $\frac{d}{dt}\alpha(t) < 0$ for $t < \mu$, and $\frac{d}{dt}\alpha'(t) > 0$ for $t > \mu$. Hence, $\alpha(t) \leq 0 \ \forall t$ and $G(\cdot; p')$ dominates $G(\cdot; p)$ in the second–order stochastic sense. Moreover, let $\omega(s_j, p)$ be the posterior valuation of a bidder with a signal equal to s_j when the bidder has observed a degree of informativeness p. Applying the law of iterated expectations to $\omega(\cdot, p)$ yields,

$$\mathbb{E}\left(\omega(s_j, p)\right) = \mathbb{E}\left(ps_j + (1-p)\mu\right)$$
$$= \mu$$

for all $p \in \mathcal{P}$.

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equilibria of subgames following histories in which both sellers choose positive degrees of informativeness. Although any equilibria of these subgames can be completely characterized by a nondecreasing and continuous function ρ with the property that any bidder with posterior valuations (ω_1, ω_2) visit seller 1 with probability one whenever $\omega_1 \geq \rho(\omega_2)$ (else the bidder visits seller 2 for sure), the question about the existence of such function is far more complicated to answer than it is in the benchmark model. In this last model, the argument –which I borrow from Troncoso-Valverde (2012)– is based on the observation that there is a zero–measure set of types which is indifferent about which seller to visit, and that the lowest such type is the one whose valuations are equal to reserve prices. In contrast, when sellers can choose intermediate values of p, this lowest type is not so easy to determine, as the following example shows. Suppose that seller 1 announces some strictly positive degree of informativeness lower than one, and seller 2 grants full access to information (i.e., chooses a p equal to one). Then, a bidder who observes a pair of signals (s_1, s_2) both equal to zero knows with certainty that the valuation of seller 2's good must be equal to zero because the signal coming from seller 2 reveals the truth with probability one. However, this same bidder will estimate a strictly positive posterior valuation of item 1 because there is a strictly positive probability that the signal is pure noise. Consequently, the lowest indifferent type must be one who observes a strictly positive signal about seller 2's good and not the $(0,0)$ type as would be the case in the benchmark model. This makes it much harder to determine the lowest indifferent type, and hence, the construction of the bidder's best response mapping, much harder than in the benchmark case. Fortunately, such characterization is available (see Theorem 4 in Troncoso-Valverde (2014)) which, together with the finiteness of the sellers' action spaces, allows us to claim existence of Perfect Bayesian Equilibrium in which bidders use symmetric participation rules.

Proposition 5.1. *Consider the extension of the benchmark game in which sellers simultaneously choose, from a finite set, the probability with which signals about their goods reveal the truth. Then, there exists a threshold number of bidders n^* such that for all $n \geq n^*$ the probability with which any seller chooses an uninformative structure in equilibrium is nil.*

Proposition 5.1 shares some of the flavour of Theorem 4.1 and Proposition 4.2 in the sense that it rules out the existence of equilibria in which

both sellers choose uninformative structures with positive probability, provided that there is a sufficiently large number of bidders in the market. In light of Theorem 4.1, having a mean lower than the median is sufficient to rule out the existence of an equilibrium in pure strategies where both sellers choose uninformative structures. Proposition 5.1 extends this argument to the cases in which sellers choose uninformative structure with positive probability. Provided that either the mean of $F(\cdot)$ is lower than the median, or there is a sufficiently large number of bidders, seller 1 can ensures a higher payoff by choosing $p = 1$ in response to $p = 0$ by seller 2. Moreover, as the expected price, obtained when seller 1 chooses the same degree of informativeness as that of seller 2, increases as the number of bidders increases, seller 1 can also guarantee a higher payoff by mimicking the choice of seller 2 instead of choosing an uninformative structure. Consequently, provided that there is a sufficiently large number of bidders in the market, choosing $p = 0$ is never a best response to any degree of informativeness chosen by seller 2 and hence, there cannot be an equilibrium in which a seller chooses $p = 0$ with positive probability.

5.2. Reserve Prices

Consider an extended version of the benchmark model in which sellers can decide on whether or not to grant full access to information as well as on what reserve prices to post. The inclusion of reserve prices as a second strategic variable introduces similar modeling issues as those arising in the model where sellers are allowed to choose intermediate degrees of informativeness, mainly about the existence and characterization of the equilibria in subgames following histories in which sellers post different reserve prices. To keep tractability, I will assume that reserve prices belong to a finite subset $[c, 1]$, $0 < c < \mu$, of the $[0, 1]$ interval, with c being the sellers' common production cost. This assumption will allow to claim existence of a continuation equilibrium in any class of subgames –which is guaranteed by the existence of a strictly positive lower bound on the set of reserve prices, see Theorem 3.3. in Troncoso-Valverde (2012), and hence existence of an equilibrium for the whole game.

It is not surprising that the characterization of the equilibrium participation rule resembles that used in the benchmark case, as the latter model can be seen as a special case of the former one when reserve prices are equal. Intuitively, for values of s_1 not too high (and reserve prices below one), bidder 1's best response operator can be characterized by some value $\rho'(s_1)$ such

that the type $(s_1, \rho'(s_1))$ is indifferent about which seller to visit. However, the fact that sellers can now post different reserve prices may lead to the existence of some subset of values of s_1 such that any type with signal about seller 1's good within this subset, strictly prefers one seller over the other. Lemma 3.4 in Troncoso-Valverde (2012) shows that even if such subset of types exists, the characterization of the bidder's best response operator can still be described by an integro-differential equation that must hold everywhere with respect to s_1 within some non–empty subset of $[0, 1]$, plus an initial condition given by the indifference of the bidder whose signals are equal to the posted reserve prices.

In a recent paper, Ganuza & Penalva (2014) have shown that the monopolist has incentives to post more restrictive reserve prices as the number of bidders increases (see their corollary 5). The intuition behind this result is simple: as a higher number of bidders means higher incentives to release information, the monopolist uses higher reserve prices to reduce the informational rents that this information generates. However, for this intuition to work, bidders cannot have an alternative to the monopolist's auction. When such alternative exists, a bidder with a valuation close to the seller's reserve price may find optimal to visit the alternative auction whenever doing so yields a higher expected rent. In a nutshell, this traffic (or market share) effect is the key behind the no surplus extraction result of competing auction models with homogeneous goods (Virag, 2010). However, in the current case there is horizontal differentiation in all subgames in which at least one seller releases information. In particular, any subgame that follows a history in which both sellers release information gives rise to a competing auction game with heterogeneous preferences. Troncoso-Valverde (2012) has shown that in such games, increasing a reserve prices not only affects the participation decisions of bidders with valuations close to the reserve price but also affects the participation decisions of high–valuation bidders who were indifferent before the change in the reserve price took place. This lost of high–valuation bidders imposes a cost to the seller who now observes a reduction in traffic and a reduction in expected price due to the drop of high–valuation bidders who stop visiting his auction. The next result confirms this intuition and shows that, contrary to the monopolist's case, as the number of bidders increase the unique equilibrium of the game is one in which sellers release information and post reserve prices equal to production costs.

Proposition 5.2. *Consider the extended game in which sellers can choose*

between granting or not access to information about the characteristics of their goods and to post reserve prices from a finite subset $[c, 1]$, $0 < c < \mu$. There exists a threshold number of bidders n^* such that if $n > n^*$, the unique equilibrium of the game is one in which both sellers grant full access to information and post reserve prices equal to sellers' production cost c.

Proposition 5.2 offers some insights about the link between competition and reserve prices in the presence of information provision. Apart from showing that competition weakens the sellers' incentives to extract surplus, this proposition also shows that competition does not necessarily mean softer price competition as it occurs in oligopoly models such as that of Damiano & Li (2007). As increasing (or decreasing) a reserve price has an effect on the participation decision of low and high–valuation types, a higher reserve price not only reduces the number of visits of low–valuation bidders but also the number of visits of high–valuation bidders who now prefer to visit the lower reserve price seller with probability one. Thus, increasing the reserve price reduces not only traffic but also expected price whereas in price competition models a higher price only affects traffic.

6. Discussion and Conclusions

This paper studies the incentives that two competing auctioneers face when deciding whether or not to grant access to bidders about information of their goods. I develop a simple two–stage model in which sellers choose to grant or prevent access to information in the first stage, and bidders simultaneously choose trading partners in the second one, and show that for many distribution functions (such as the uniform or the normal distribution), this game possesses a unique symmetric equilibrium in which sellers release all available information. I also consider an extension of the baseline model in which sellers can choose the probability with which signals reveal the truth about bidders' valuations. Similar to the baseline case, I find that for a sufficiently large number of bidders, the probability with which any seller chooses an uninformative structure in equilibrium is nil. When sellers can use reserve prices as a second strategic variable in the model, I find that the game has an equilibrium in which sellers release information and post reserve prices equal to production costs provided that there is a sufficiently large number of bidders in the market. Thus, the main finding of the paper is to show how competition between auctioneers may improve informational

efficiency relative to monopoly even in cases where sellers can use reserve prices to extract (part of) the surplus that information generates.

The model presented in this paper is highly stylized, but it provides a simple framework in which to study the relation between competition and the seller's incentives to release information in competing auctions. The assumption of two sellers makes the characterization of the equilibrium in subgames where sellers release information tractable because it simplifies the associated fixed–point problem to a single integro–differential equation that results from the indifferent condition that must hold for some zero-measure subset of types. Adding more sellers not only increases the number of these integro–differential equations but also raises their complexity. Although it is possible to show existence of an equilibrium in this more complicated case, the complexity of its characterization obscures the analysis of the sellers' incentives to release information, which is the core of this paper.

The assumption about the stochastic structure of signals and valuations play a central role in our analysis. Firstly, I exploit the stochastic independence of valuations across bidders to obtain a tractable expression for the indifference condition that characterizes the participation rule used by bidders in equilibrium. Letting bidders' valuations be correlated is likely to make the problem intractable because bidders' payoffs can no longer be written as the integral of the respective trading probability, a property that I use to establish equilibrium existence in subgames where both sellers release information. Secondly, the stochastic independence of valuations across sellers, an assumption that aims to capture in the simplest possible way the idea that information is the only source of differentiation in the model, is used to rule out the possibility that bidders use the information released by some seller to make inferences about their valuations of the other seller's good. Notice this is consistent with the private value assumption when applied to our competitive environment as bidders cannot learn anything about their valuations of a good unless the seller offering this good releases information.

This last assumption, together with the common distribution function used to draw valuations, imposes strong symmetry across bidders because it forces them to share the exact same estimate of the two sellers' goods when no information is released, even though these bidders have different and completely independent preferences. Similar to Eső & Szentes (2007), I could, in principle, consider environments in which bidders have private information before any interaction with the sellers, and where sellers can use additional information to refine these initial estimates. The main difficulty with this

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approach is the characterization of sellers' best responses as every equilibrium of the bidders' stage game is now described in terms of cutoff functions for which no closed-form solution is available. An outstanding conjecture is that similar results to the ones in this paper could be obtained after some restrictions on the structure of the distribution functions of valuations and signals. However, I believe that the current setup provides a simpler and more transparent environment to analyze the problem of information provision in competing auction.

There are some alternative equilibria to the symmetric equilibria used to described the continuation play in the bidders' stage game. Some of these continuation equilibria can be used to support equilibria in which only one seller releases information, at least when the number of bidders is small. For instance, consider the three–bidder case in which the distribution of bidders' valuations is uniform. It is straightforward to check that the following participation rule induces a Bayes-Nash equilibrium in every subgame: (i) in subgames where no seller releases information, bidder 1 visits seller 1, and bidders 2 and 3 visit seller 2 for sure; (ii) in subgames where one seller releases information, bidders use the common cutoff $s = 1/2$; (iii) in subgames where both sellers release information, bidders use the identity function as cutoff function. This continuation equilibrium can be used to support an equilibrium where seller 1 releases information and seller 2 does not. First, releasing information must be seller 1's best response when seller 2 does not release information because this action yields a strictly positive payoff instead of a payoff equal to zero, which is what the seller obtains by not releasing information. As for seller 2, he expects a payoff equal to a half (the mean of the uniform distribution) if he does not release information because he is sure to trade (seller 2 is visited bidders 2 and 3 with probability one in the continuation game). On the other hand, if seller 2 releases information, his payoff falls to 0.2858 since he now expects a positive price –equal to the expected value of the second highest valuation– only when two or more bidders visit. Hence, the fact that seller 2 expects no visitor with positive probability when he releases information leads him to respond by not releasing information whenever seller 1 is expected to release information.

The previous example exploits the fact that bidders sort themselves deterministically in subgames where no seller releases information in such a way that one of the sellers receives a single visitor. This seller is clearly better off releasing information since this yields a positive payoff with strictly positive probability whereas the other seller cannot improve his payoff by

releasing information because traffic falls too much to be compensated by a higher expected price. However, these continuation equilibria are likely to require some sort of coordination among bidders, which presumably is not what one expects to see in markets with trading frictions. Furthermore, the fact that bidders sort themselves deterministically in subgames where some seller releases information is not a consequence of any of our assumptions but a property of the equilibrium set of these subgames. In this sense, the symmetry assumption can be seen as a simplifying assumption that helps focus on a more decentralized type of equilibrium in subgames where no information is released.

Appendix A. Proofs

Lemma Appendix A.1. *Let $\bar{\pi}^i$ be a bidder i's best response strategy to π^{-i}. There exists a value $s^{*i} \in [0,1]$ and a strategy π^{*i} such that for any pair of signals (s_1, s_2) observed by bidder i, $\pi^{*i}(s_1, s_2) = 1$ if and only if $s_1 \geq s^{*i}$, and $\pi^{*i}(s_1, s_2) = 0$ if and only if $s_1 < s^{*i}$, and such that π^{*i} is also a best response to π^{-i} in any subgame in which only one seller releases information.*

Proof. Without loss of generality, suppose that seller 1 releases information. Let $Q_1(s_1, s_2; \pi^{-i})$ be bidder i's reduced–form probability of trading with seller 1 when bidder i's signals are (s_1, s_2) and other bidders use the array of participation rules π^{-i}. Since bidding is truthful and signals are i.i.d. both across bidders and across sellers, $Q_1(s_1'; \pi^{-i}) := Q_1(s_1, s_2; \pi^{-i}) = Q_1(s_1, s_2'; \pi^{-i})$ for all s_2 and s_2' in $[0, 1]$. Hence, bidder i's expected payoff if bidding in auction 1 can be written as a function of signal s_1 alone,

$$\mathcal{U}_1(s_1; \pi^{-i}) = \int_0^{s_1} Q_1(t; \pi^{-i}) dt$$

where $\mathcal{U}_1(0; \pi^{-i}) = 0$ irrespective of π^{-i}. On the other hand, every bidder has a valuation of seller 2's good equal to μ because seller 2 does not release information. Furthermore, the probability of trading with seller 2 can only depend on the number of visitors in the auction because every bidder values seller 2's good the same. Thus, bidder i's expected payoff if bidding in auction 2 must be independent of her signals and $\mathcal{U}_2(s_1, s_2; \pi^{-i}) = \mathcal{U}_2(s_1', s_2'; \pi^{-i})$ for all (s_1, s_2) and (s_1', s_2') in $[0, 1]^2$. Let $\bar{\mathcal{U}}_2(\pi^{-i})$ denote bidder i's payoff if bidding in auction 2. Clearly, $\bar{\mathcal{U}}_2(\pi^{-i}) \geq 0$. Define $\Psi(s_1; \pi^{-i})$ as follows:

$$\Psi(s_1; \pi^{-i}) := \mathcal{U}_1(s_1; \pi^{-i}) - \bar{\mathcal{U}}_2(\pi^{-i})$$

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It is almost immediate that $\Psi'(\cdot; \pi^{-i}) := Q_1(\cdot; \pi^{-i}) > 0$ for all $s_1 \in (0, 1)$, and hence, Ψ is strictly increasing with respect to s_1 for any given π^{-i}. Suppose that $\bar{\mathcal{U}}_2(\pi^{-i}) = 0$. Then, $\Psi(0; \pi^{-i}) := \mathcal{U}_1(0; \pi^{-i}) = 0$ and $\Psi(1; \pi^{-i}) := \mathcal{U}_1(1; \pi^{-i}) > 0$. Therefore, the strategy $\pi^{*i}(s_1, s_2) = 1$ iff $s_1 \geq s^* = 0$, and $\pi^{*i}(s_1, s_2) = 0$ iff $s_1 < s^* = 0$, $s_2 \in [0, 1]$, must be bidder i's best response to π^{-i}. If $\Psi(s_1; \pi^{-i}) \leq 0$ for all $s_1 \in [0, 1]$ then the strategy $\pi^{*i}(s_1, s_2) = 1$ iff $s_1 \geq s^* = 1$, and $\pi^{*i}(s_1, s_2) = 0$ iff $s_1 < s^* = 1$, $s_2 \in [0, 1]$, must be bidder i's best response to π^{-i}. In all other cases, continuity of Ψ plus the facts that $\Psi(0; \pi^{-i}) < 0$, $\Psi(1; \pi^{-i}) > 0$ and $\Psi'(s_1) > 0$, $s_1 \in (0, 1)$, ensures the existence of a unique value $s^{*i} \in (0, 1)$ such that the strategy $\pi^{*i}(s_1, s_2) = 1$ iff $s_1 \geq s^{*i}$, and $\pi^{*i}(s_1, s_2) = 0$ iff $s_1 < s^{*i}$, is bidder i's best response to π^{-i}, $s_2 \in [0, 1]$. The value of s^{*i} is uniquely determined by $\Psi(s^{*i}; \pi^{-1}) = 0$. □

Lemma Appendix A.2. *Let $\hat{\pi}$ be a bidder i's best response strategy to the symmetric participation rule π. Then, there exists a participation rule π^* and a nondecreasing and continuous function $\rho : [0, 1] \to \mathbb{R}$ with the property that $\pi^*(s_1, s_2) = 1$ iff $s_2 \leq \rho(s_1)$, and $\pi^*(s_1, s_2) = 0$ iff $s_2 > \rho(s_1)$, and such that π^* is also a best response to π in any subgame in which both sellers release information.*

Proof. Let $Q_j(s_j; \pi)$ be the reduced–form probability with which bidder i with signal s_j about seller j's good trades with this seller in any subgame where both sellers release information, and all other bidders use the symmetric participation rule π, and $\mathcal{U}_j(\cdot; \pi)$ be the corresponding reduced–form payoff. A standard incentive compatibility argument implies that:

$$\mathcal{U}_j(s_j; \pi) - \mathcal{U}_j(\hat{s}_j; \pi) \geq Q_j(\hat{s}_j; \pi)(s_j - \hat{s}_j)$$

Clearly, the right–hand side of this expression is strictly positive so long as $s_j > \hat{s}_j$ because $Q_j(\hat{s}_j; \pi) > 0$ as there is a strictly positive probability that conditional on visiting, bidders come with valuations strictly lower than $\hat{s}_1$. Therefore, $\mathcal{U}_j$ must be strictly increasing with respect to s_j, and $\frac{d\mathcal{U}_1(s_1; \pi)}{ds_1} = Q_1(s_1; \pi)$. Furthermore, as $Q_j(\cdot; \pi)$ is monotonic, it is Riemann integrable. Therefore,

$$
\begin{aligned}
\mathcal{U}_j(s_j; \pi) &= \mathcal{U}_j(0; \pi) + \int_0^{s_j} Q_j(\xi; \pi)d\xi \\
&= \int_0^{s_j} Q_j(\xi; \pi)d\xi
\end{aligned}
$$

because $\mathcal{U}_j(0;\pi) = 0$. This function is continuous (because its derivative is Lebesgue integrable) for all $s_j \in [0,1]$ and hence, bounded. Let $I_1 = [0,\bar{u}_1]$ and $I_2 = [0,\bar{u}_2]$ be the compact image of $\mathcal{U}_1(\cdot;\pi)$ and $\mathcal{U}_2(\cdot;\pi)$ on $[0,1]$ respectively. Clearly $I_1 \cap I_2 \neq \emptyset$ because $\mathcal{U}_j(0;\pi) = 0 \in I_j$, $j = 1,2$, regardless of π. Hence, suppose that $I_1 \subseteq I_2$. From the intermediate value theorem, it is possible to assign to every $s_1 \in [0,1]$ another number $\rho(s_1) \in [0,1]$ such that $\mathcal{U}_1(s_1;\pi) = \mathcal{U}_2(\rho(s_1);\pi)$. If I_2 is a proper subset of I_1, then this assignment can be done for every s_1 in $[0,\bar{s}_1]$, where $\bar{s}_1$ is implicitly defined by $\mathcal{U}_1(\bar{s}_1;\pi) = \mathcal{U}_2(1;\pi)$. For values of s_1 greater than $\bar{s}_1$, let $\rho(s_1) = 1$. It is immediate that this function ρ mapping numbers from the $[0,1]$ interval into $[0,1]$ can be used to define bidder i's best response strategy as follows:

$$\pi^{*i}(s_1,s_2) = \begin{cases} 1 & \text{iff } s_2 < \rho(s_1) \\ \in [0,1] & \text{iff } s_2 = \rho(s_1) \\ 0 & \text{iff } s_2 > \rho(s_1) \end{cases}$$

because $\mathcal{U}_2$ increasing with respect to s_2 implies that $\mathcal{U}_2(s_2;\pi) \geq (<)\mathcal{U}_2(\rho(s_1);\pi) = \mathcal{U}_1(s_1;\pi)$ whenever $s_2 \geq (<)\rho(s_1)$, $s_1 \in [0,\bar{s}_1]$, and $\mathcal{U}_1(s_1;\pi) > \mathcal{U}_2(\rho(s_1);\pi)$ whenever $s_1 > \bar{s}_1$. □

Lemma Appendix A.3. *Consider any subgame in which both sellers release information and let T be bidder i's best response mapping defined as follows:*

$$T\rho(s_1) = \max\{s_2 \in [0,1] : \mathcal{U}_2(s_2;\rho) \leq \mathcal{U}_1(s_1;\rho)\}$$

where $\mathcal{U}_1(s_1;\rho)$ and $\mathcal{U}_2(v_2;\rho)$ are bidder i's payoffs if bidding in auction 1 and auction 2 respectively, when other bidders use a symmetric strategy characterized by the cutoff function ρ. Then, the participation rule π^* induces a Nash equilibrium in any subgame where both sellers release information if and only if the function ρ^* used to characterize π^* in subgames where both sellers release information is a fixed point of T.

Proof. First, suppose that ρ^* is a fixed point of T and consider the strategy π^* that prescribes the following participation rule in any subgame where both sellers release information,

$$\pi^*(s_1,s_2) = \begin{cases} 1 & \text{iff } s_2 \leq \rho^*(s_1) \\ 0 & \text{iff } s_2 > \rho^*(s_1) \end{cases}$$

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$(s_1, s_2) \in [0, 1]^2$. Since ρ^* is a fixed point of T, $T\rho^*(s_1) = \rho^*(s_1)$ for every $s_1 \in [0, 1]$, and $\mathcal{U}_2(\rho^*(s_1); \rho^*) \leq \mathcal{U}_1(s_1; \rho^*)$, $s_1 \in [0, 1]$. It is readily seen that π^* must induce a Nash equilibrium in any subgame where seller release information provided that $\mathcal{U}_2(\rho^*(s_1); \rho^*) = \mathcal{U}_1(s_1; \rho^*)$ holds for every $s_1 \in [0, 1]$. Hence, suppose that there is some s_1 such that $\mathcal{U}_2(\rho^*(s_1); \rho^*) < \mathcal{U}_1(s_1; \rho^*)$. Then $\rho^*(s_1)$ must be equal to one as otherwise there would be some $\tilde{s}_2$ such that $\rho^*(s_1) < \tilde{s}_2 < 1$ and $\mathcal{U}_2(\rho^*(s_1); \rho^*) < \mathcal{U}_2(\tilde{s}_2; \rho^*) \leq \mathcal{U}_1(s_1; \rho^*)$, contradicting the fact that $\rho^*(s_1) = T\rho^*(s_1)$ is the highest such number. Therefore, it must be true that $s_2 \leq \rho^*(s_1)$ and $\mathcal{U}_2(s_2; \rho^*) \leq \mathcal{U}_2(\rho^*(s_1); \rho^*)$ for all $s_1 \in [0, 1]$ (because $\mathcal{U}_2$ is increasing in s_2), from where it follows that the participation rule π^* defined above induces a Nash equilibrium in any subgame where both sellers release information.

Second, suppose that the participation rule π^* defined above induces a Nash equilibrium in any subgame where both sellers release information. From lemma Appendix A.2, the set of bidder i's best responses to π^* must contain a strategy π' that can be characterized by a nondecreasing and continuous function ρ' such that $\pi'(s_1, s_2) = 1$ iff $s_2 \leq \rho'(s_1)$ and $\pi'(s_1, s_2) = 0$ iff $s_2 > \rho'(s_1)$. Therefore, π^* and π' must be payoff–equivalent in the class of subgames where both sellers release information and hence, $\rho^*(s_1) = \rho'(s_1)$ for all $s_1 \in [0, 1]$ (otherwise, there would be some type $(\hat{s}_1, \hat{s}_2)$ with $\hat{s}_2$ strictly between $\rho^*(\hat{s}_1)$ and $\rho'(\hat{s}_1)$ who would prefer to visit seller 1 with a different probability from that prescribed by π^*). Moreover, the function ρ^* must be the highest number satisfying $\mathcal{U}_2(\rho^* s_1; \rho^*) \leq \mathcal{U}_1(s_1; \rho^*)$ because otherwise there would exist some type (s_1, s_2') with $\rho^*(s_1) < s_2' < 1$ who would prefer to visit seller 1 with probability one, contradicting the fact that π^* induces a Nash equilibrium. Consequently, the function ρ^* must necessarily satisfy $T\rho^*(s_1) = \rho^*(s_1)$ for all $s_1 \in [0, 1]$, which implies that ρ^* must be a fixed point of T. $\qquad \square$

Lemma Appendix A.4. *For every continuous and nondecreasing cutoff function ρ, there exists a number $\bar{s}_1$ satisfying $0 < \bar{s}_1 \leq 1$, such that $\mathcal{U}_2(\rho(s_1); \rho) = \mathcal{U}_1(s_1; \rho)$ for all $s_1 \in [0, \bar{s}_1]$.*

Proof. Let $\bar{s}_1 = \sup\{s_1 : \mathcal{U}_1(s_1; \rho) \leq \mathcal{U}_2(1, \rho)\}$. Clearly, $\bar{s}_1 \in [0, 1]$ because $\mathcal{U}_2(1; \rho) \in [0, 1]$, and $\mathcal{U}_1$ is a continuous and increasing satisfying $\mathcal{U}_1(0; \rho) = 0$ regardless ρ. Moreover, $\bar{s}_1 > 0$ as otherwise, $\mathcal{U}_2(1, \rho) = 0$ and hence, $\mathcal{U}_2(s_2; \rho) = 0$ for all $s_2 \in [0, 1]$ because $\mathcal{U}_2(\cdot; \rho)$ is increasing in s_2. However, expression (3) reveals that this is possible only if the term inside the integral is zero, which cannot happen because by lemma Appendix A.2 the function

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ρ is nondecreasing and continuous in s_1. Therefore, $\bar{s}_1 > 0$ and the set $[0, \bar{s}_1]$ is nonempty. Hence, from the intermediate value theorem there must exist a number $s_2^* \in [0, 1]$ such that for every $s_1 \in [0, \bar{s}_1]$, $\mathcal{U}_1(s_1; \rho) = \mathcal{U}_2(s_2^*; \rho)$. As $\mathcal{U}_2$ is continuous and increasing with respect to $s_2 \in (0, 1)$, and $\mathcal{U}_1$ does not depend on s_2, the number s_2^* must be unique. Hence, $T\rho(s_1) = s_2^*$ because $T\rho(s_1)$ delivers the maximum such number and thus, $\mathcal{U}_2(T\rho(s_1); \rho) = \mathcal{U}_1(s_1; \rho)$ for all $s_1 \in [0, \bar{s}_1]$ as claimed. □

Proof of Proposition 4.1. The proof of part (1) is straightforward and hence omitted. To prove part (2) of the Proposition, let $\pi^i(\cdot, \cdot) = \pi^k(\cdot, \cdot) \equiv \pi(\cdot, \cdot)$, $i \neq k$ such that bidders use a symmetric participation rule. Then, the mapping $\Psi(s; \pi)$ defined in lemma Appendix A.1 becomes $\Psi(s; \pi) := sF(s)^{n-1} - \mu(1 - F(s))^{n-1}$ and bidder i's best response in any subgame where only one seller releases information can be characterized by a common cutoff value s^* that is the unique solution to,

$$sF(s)^{n-1} - \mu(1 - F(s))^{n-1} = 0$$

To prove part (3), consider any subgame in which both sellers release information. It is readily seen that the function $\rho^*(s_1) = s_1$ described in the main text is a fixed point of bidder i's best response T given in lemma Appendix A.3. Therefore, the strategy characterized by the function ρ^* must induce a Nash equilibrium in this class of subgames. Thus, all that remains is to show that the function ρ^* is unique. To do so, suppose that contrary to this statement, there is another function ρ' such that $T\rho' = \rho'$ and $\rho'(s_1) \neq \rho^*(s_1)$ for s_1 within some nonempty interval Ω. From lemma Appendix A.4, there exists a value s_1' such that $\mathcal{U}_2(\rho'(s_1); \rho') = \mathcal{U}_1(s_1; \rho')$ for all $s_1 \in [0, s_1']$ implying that the slopes of these two functions must also be equal within this interval. Let $Q_j(\cdot; \rho')$ denote the probability with which bidder i trades with seller j when the other bidders use the symmetric strategy characterized by the cutoff function ρ'. Then, the slope of ρ' at any $s_1 \in (0, s_1')$ must be equal to

$$\frac{d\rho'(s_1)}{ds_1} = \frac{Q_1(s_1; \rho')}{Q_2(\rho'(s_1); \rho')}$$

Suppose that $\Omega = [0, s_1']$ and that $\rho'(s_1) > \rho^*(s_1)$ (the same argument applies

if $\rho'(s_1) < \rho^*(s_1)$. Then $\frac{s\rho'(s_1)}{ds_1} < 1$ within this interval. However,

$$\rho'(s_1) = \int_0^{s_1} \frac{d\rho'(t)}{dt} dt$$
$$< \int_0^{s_1} dt$$
$$= \rho^*(s_1)$$

if $s_1 \in \Omega$, a contradiction. Second, suppose that Ω is a strict subset of $[0, s_1']$. As $\rho'(0) = \rho^*(0) = 0$ and ρ' is continuous and nondecreasing function (because it is a fixed point of T), Ω must take the form $(0, \hat{s}_1)$, where $\hat{s}_1$ is the value at which the function ρ' crosses the 45 degree line (i.e., it crosses the function ρ^*). Clearly $\hat{s}_1 < s_1'$. For simplicity, suppose that $\rho'(s_1) < \rho^*(s_1)$ if $s_1 \in (0, \hat{s}_1)$, and $\rho'(s_1) > \rho^*(s_1)$ if $s_1 \in (\hat{s}_1, 1)$. Then,

$$\left. \frac{d\rho'(s_1)}{ds_1} \right|_{s_1 = \hat{s}_1} = \frac{Q_1(\hat{s}_1; \rho')}{Q_2(\rho'(\hat{s}_1); \rho')}$$
$$= \left(\frac{1 - \int_{\hat{s}_1}^1 F(\rho'(t)) f(t) dt}{F(\rho'(\hat{s}_1)) F(\rho'^{-1}(\rho'(\hat{s}_1))) + \int_{\rho'^{-1}(\rho'(\hat{s}_1))}^1 F(\rho'(t)) f(t) dt} \right)^{n-1}$$

because $\hat{s}_1 \in [0, s_1']$. However,

$$\left. \frac{d\rho'(s_1)}{ds_1} \right|_{s_1 = \hat{s}_1} < \left(\frac{1 - \int_{s_1}^1 F(\rho^*(t)) f(t) dt}{F(\rho^*(s_1)) F(\rho^{*-1}(\rho^*(s_1))) + \int_{\rho^{*-1}(\rho^*(s_1))}^1 F(\rho^*(t)) f(t) dt} \right)^{n-1}$$

because $\rho'(s_1) > \rho^*(s_1)$ when $s_1 > \hat{s}_1$, and $\rho'(\hat{s}_1) = \rho^*(\hat{s}_1)$. Thus, ρ' must cut the 45 degree line from above, which contradicts the fact that $\rho'(s_1) < \rho^*(s_1)$ if $s_1 < \hat{s}_1$. Therefore, $\rho^*(s_1) = s_1$ for all $s_1 \in [0, 1]$ must be the unique fixed point of T. $\qquad\square$

Proof of Lemma 4.1. Suppose that $\mu \leq m$. As $F(\cdot)$ is increasing, then $F(\mu) \leq \frac{1}{2}$. Let $\psi(s) := sF(s)^{n-1} - \mu(1 - F(s))^{n-1}$. It is immediate that $\psi'(\cdot) > 0$ and hence, ψ is an increasing function of $s \in (0, 1)$, and that $\psi(s^*) = 0$. To show sufficiency, observe that

$$\psi(\mu) = \mu F(\mu)^{n-1} - \mu(1 - F(\mu))^{n-1}$$
$$= \mu \left(F(\mu)^{n-1} - (1 - F(\mu))^{n-1} \right)$$
$$\leq 0$$
$$= \psi(s^*)$$

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because $F(\mu) \leq 1/2$ implies that $F(\mu)^{n-1} - (1 - F(\mu))^{n-1} \leq 0$, and,

$$\psi(m) = mF(m)^{n-1} - \mu(1 - F(m))^{n-1}$$
$$= (m - \mu)\left(\frac{1}{2}\right)^{n-1}$$
$$\geq 0$$
$$= \psi(s^*)$$

because $\mu \leq m$. Therefore, $\mu \leq s^* \leq m$ and $F(s^*) \leq 1/2$. To show necessity, suppose that s^* satisfies $\mu \leq s^* \leq m$. Since $\psi(s)$ is increasing, we must have that $\psi(\mu) \leq \psi(s^*) \leq \psi(m)$, from where it follows that $\psi(\mu) \leq 0$. This in turn implies that the ratio $\frac{F(\mu)}{1-F(\mu)}$ must be less than or equal to one, which is equivalent to $F(\mu) \leq \frac{1}{2}$. The proof of the case where $\mu > m$ is similar and hence, omitted. $\square$

Proof of Lemma 4.2. Consider any subgame in which both sellers release information. Let $G(s)$ denote the probability that a bidder who has chosen to participate in auction 1 has a valuation lower than s. Then, $G(s) := F^2(s)$, $s \in [0, 1]$, because bidders who visit auction 1 do so provided that their valuations of seller 1's good are greater than their valuations of seller 2's good. Let $H_k(s)$ be the cumulative distribution function of the second highest valuation conditional on seller 1 being matched with exactly k bidders, $k = 2, \ldots, n$,

$$H_k(s) = [F^2(s)]^k + k[F^2(s)]^{(k-1)}[1 - F^2(s)], \tag{A.1}$$

and μ_k be the expected value of the second highest valuation when seller 1 is matched with k bidders,

$$\mu_k = \int_0^1 t dH_k(t)$$
$$= 1 - \int_0^1 H_k(t)dt$$

where the second line follows from integration by parts. It is not difficult to see that $H_{k+1}(s)$ dominates $H_k(s)$ in the first–order stochastic sense, $k = 3, \ldots, n$, and hence, $\mu_{k+1} \geq \mu_k$. Then, in order to prove the lemma it is sufficient to show that $\mu_2 > \mu$.

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Let $u := F(s)$, $du = f(s)ds$, $\alpha(u) := F^{-1}(u)$, with F^{-1} the (right) inverse of F, and $\alpha'(u) := \frac{d}{du}\alpha(u) = \frac{1}{f(s)}$. Then,

$$\mu_2 - \mu = \int_0^1 (F(s) - 2F^2(s) + F^4(s))ds$$

$$= \int_0^1 (u - 2u^2 + u^4)\alpha'(u)du$$

Let $\beta(t) = \int_0^t (u + u^4 - 2u^2)du$, $t \in [0,1]$, and $\alpha''(u) = \frac{d^2\alpha(u)}{du^2} = -\frac{f'(s)}{f^2(s)}$, $u = F(s)$. Integration by parts yields,

$$\mu_2 - \mu = \alpha'(s)\beta(s)|_0^1 - \int_0^1 \beta(u)\alpha''(u)du$$

$$= \beta(1)\alpha'(1) - \int_0^1 \beta(u)\alpha''(u)du$$

It is readily seen that $\beta(t) \geq 0$ for all $t \in [0,1]$. Furthermore, $\alpha'(1) = \frac{1}{f(1)} > 0$ and hence, $\beta(1)\alpha'(1) > 0$. If $f'(\cdot) > 0$ then $\alpha''(u)$ is nonpositive, and the integral term is nonnegative. Consequently, the whole expression is greater than zero provided that $f'(\cdot) \geq 0$. Alternatively, suppose that $f'(1) \geq 0$ and $f(\cdot)$ is log–concave. Consider the the second (integral) term of the above expression. Plugging $u := F(s)$ and $\alpha''(u)du = -\left(\frac{f'(s)}{f^2(s)}\right)f(s)ds$ back into this term gives,

$$-\int_0^1 \beta(u)\alpha''(u)du = \frac{1}{3}\int_0^1 (6F^5(s) - 20F^3(s) + 15F^2(s))\left(\frac{f'(s)}{f(s)}\right)ds$$

Let $\lambda(s) = \int_0^s (6F^5(t) - 20F^3(t) + 15F^2(t))dt$, and $l(s) = \frac{d}{ds}\left(\frac{f'(s)}{f(s)}\right)$. Integrating by parts once again yields,

$$-\int_0^1 \beta(u)\alpha''(u)du = \left(\frac{f'(1)}{f(1)}\right)\left(\int_0^1 \lambda(x)dx\right) - \int_0^1 \left(\int_0^s \lambda(t)dt\right)l(s)ds$$

It is not difficult to see that $\lambda(s)$ is nonnegative for all $s \in [0,1]$. Hence, the first term in the above expression is nonnegative because $f'(1) \geq 0$ and $f(\cdot) > 0$. Furthermore, as $f(\cdot)$ is log–concave then $l(s) = \frac{d}{ds}\left(\frac{f'(s)}{f(s)}\right) \leq 0$ and the integral term is nonpositive. Consequently, $\mu_2 \geq \mu$ whenever $f'(\cdot) \geq 0$ or $f'(1) \geq 0$ and f is log–concave. $\qquad\square$

Proof of Theorem 4.1. Without loss of generality, take seller 1's perspective. First, suppose that seller 1 expects seller 2 not to release information. By not releasing information seller 1 induces a continuation equilibrium in which each bidder visits with probability a half, and bid an amount equal to μ. By releasing information, this seller induces a continuation equilibrium in which a bidder visits seller 1's auction if the bidder's signal is above the common cutoff value s^* that solves equation (4). From lemma 4.1, $\mu \leq m$ is sufficient for this cutoff value to satisfy $\mu \leq s^* \leq m$. Therefore, $1 - F(s^*) \geq \frac{1}{2}$ because $F(s^*) \leq \frac{1}{2}$, and conditional on visiting every bidder does so with a valuation that is never lower than μ. This implies that both traffic and price increase when seller 1 releases information compared to those that seller 1 would obtain by not releasing information. Second, suppose that seller 1 expects seller 2 to release information. By releasing information, seller 1 induces a continuation equilibrium in which each bidder visits his auction with probability equal to a half because proposition 4.1 ensures that a visit occurs whenever the bidder's signal about seller 1's good is greater than or equal to the bidder's signal about seller 2's good. Moreover, the expected price must be greater than μ because the density function satisfies one of the conditions of lemma 4.2 and thus, traffic cannot fall and price increases when seller 1 releases information in response to a seller 2 who does not release information. Therefore, releasing information must yield seller 1 a higher payoff than not releasing information regardless of the action adopted by seller 2's, which implies that the unique symmetric equilibrium must be one in which both sellers release information. □

Proof of Proposition 4.2. Let $G_1(s_1)$ be the probability that a bidder with signals (s_1, s_2) who participates in auction 1 trades with seller 1 in any sub-game where both sellers release information,

$$G_1(s_1) = \left(\frac{1}{2} + \frac{F^2(s_1)}{2} \right)$$

Then, seller 1's payoff when he and his competitor release information can be written as follows:

$$V_1(1,1) = n(n-1) \int_0^1 sG_1^{n-2}(s)(1 - G(s))dG(s)$$

Define $H(s)$ by:

$$H_1(s) = G_1^n(s) + nG_1^{n-1}(s)(1 - G_1(s))$$

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Then, $dH_1(s) = n(n-1)G_1^{n-2}(s)(1-G(s))$ and thus,

$$V_1(1,1) = 1 - \int_0^1 H_1(t)dt$$

after integration by parts. Since $G^n(s)$ and $nG_1^{n-1}(s)$ tends to zero when n tends to infinity, there must exist a n_0 such that

$$H_1(s) - F(s) = G_1^n(t) + nG_1^{n-1}(s)(1-G_1(s)) - F(s)$$
$$\leq 0$$

$s \in [0,1]$, provided that $n > n_0$. Thus,

$$V_1(1,1) - \mu \;\; > 1 - \int_0^1 H_1(s)ds - \left(1 - \int_0^1 F(s)ds\right)$$
$$= \int_0^1 \{F(s) - H_1(s)\}\, ds$$
$$> 0$$

if $n > n_0$. Since seller 1's payoff when he is the only seller who does not release information is bounded above by μ, releasing information must be the best response of this seller to information by seller 2.

Suppose that seller 2 is expected not to releases information. If $\mu \leq m$ holds, both traffic and expected price must be higher when seller 1 releases information than when this seller does not release information because the continuation equilibrium of any subgame in which only seller 1 releases information is characterized by some common cutoff value s^* that satisfies $\mu \leq s^* \leq m$. Hence, consider the case in which $\mu > m$ holds. As bidders visit seller 1's auction one provided that their valuations are above a cutoff value s^*, the probability with which any visitor with signals (s_1, s_2) trades with seller 1 must be equal to $F(s_1)$. Thus, seller 1's payoff when he is the only seller releasing information, $V_1(1,0)$, can be written as,

$$V_1(1,0) = 1 - s^* \tilde{H}_1(s^*) - \int_{s^*}^1 \tilde{H}_1(s)ds$$

where $\tilde{H}_1(s) = F^n(s) + nF^{n-1}(s)(1-F(s))$, $s \in [s^*, 1]$.

Lemma Appendix A.5. *Suppose that $\mu > m$ holds and let s_n^* denote the unique solution to Eq. (4) when the number of bidders is equal to n. Then, $s_n^* > s_{n+1}^*$.*

Proof. Rewrite Eq. (4) as follows:

$$s_n^* \varphi(s_n)^{n-1} = \mu$$

where $\varphi(s_n^*) = \left(\frac{F(s_n^*)}{1-F(s_n^*)}\right)$. Contrary to the statement in the lemma, suppose that $s_n^* \leq s_{n+1}^*$ for some $n \geq 2$. From lemma 4.1 $\mu > m$ implies $F(s_n^*) > {}^1\!/_2$ for all n, from where it follows that $\varphi(s_n) > 1$ and $\varphi(s_n^*)^{n-1} < \varphi(s_n^*)^n$ for any n, $n \geq 2$. Furthermore, since $\varphi(s)$ is increasing in s on $[0,1]$, $\varphi(s_n^*)^n \leq \varphi(s_{n+1}^*)^n$. Therefore,

$$\mu = s_n^* \varphi(s_n^*)^{n-1} < s_{n+1}^* \varphi(s_{n+1}^*)^n = \mu$$

a contradiction. $\qquad\square$

As $\frac{dV_1(1,0)}{ds^*} < 0$, lemma Appendix A.5 guarantees that seller 1's payoff when he is the only seller releasing information can only increase with the number of bidders. Let n_1 be such that,

$$F(s) - F^n(s) - nF^{n-1}(s)(1 - F(s)) \geq 0$$

holds for $s \in [0,1]$, $n > n_1$. Then, if $n > n_1$,

$$
\begin{aligned}
V_1(1,0) - \mu &= 1 - s^* \tilde{H}_1(s^*) - \int_{s^*}^1 \tilde{H}_1(s)ds - 1 + \int_0^1 F(s)ds \\
&= \int_0^{s^*} F(s)ds - s^* \tilde{H}_1(s^*) + \int_{s^*}^1 \left\{F(s) - \tilde{H}_1(s)\right\} ds \\
&= \int_0^{s^*} \left\{F(s) - \tilde{H}_1(s^*)\right\} ds + \int_{s^*}^1 \left\{F(s) - \tilde{H}_1(s)\right\} ds \\
&\geq 0
\end{aligned}
$$

and releasing information must also be seller 1's best response to a seller 2 that does not release information. Set $n_2 = 2$ if $\mu \leq m$, and $n_2 = n_1$ if $\mu > m$ and let $n^* = \max\{n_0, n_2\}$. Then, releasing information must be a strictly dominant strategy for each seller whenever $n > n^*$, and thus the game must have a unique equilibrium in which both sellers release information if $n > n^*$. $\qquad\square$

Proof of Proposition 5.1. Let $V(p, p')$ be seller 1's expected payoff when he and seller 2 choose the information structure indexed by p and p' respectively.

Suppose that seller 2 chooses $p' = 0$ with positive probability. Then,

$$V(0,0) = \left(1 - (n+1)\left(\frac{1}{2}\right)^n\right)\mu$$

and,

$$V(p,0) = \overline{\omega} - \omega^* Z(\omega^*; p) - \int_{\omega^*}^{\overline{\omega}} Z(\omega; p)d\omega$$

where $Z(\omega; p) = G^n(\omega; p) + nG^{n-1}(\omega; p)(1 - G(\omega; p))$, $G(\cdot; p)$ is given by Eq. (8), ω^* is the cutoff value that solves $\omega^* G^{n-1}(\omega^*; p) = \mu(1 - G(\omega^*; p))^{n-1}$, and $\underline{\omega} \leq \omega \leq \overline{\omega}$, $\underline{\omega} = (1-p)\mu$, $\overline{\omega} = p + (1-p)\mu]$. It is not difficult to show that $\mu \leq m$ implies $\mu \leq \omega^* \leq m$ and hence, $V(p,0) > V(0,0)$. Therefore, suppose that $\mu > m$. Set $p = 1$ such that $G(\cdot; 1) = F(\cdot)$, and define n_3 to be the threshold number of bidders such that for all $n > n_3$,

$$F(s) - F^n(s) - nF^{n-1}(s)(1 - F(s)) > 0$$

for all $s \in [0,1]$. Then,

$$\begin{aligned}
V(1,0) - \mu &= 1 - s^* Z(s^*) - \int_{s^*}^{1} Z(s)ds - 1 + \int_{0}^{1} F(s)ds \\
&= \int_{0}^{s^*} F(s)ds - s^* Z(s^*) + \int_{s^*}^{1} \{F(s) - Z(s)\}\, ds \\
&= \int_{0}^{s^*} \{F(s) - Z(s^*)\}\, ds + \int_{s^*}^{1} \{F(s) - Z(s)\}\, ds \\
&> 0
\end{aligned}$$

and $V(1,0) > \mu \geq V(0,0)$ whenever $n > n_3$. Next, suppose that seller 2 chooses $p' > 0$ with positive probability. By setting $p = 0$, seller 1 attracts bidders whose valuations (of seller 2's good) are below the cutoff value ω^* that solves $\omega G^{n-1}(\omega^*; p') = \mu(1 - G(\omega^*; p'))^{n-1}$. Then,

$$V_1(0, p') = \left(1 - (1 - G(\omega^*; p'))^n - nG(\omega^*; p')(1 - G(\omega^*; p'))^{n-1}\right)\mu$$

By mimicking seller 2's choice (i.e., by choosing $p = p'$), seller 1 expects a payoff of,

$$V_1(p', p') = n(n-1)\int_{\underline{\omega}_1}^{\overline{\omega}_1} \omega \left(\frac{1}{2} + \frac{G^2(\omega; p')}{2}\right)^{n-2}\left(\frac{1}{2} - \frac{G^2(\omega; p')}{2}\right)G(\omega; p')dG(\omega; p')$$

because $p = p'$ implies that bidders use the identity function as the cutoff function (the proof of this claim is similar to the proof of part 3 in Theorem 4.1 with $G(\cdot; p')$ playing the role of $F(\cdot)$). Using a change of variable, rewrite $V(p', p')$ as follows:

$$V(p', p') = n(n-1)\frac{1}{p'}\int_0^1 (p's + (1-p')\mu)H(s)^{n-2}(1 - H(s))F(s)f(s)ds$$

where $H(s) = \left(\frac{1}{2} + \frac{F^2(s)}{2}\right)$. Write $Z(s) = H^n(s) + nH^{n-1}(s)(1 - H(s))$. Then,

$$V(p', p') = \int_0^1 t dZ(t) + \left(\frac{1-p}{p}\right)\mu\int_0^1 dZ(t)$$

Let n_4 be defined such that

$$Z^n(s) + nZ^{n-1}(s)(1 - Z(s)) - F(s) < 0$$

for every $s \in [0, 1]$ whenever $n > n_4$. Then,

$$V(p', p') - \mu = \left(1 - \int_0^1 Z(t)dt\right) + \left(\frac{1-p}{p}\right)\mu\int_0^1 dZ(t) - \left(1 - \int_0^1 F(t)dt\right)$$

$$= \int_0^1 (F(t) - Z(t))dt + \left(\frac{1-p}{p}\right)\mu\int_0^1 dZ(t)$$

$$> 0$$

if $n > n_4$. Hence, $V(p', p') > \mu \geq V(0, p')$ provided that the number of bidders is above n_3. Thus, it is never optimal for seller 1 to put positive probability on $p = 0$ if seller 2 chooses $p' > 0$ for a sufficiently large n. Since this also holds when seller 2 chooses $p' = 0$ with positive probability, it must be the case that in equilibrium sellers never put positive probability on the uninformative structure provided that n is greater than $\max\{n_3, n_4\}$. □

Proof of Proposition 5.2. Consider any subgame in which seller 1 and seller 2 release information and post reserve prices $r_1 \in [c, 0]$ and $r_2 \in [0, 1]$ respectively. From Proposition 3.1 and Theorem 3.3 in Troncoso-Valverde (2012), the equilibrium participation rule can be described by a continuous and non-decreasing cutoff function $\rho : [0, 1] \to [0, 1]$ defined by:

$$\rho(s_1) = \begin{cases} \min\{1, r_2\} & \text{if } \max\{r_1, r_2\} = 1 \\ \varphi(s_1) & \text{if } \max\{r_1, r_2\} < 1 \end{cases}$$

where,

$$\varphi(s_1) = \begin{cases} r_2 & \text{if } s_1 < r_1 \\ \min\{z(s_1); 1\} & \text{if } s_1 \geq r_1 \end{cases}$$

and the function z solves the following equation:

$$\frac{d}{dt} z(t) = \left(\frac{1 - \int_t^1 F(z(\hat{t})) f(\hat{t}) d\hat{t}}{F(z(t)) F(t) + \int_t^1 F(z(\hat{t})) f(\hat{t}) d\hat{t}} \right)^{n-1} \qquad t \in [r_1, 1]$$

with initial condition $z(r_1) = r_2$. Let $G(s_1; \rho)$ be the probability with which a bidder with a signal s_1 from seller 1 trades with this seller when reserve prices are $r = (r_1, r_2) \in [c, 1]^2$, and bidders use the cutoff function ρ,

$$G(s_1; \rho) = 1 - \int_{s_1}^1 F(\rho(t)) dF(t) \qquad s_1 \in [r_1, 1]$$

with $G(s_1; \rho) = G(r_1; \rho)$ whenever $s_1 \in [0, r_1)$. Then, seller 1's payoff is,

$$V(r_1, r_2; \rho) = \begin{cases} 0 & \text{if } r_1 = 1 \\ V^+(r_1, r_2; \rho) & \text{if } c \leq r_1 < 1 \end{cases}$$

where,

$$V^+(r_1, r_2; \rho) = n r_1 q \left(1 - q\right)^{n-1} + \int_{r_1}^1 t \, dZ(t; \rho) - c$$

$q = \int_{r_1}^1 F(\rho(t)) dF(t)$, and $Z(s_1; \rho) = G^n(s_1; \rho) + nG^{n-1}(s_1; \rho)(1 - G(s_1; \rho))$. Notice that $Z(r_1; \rho) = (1 - q)^n + nq(1 - q)^{n-1}$ and hence,

$$V^+(r_1, r_2; \rho) = r_1(Z(r; \rho) - (1 - q)^n) + \int_{r_1}^1 t \, dZ(t; \rho) - c$$

Integrating the second term by parts yields,

$$V^+(r_1, r_2; \rho) = 1 - r_1(1 - q)^n - \int_{r_1}^1 Z(t; \rho) - c$$

Let ρ^* be the cutoff function when seller 1 also posts a reserve price r_1 equal to c, and $\hat{\rho}$ be the function when seller 1 posts some reserve price $\hat{r}_1 \in$

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$(c, 1)$ (posting a reserve price equal to one cannot be part of an equilibrium). From Proposition 3.5 in Troncoso-Valverde (2012), the function ρ is nonincreasing with respect to seller 1's reserve price. Therefore, $\rho^*(s_1) \geq \hat{\rho}(s_1)$, $G(s_1; \hat{\rho}) \geq G(s_1; \rho^*)$, and $Z(s_1; \hat{\rho}) \geq Z(s_1; \rho^*)$ for all $s_1 \in [0, 1]$, with strict inequality for some nonempty subset of this interval. Therefore,

$$V(c, c; \rho^*) - V(\hat{r}_1, c; \hat{\rho}) =$$

$$\hat{r}_1(1 - \hat{q})^n - c(1 - q^*)^n - \int_c^{\hat{r}_1} Z(t; \rho^*)dt + \int_{\hat{r}_1}^1 \{Z(t; \hat{\rho}) - Z(t; \rho^*)\} \, dt$$

Since $Z(\cdot; \hat{\rho}) \geq Z(\cdot; \rho^*)$, with strict inequality for some subset of $[0, 1]$, the last term in this expression is nonnegative for all n. Consider the first integral term,

$$\int_c^{\hat{r}_1} Z(t; \rho^*)dt \leq (\hat{r}_1 - c)Z(\hat{r}_1; \rho^*)$$

because $Z(\cdot; \rho^*)$ is increasing. Let $\varphi(n) := c(1-q^*)^n + (\hat{r}_1 - c)Z(\hat{r}_1; \rho^*)$. It can be readily seen that $\varphi(n)$ grows to zero at a much faster rate than $r(1 - \hat{q})^n$. Therefore, there must exist some n_5 such that the difference of the first three terms is positive provided that $n > n_5$. Thus, $V(c, c; \rho^*) > V(\hat{r}_1, c; \hat{\rho})$ if $n > n_5$.

By deviating to not providing information, seller 1 expects a payoff that is never greater than μ. Let $\epsilon = \int_0^c F(t)dt > 0$. Then, it is clear that there must be some n_6 such that $|c(1 - q)^n| < \epsilon$ when $n > n_6$. Moreover, for some n_7, $F(t) - Z(t; \rho^*) > 0$ if n_7. Therefore, if $n > \max\{n_6, n_7\}$, deviating to not providing information (and posting any reserve price) cannot be profitable for seller 1. It follows that for a sufficiently large value of n, the game has an equilibrium in which both sellers release information and post reserve prices equal to production costs. □

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