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ENTREPRENEURIAL BEHAVIOR UNDER RISK AND UNCERTAINTY

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Chapter 1

Introduction

Entrepreneurship is a multidisciplinary research domain, extensively examined through the perspectives of business studies and economics, attracting the attention of various researchers and governments due to the significant role entrepreneurs play in the economy by concentrating a substantial share of business control, production, and wealth throughout multiple countries. Their involvement contributes specialized goods and services to the economy and enhances productivity through their dynamic entry and exit, fostering economic development ([Parker, 2018a](#)). Entrepreneurs make choices that influence the economies of the countries in which they operate. These choices are made between alternatives that differ in degree of risk among other factors ([Friedman and Savage, 1948](#)), as risk behavior of entrepreneurs, and in the context of uncertainty ([Knight, 1921](#)).

The influence of risk preferences on the decision to become an entrepreneur is acknowledged, although consensus on this matter is lacking. Since [Knight, \(1921\)](#) and [Kihlstrom and Laffont \(1979\)](#), the literature has argued that less risk-averse individuals are more prone to become entrepreneurs, willing to accept the substantial risk inherent in starting a new firm, while the more risk-averse individuals worked as laborers. The observed differences arise from variations in wealth; consequently, wealthier individuals are more inclined to accept greater risks than their less wealthy counterparts and

are more likely to engage in entrepreneurial activities. [Evans and Jovanovic \(1989\)](#) asserted there is a positive correlation between the probability of starting a business and assets if and only if there are liquidity constraints, while [Cressy \(2000\)](#) demonstrated that under the assumption of decreasing absolute risk aversion (DARA), optimal capital and the relative return to entrepreneurship are increasing in assets. DARA constitutes the fundamental premise supporting the shift towards entrepreneurship. Contrary to this assumption, [Bonilla and Vergara \(2013\)](#) found that the risk attitude required to maintain [Cressy's](#) findings is prudence, which is consistent with DARA, IARA, or CARA preferences. Unlike [Bonilla and Vergara](#), [Vereshchagina and Hopenhayn](#) developed an endogenous risk-taking model that, contrary to [Cressy's](#) conclusions, predicts that the combination of the outside option and financing constraints implies a desire for risk at low wealth levels ([Vereshchagina and Hopenhayn, 2009, pp. 99](#)).

The literature on risk and choice provides valuable insights into decision-making under uncertainty. This body of literature has expanded since [Leland's \(1968\)](#) seminal work on precautionary saving (see, e.g., [Sandmo, 1970](#); [Leland, 1968](#); [Drèze and Modigliani, 1972](#); [Kimball, 1990](#)), which investigates the influence of future income uncertainty on saving decisions ([Bommier and Le Grand, 2019](#)) to the application to other economic decision problems as in the theory of the competitive firm (see, e.g., [Sandmo, 1971](#); [Leland, 1972](#); [Batra and Ullah, 1974](#); [Broll and Wong, 2013](#), [Chavas, 1985](#); [Viaene and Zilcha, 1998](#); [Wong, 1996](#)), where risk-averse firm under uncertainty chooses its optimal output; the optimal insurance decisions see e.g. [Alarie et al., 1992](#)); the literature on labor supply (see e.g. [Knight, 1921](#); [Tressler and Menezes, 1980](#); [Dionne and Eeckhoudt, 1987](#); [Dardanoni, 1988](#); [Parker et al., 2005](#)); and optimal portfolio choices ([Meyer and Ormiston, 1989](#); [Laffont, 1989](#)). The results of these studies demonstrate how risk attitudes (risk aversion, prudence, temperance, aversion to higher degree risks, and cross-derivates of the utility function) influence decision-making. To our knowledge, the existing research on decision-making under uncertainty has not yet explored optimal entrepreneurial decisions.

This thesis aims to analyze the behavior of entrepreneurs in contexts of risk and

uncertainty. This analysis explores various situations to examine how entrepreneurial optimal decisions are affected by risk and uncertainty, along with the underlying risk attitudes or characteristics that drive these decisions. This thesis considers the development of two articles and is structured as follows: The second chapter presents the paper titled Hybrid Entrepreneurship and Risk, which addresses how hybrid entrepreneurs allocate time between paid employment (labor) and working on a venture (entrepreneurship) in the face of increased risk. The third chapter introduces the article entitled Uncertainty and Export Behavior: The SMEs Case, and it examines the influence of background risk on small and medium-sized enterprises (SME) export behavior. Finally, the last chapter presents the conclusions of the thesis.

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Chapter 2

Hybrid entrepreneurship and risk

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2.1 Abstract

In this paper we study the impact of risk on time allocation decisions between occupations by modeling a hybrid entrepreneur who must decide how to allocate time between paid employment (labor) and working on a venture (entrepreneurship). We argue that hybrid entrepreneurs self-insure in response to income risk by managing the time they allocate between both occupations. We provide the conditions under which an uninsurable risk (in paid employment or the entrepreneurial sector) has a non-ambiguous precautionary effect on the optimal time allocated to each occupation, and these conditions are based on the strengths of risk aversion and downside risk aversion. We focus on three cases: when risk affects only the entrepreneurial sector, which is the classical case studied in the occupational choice literature; when risk affects only the paid employment sector; and finally, when risk affects both sectors, as we experienced during the recent pandemic.

2.2 Plain English Summary

Hybrid entrepreneurs allocate their time between paid work (labor) and working on a venture. Traditionally, it is thought that the paid job sector is stable throughout time, and so all risks are concentrated in the entrepreneurial sector. However, the disruption of COVID-19 affected a wide variety of productive activity, and the labor market in particular became extremely turbulent for many people. As a result, it's vital to understand how risk influences how hybrid entrepreneurs allocate their time between different occupations using an optimization technique that takes into account the risk connected with each activity. In this research, we investigate the precautionary behavior of a hybrid entrepreneur in two scenarios in which entrepreneurship and labor function as substitutes or complements. The precautionary behavior is associated with the contemporary risk literature's concept of harm disaggregation, which is directly related to risk aversion and downside risk aversion, both of which are frequent behavior found in the real world. The

method utilized in this study has the primary advantage of permitting the investigation of simultaneous risks, which can be tested in the future through experimentation. This link brings up a plethora of possibilities for exploration in the study of entrepreneurship activities utilizing modern decision theory methods.

Keywords: Hybrid entrepreneurship, risk, risk apportionment, precautionary effect.

JEL Classification: D81, L26, M13

2.3 Introduction

The COVID-19 disruption has changed the way we see work for good. In many activities and professions individuals now enjoy more flexible employment relationships that provide more freedom in the use of their time. In this new context, the idea that some individuals have the possibility to choose how to divide their working time between paid employment and developing a venture (becoming an entrepreneur) is a reality¹. Therefore, the study of hybrid entrepreneurs regains relevance and realism. Following the time allocation theory initially developed by [Becker \(1965\)](#), this paper studies the role of risk in precautionary time allocation by hybrid entrepreneurs.

The nature of entrepreneurship and wage employment is distinct. As asserted by [Burmeister-Lamp et al., \(2012\)](#), a key determinant of the time allocation decision between working on a new venture and a wage job is the sense of certainty associated with wages coming from the paid employment sector versus the risk surrounding the potential returns from entrepreneurship. Traditionally, entrepreneurship has been described as a risky activity, while paid employment is an activity that enjoys a certain degree of stability over time. However, assuming that risk is concentrated only in the

¹When we discuss allocating time between paid employment and developing a venture (allocating time to an entrepreneurial endeavor), our analytic context is the time-allocation decision of a hybrid entrepreneur.

entrepreneurial sector is unrealistic. Risk is a transversal phenomenon that affects a wide range of productive activities: evidence of this is the COVID-19 pandemic and its consequent economic impact, which is an event of greater social and economic magnitude than the typical event we were used to experiencing. The different policy decisions adopted worldwide to fight COVID-19, such as lockdowns, business operating restrictions, hygiene protocols, and social distancing, among other prevention practices, had an adverse effect on labor markets, causing a decline in working hours, an increase in job lost, inactivity and lost income. According to the [ILO](#), in 2020 the consequences of pandemic disruption on labor markets around the world were on an historically unprecedented scale, so an obvious response to such economic hardships, derived from a weakened and more risky labor market, was to develop small new ventures as hybrid entrepreneurs.

Hybrid entrepreneurs represent a significant share of all entrepreneurial activity ([Folta et al., 2010](#)). Boosting entrepreneurial activity after the COVID-19 pandemic is pivotal because of the role entrepreneurs play in economic recovery through technological innovations and job creation (e.g. [Faria et al., 2009](#); [Faria, 2015](#); [Glaeser et al., 2015](#)). Therefore, it is important to understand how risk impacts the way hybrid entrepreneurs allocate their time to different activities by a process of optimization that takes into account the risk involved in each activity.

But, what is time management and why is it so important?. A response to this question is easy to find in management literature. For example, [Levesque and Stephan \(2020\)](#) defined it as the systematic process accounting for time and then allocating it more efficiently among a series of prioritized tasks and activities. While [Aeon and Aguinis \(2017\)](#) defined it as a form of decision-making used by individuals to structure, protect, and adapt their time to changing conditions. As asserted by [Levesque and Maccrimmon \(1998\)](#), time is one of the most important inputs contributed by the entrepreneur and the most valuable and scarcest resource of all in business ([Zachary et al., 2015](#)). The relevance of efficiently allocating time between occupations is that it constitutes a control variable that individuals – as hybrid entrepreneurs – have to buffer

the adverse effects of negative shocks in the economy.

The buffer is known as precautionary behavior in the economic literature. For example, an exogenous shock such as the pandemic creates risk about future labor income, which risk-averse consumers see as "bad" and therefore risk-averse individuals want less of it. On the other hand, precautionary behavior leads into more working hours and, as a result, a higher expected payment, i.e., a "good" that allows consumers to better face risk in the new scenario. This indicates that consumers can utilize work-hour reallocation as a control variable to maximize expected utility by mitigating the pain associated with future income risk. Essentially, they blend a bad with a good in order to improve their well-being.

In essence, the hybrid entrepreneur divides his or her time between two potentially risky activities: labor and entrepreneurship. And, on the one hand, risk aversion causes the hybrid entrepreneur to reduce the time allocated to a risky activity, whereas prudence causes the entrepreneur to increase the time allocated to the risky activity as a way to deal with increased risk. The proportional importance of these forces will determine the final outcome.

As we will see in this paper, risk aversion is not a concept that exhaustively describes the decision makers' risk preferences. It is just one piece of the puzzle and needs to be complemented with higher-order risk preferences such as prudence which produces the precautionary effect. A risk-averse decision maker dislikes mean-preserving increases in risk (second-order risk-aversion), while a prudent decision-maker dislikes transferring risk from a good to a bad state of nature. In this case, it is said that he is averse to downside risk (third-order risk-aversion). Both risk preference orders will be present in our model and act as forces operating in opposite directions.

In this paper we study how risk affects time allocation and under what conditions a hybrid entrepreneur will show precautionary behavior in entrepreneurial time allocation. Our analysis yields several results. First, we show that precautionary behavior is affected by the relationship between entrepreneurship and labor as substitutes or com-

plements. And second, the sufficient condition for precautionary behavior is related to the relative strength of downside risk aversion of entrepreneurship, labor and leisure.

In our analysis, the precautionary behavior of hybrid entrepreneurs is associated with the idea of harm disaggregation recently developed in the risk literature. Therefore, we show a direct connection between the study of the economics of entrepreneurship and the classical study of decision-making under risk. One of the primary benefits of this approach is that it enables us to develop an intuitive method for working with bivariate risks that can be tested experimentally in the future. The works of [Deck and Schlesinger \(2014\)](#), [Ebert and Wisen \(2014\)](#), and [Krieger and Mayrhofer \(2017\)](#) are examples of laboratory experiments based on this behavioral approach of harm disaggregation² that emphasizes risk preferences. In addition, the behavioral approach may provide new insights that the traditional expected utility approach cannot capture ([Bonilla and Vergara, 2021](#)), implying that policy implications regarding entrepreneurship based solely on expected utility must be evaluated with caution. Particularly when the behavioral approach accurately reflects the individual's decision-making process. This connection opens up multiple possibilities for exploration in the study of entrepreneurship using the modern decision theory tools. We believe that our paper takes advantage of this unexploited connection and provides new insights into entrepreneurship literature.

The study of hybrid entrepreneurship is the study of future labor interactions. Advances in communication technology and process automation create greater opportunities for traditional workers to transition from traditional 9-5 jobs to hybrid entrepreneurship. Decoding hybrid entrepreneurship is comparable to decoding labor's future. It's all about adaptability, overcoming uncertainty, and making the most of a variety of opportunities that will be available to most people.

²In the economics of risk literature, the behavioral approach of harm disaggregation refers to the study of decision making under risk that can be analyzed through lotteries, and thus can be experimentally tested. Therefore, the behavioral characteristic is affixed to decision-making with lotteries and it does not necessarily imply a departure from traditional expected utility theory and rationality like the case of behavioral economics.

The remainder of this paper is organized as follows. Section 2 discusses some related literature on risk and entrepreneurship. Section 3 develops the optimal decision problem faced by a hybrid entrepreneur and provides a graphical interpretation of the equilibrium. Section 4 defines precautionary behavior in our context and harm disaggregation following the behavioral approach. Section 5 presents the optimal time allocation for hybrid entrepreneurs and the main results of the paper. In section 5 we analyze optimal behavior under three specific sources of risks: when risk affects only the returns of the entrepreneurial sector, when risk affects only the paid employment sector, and when risk affects both occupations. In the last section we provide the main conclusions of the paper.

2.4 Related literature

Our paper relates first to the entrepreneurship and occupational choice literature, since the question of who becomes an entrepreneur and the role that risk plays, is a long-standing question in the economics of entrepreneurship literature. Much of the literature has treated this decision as a discrete choice between entrepreneurship and paid employment. Traditionally, some authors have argued that this decision depends on the individual's risk preferences ([Kihlstrom and Laffont, 1979](#)), credit constraints ([Evans and Jovanovic, 1989](#)), and differences in wealth under DARA preferences ([Cressy, 2000](#)). They all assumed that risky is only present in the entrepreneurial sector, which is the traditional perspective found in the literature of occupational choice.

However, other authors have also addressed the role that wage risk plays in the decision to engage in self-employment or entrepreneurship. For instance, [Parker et al. \(2005\)](#) focused on the impact that wage risk has on the labor supply of self-employment workers. When wage risk is specified as an additive shock, they found that prudence ($u''' > 0$) is the condition explaining a greater labor supply of self employees in response to a greater risk since the self employed self-insure against this risk by working harder

to make the deterministic part of their income relatively higher. However, they could not extend this argument to a multiplicative shock. They found inconclusive results and made an unclear prediction about how self-employed labor supply responds to greater multiplicative wage risk. Given these theoretical results, they conducted an empirical analysis of self-employed American workers and found that wage risk is a key influence in explaining self-employed labor supply. [Berkhout et al. \(2016\)](#) showed that paid employment wage distribution affects the probability of becoming an entrepreneur. Individuals with lower mean wages, a higher wage variance, and a lower-wage skew are more likely to leave paid employment and choose entrepreneurship. Our paper extends the contribution of this literature by considering two types of risk sources. We argue that labor income risk about wages and risky returns from self-employment, play a key role in a hybrid entrepreneur's decision to allocate more time to entrepreneurship.

Nowadays, in a more flexible labor market with more flexible working arrangements since the COVID-19 pandemic, there is no doubt that the decision to become an entrepreneur transcends the employee versus entrepreneur dichotomy and is instead a continuous choice, reviving interest in the phenomenon of hybrid entrepreneurship. [Folta et al. \(2010\)](#) defined hybrid entrepreneurs as individuals who engage in self-employment while also holding a primary job in wage work, which turns out to be a more widespread practice than thought ([Thorgren, Nordstrom, and Wincent, 2014](#)), with a large and growing portion of individuals starting their businesses while holding a primary job in wage work ([Folta et al., 2010](#) ; [Burke et al., 2008](#)). Like other authors, we consider that hybrid entrepreneurship is a permanent condition for many individuals. Therefore, we are only interested in understanding the entrepreneurial behavior of these individuals that persists in both simultaneous activities and we are not looking into a transition to full-time entrepreneurship.

According to related research, these individuals pursue hybrid entrepreneurship as an alternative and safer way to entrepreneurship than full-time entry. Since hybrid entrepreneurs remain salaried, this entry mode allows them to experience entrepreneurship and test the entrepreneurial waters, on the one hand ([Folta et al., 2010](#)). While on

the other hand, to reduce the downside risk ([Raffiee and Feng, 2014](#); [Folta et al., 2010](#)). Prior research understands downside risk related to the real options theory, where the potential for downside loss has a limit due to hybrid entry provides individuals the flexibility to start a venture at a slower pace with fewer sunk costs. However, to the best of our knowledge, downside risk has not yet been incorporated into hybrid entrepreneurship research through the lens of decision-making theory under risk. Therefore, acknowledging that an increase in downside corresponds to a dispersion transfer from higher to lower income levels and implies a reduction in skewness ([Menezes, Geiss, and Tressler, 1980](#)) adds to the growing body of literature by providing insights into how the heterogeneity individuals' behavioral response to the risk exists and depends on their heterogeneous risk preferences.

In accordance with the current idea of risk aversion, individuals who are less risk-averse are more likely to start their own full-time ventures than those who are not. [Raffiee and Feng \(2014\)](#) confirmed this and also suggested for the first time that risk preferences may affect how rather than whether someone enters the entrepreneurial world. They discovered that risk aversion influences the process of how an individual decides to start a business (i.e., full-time versus hybrid).

The literature on occupational choice and entrepreneurship is also relevant to our paper. In recent years, the literature has examined the antecedents of becoming a serial or portfolio entrepreneur ([Carbonara et al., 2020](#); [Plehn-Dujovich, 2010](#); [Parker, 2014](#)). For example, [Carbonara et al. \(2020\)](#) use a Vietnamese database to conclude that human capital and a higher quality of the new business indicate a greater likelihood that a business owner will become a serial or portfolio entrepreneur. In equilibrium, a high-skilled entrepreneur closes a low-quality business and becomes a serial entrepreneur until a high-quality business is found, whereas a low-skilled entrepreneur closes a low-quality business and enters the labor market. [Parker \(2014\)](#) models the career decisions of serial, portfolio, and first-time entrepreneurs and derives behavioral predictions for each group. [Poschke \(2013\)](#) develops a dynamic model that is consistent with empirical evidence and demonstrates that entrepreneurs are more likely

to originate from the extremes of the education and wage distributions, resulting in a U-shaped relationship with entrepreneurship.

Furthermore, serial entrepreneurs regularly switch industries (Eggers and Song, 2015) since they can use their prior skills to launch a new venture in a different industry (Kuluvainen, 2010). However, another body of research has found that successful serial entrepreneurs typically stay within their industry and attain economic success with less inventive performance (Dabic et al., 2023). To the best of our knowledge, not enough attention has been paid in the study of hybrid entrepreneurship to this critical issue, namely the relationship between the industry in which hybrid entrepreneurs build their business ventures and the sector in which they are currently employed (see Demir et al., 2020).

Examples include Steve Wozniak and Sara Blakely, co-founders of Apple and SPANX, who both launched their businesses while working full-time. Steve Wozniak worked at Hewlett Packard, designing scientific calculators, while also creating the Apple I, a home-brew personal computer (Wozniak and Smith, 2006). His two jobs were both in the same industry. Steve brought the prototype to HP before co-founding Apple and begged them to build it, but they refused (Chu, 2017). Similarly to Steve, Sara Blakely began creating her body-slimming women's underwear endeavor while working as a door-to-door fax machine salesperson for the office supply company Danka (Wetzel and Wetzel, 2020).

Urbig et al. (2021) defined this relationship as business proximity, defined as the extent to which the employee's venture is related to those of his employer, whereas Kylver (2021) defines it as business related to their current industry and uses his concept as a proxy to study industry experience.

Our model recognizes this characteristic of hybrid entrepreneurs by taking into account the likelihood that the two activities interact as substitutes or complements. Recognizing that committing time to entrepreneurship within your current industry will help both activities, however developing a venture outside of it may have negative conse-

quences

Another significant aspect is that a share of the individuals who use hybrid entry mode to entrepreneurship establish their ventures in knowledge-intensive, innovative, and high-technology industries. For example, Apple co-founder Steve Wozniak, eBay founder Pierre Omidyar, and Detroit Automobile Group founder Henry Ford were all workers while launching their ventures ([Raffiee and Feng, 2014](#)). Considering that the tech industry does not have low levels of risk, hybrid entrepreneurship is an entry-level option and viable strategy for developing entrepreneurship. To date, research has overlooked the fact that risk exists not only in the self-employment sector but also in the paid employment sector, thus ignoring the role that risk both-sided plays in hybrid entry choice and how hybrid entrepreneurs choose to allocate their working time between entrepreneurship and paid employment.

The studies by [Parker \(1997\)](#) and [Ganser-Stickler, Schulz, and Schwens \(2022\)](#) are the exceptions. Parker³ assumed that income risk is concentrated not only in self-employment but also in the paid-employment. He acknowledged that the assumption that risk focuses just on the self-employment sector is not harmless and affects the predicted direction that sectoral risk has on self-employment. He investigated the effects of sectoral risk on self-employment participation using a general equilibrium model based on heterogeneous risk-averse agents with no switching costs when changing sectors demonstrating that the sectoral risk effects on the optimal self-employment decision are ambiguous for hybrid entrepreneurs.

In a similar way to Parker, [Ganser-Stickler, Schulz, and Schwens \(2022\)](#) studied how risk in paid employment and in entrepreneurship individually and jointly influence an individual's choices, particularly the decision to enter entrepreneurship in a hybrid mode rather than as a full-time entrepreneur. Using real options reasoning, they argued that

³Unlike [Parker's \(1997\)](#) findings, our paper provides the conditions under which an uninsurable risk (in paid employment or the entrepreneurial sector) has a non-ambiguous precautionary effect on the optimal time allocated to each occupation, and these conditions are based on the strengths of risk aversion and downside risk aversion.

the hybrid portfolio created by holding both real options is less valuable than starting a full-time business due to the increased volatility in paid employment. Hence, paid employment risk reduces the individual's likelihood of choosing hybrid mode over full-time.

Unlike previous studies, our methodological approach allows us to improve realism by acknowledging how hybrid entrepreneurs allocate more time to entrepreneurship as a multi-dimensional decision. We do so by considering that entrepreneurs face two sources of risk and that their preferences depend on two attributes: consumption and leisure. From an economics perspective, we concentrate on comprehending entrepreneurial behavior following entry by analyzing the marginal decisions made by those individuals who continue in both simultaneous activities. Our analysis therefore focuses on both the hybrid entrepreneurs who aim to become full-time entrepreneurs and those who do not. In contrast with the findings of [Parker \(1997\)](#), our paper shows that the effects that the uninsurable risk in paid employment and in the entrepreneurial sector has on the optimal time allocated to each occupation are not ambiguous. We find that the conditions explaining precautionary entrepreneurial time allocation and precautionary paid-employment time allocation are related to the strengths of downside risk aversion.

Our paper also relates to the recent literature on risk. We build from the seminal work of [Eeckhoudt et al. \(2006\)](#) who developed the idea of preferences for harm disaggregation under a behavioral approach and showed that risk apportionment relates to the differentiable utility function u , which satisfies the additive risk apportionment of order n if $(-1)^{n+1}u^{(n)} \geq 0$ ⁴. Working with the behavioral approach enables us to an-

⁴The function $u^{(n)}$ denotes the n th order derivative of utility function u . Thus the mathematical expression $(-1)^{n+1}u^{(n)} \geq 0$ describes a n th degree risk averse decision-maker. This expression encompasses the concepts of risk-aversion ($n = 2$) and downside risk aversion ($n = 3$), which are used in this paper to describe the behavior of the hybrid entrepreneur (see [Ekern, 1980](#) for the more technically inclined readers). These concepts have been tested in laboratory experiments by [Deck and Schlesinger \(2014\)](#) and [Ebert and Wiesen \(2014\)](#), among others. The first work tested the direction of third-order risk preferences (prudence), while the second developed a method to measure its intensity or strength.

alyze entrepreneurship predominantly as an application of decision theory under risk by employing lottery pairs that perfectly connect with higher-order properties of utility functions that are particularly amenable to experimentation. The main intuition is that decision-makers favor combining relatively good assets with relatively poor ones, where poor is defined by n th-order stochastic dominance. We will apply these concepts to our analysis beginning with the simplest exercise conceivable in Section 4.2.1.

Our paper is also related to the multiplicative risk apportionment literature (e.g. [Eeckhoudt et al., 2009](#), [Wang and Li, 2010](#); [Chiu et al., 2012](#)) and to the bivariate risk apportionment idea, first developed by [Eeckhoudt et al. \(2007\)](#) who introduced this notion by acknowledging that decisions under risk are often multidimensional and, therefore, the decision-maker preferences depend on several attributes. [Jokung \(2011\)](#) describes this concept as the implications of preferring a bivariate lottery with one bad and one good attribute over a lottery with a 50-50 probability of either two good or two bad attributes. [Denuit and Rey \(2013\)](#) refer to this as cross risk apportionment while [Crainich et al. \(2020\)](#) refined the concept of bivariate risk apportionment by highlighting that this preference for dispersing risks is associated with two aspects of an individual's well-being in different states of the world. We use the risk apportionment technology to derive an optimal decision of a hybrid entrepreneur who faces multiple risks in their alternative occupations. We provide the conditions under which an uninsurable risk (in paid employment or the entrepreneurial sector) has a non-ambiguous precautionary effect on the optimal time allocated to each occupation, and these conditions are based on the strengths of risk aversion and downside risk aversion.

2.5 The basic model of time allocation

We developed an economic model of entrepreneurship in a context of risk where hybrid entrepreneurs choose how to allocate their working time between paid employment (labor) and entrepreneurship (working on a side venture or self-employment). We used a

single-agent, single-period occupational choice bivariate model and followed a methodological approach similar to [Liu and Menegatti \(2019\)](#) but we added a risk apportionment framework ([Eeckhoudt and Schlesinger, 2006](#); [Chiu et al., 2012](#); [Eeckhoudt et al. 2009](#)) that allows us to represent the decision under risk of changing time allocation between competing activities.

The hybrid entrepreneur's preferences are defined over consumption c and leisure l and they are represented by a Von Neumann-Morgenstern utility function $u(c, l)$. The utility function is assumed to be continuous, increasing and concave in both c and l . That is, $u_c > 0$ and $u_l > 0$ (non-satiation), $u_{cc} < 0$ and $u_{ll} < 0$ (risk aversion) However, u_{cl} can be negative or positive. When $u_{cl} > 0$, it means that the marginal utility of one good increases with the consumption of the other good. Alternatively, when $u_{cl} < 0$, it means that the marginal utility of one good decreases with the consumption of the other (pair-wise risk aversion), and both cases are possible in our model.

The decision-maker (DM) is endowed with total time T , which he or she distributes between leisure and productive activities (labor and entrepreneurship). Let L be the working hours that they devote to paid employment, while e describes the workings hours devoted to entrepreneurship. The total number of hours devoted to leisure time is $l = T - L - e$.

The individual has a wealth level a (exogenous assets that, without loss of generality, we will set to zero) and earns a fixed wage w for each time unit devoted to paid employment. This w is non-random (for now) and non-negative. On the other hand, the income of the entrepreneurial activity is described by a positive random variable $\tilde{\theta}$ that represents the random returns generated by the time unit devoted to entrepreneurship. Therefore, the hybrid entrepreneur has to choose the optimal time allocations for both: the number of hours allocated to paid employment and the number of hours allocated to entrepreneurship. In this context, the hybrid entrepreneur's optimization problem is:

$$\max_{L,e} V(e, L) \quad (2.1a)$$

$$\text{where, } V(e, L) = Eu(wL + \tilde{\theta}e, T - L - e) \quad (2.1b)$$

The first and second order conditions of the maximization problem are:

$$V_e = E[\tilde{\theta}u_c] - Eu_l = 0 \quad (2.2)$$

$$V_L = wEu_c - Eu_l = 0 \quad (2.3)$$

$$V_{ee} = E[(\tilde{\theta})^2u_{cc}] - 2E[\tilde{\theta}u_{cl}] + Eu_{ll} < 0 \quad (2.4)$$

$$V_{LL} = w^2Eu_{cc} - 2wEu_{cl} + Eu_{ll} < 0 \quad (2.5)$$

$$H = V_{ee}V_{LL} - (V_{eL})^2 > 0 \quad (2.6)$$

where,

$$V_{eL} = wE[\tilde{\theta}u_{cc}] - E[\tilde{\theta}u_{cl}] - wEu_{lc} + Eu_{ll} \quad (2.7)$$

The joint optimum (e^*, L^*) is obtained when both eq. (2.2) and eq. (2.3) are satisfied. Also, note that V_{eL} indicates if the two occupations are substitutes ($V_{eL} < 0$) or complements ($V_{eL} > 0$) and this difference will comprise an important part of the results in this article. In order to conceptualize this difference for the case of complements, we consider the case where a DM starts a venture that is related to their current employment, so any time devoted to the venture will produce positive effects in their performance as an employee and any professional improvement and better knowledge of their current craft will positively impact their venture. On the contrary, when the occupations are substitutes, developing a venture in an entirely different industry will have adverse effects on their current occupation. In this case we considered a DM whose focus and energy are limited, so focusing on industry A takes away focus and efficiency from industry B.

2.6 Preliminaries and definitions

2.6.1 Precautionary behavior

In this paper we study precautionary entrepreneurial and labor time allocations. We define these concepts as the extra working time allocated to either of those activities when the DM is facing an increase in risk.

We determined the precautionary effect by the difference between the optimal choice in problem (1a)⁵ and the optimal choice made when we assumed that there is no risk to entrepreneurship and consequently, the return on entrepreneurship is replaced by its non-random expectation ($E[\tilde{\theta}]$), that is,

$$\max_{L,e} u(wL + E[\tilde{\theta}]e, T - L - e) \quad (2.8)$$

Let e^* and L^* be the optimal choice in problem (1a) and e^{**} and L^{**} be the optimal choice in problem (8). We thus have the following two definitions:

Definition 1. *We have precautionary entrepreneurial time allocation when $e^* > e^{**}$, and precautionary entrepreneurial time deallocation when $e^* < e^{**}$.*

Definition 2. *We have precautionary labor time allocation when $L^* > L^{**}$, and precautionary labor time deallocation when $L^* < L^{**}$.*

We will use these definitions in the next section where we study precautionary time allocation induced by different sources of risk (not only risk in the entrepreneurial sector). For now, let us focus on the equilibrium conditions of problem 1(a) that will be represented by Figure 1. First, note that if $V_{eL} < 0$, from eq. (2.6) we can infer that,

⁵Problem (1a) represents the classical model of self-selection of occupations in which entrepreneurship provides a risky payment while labor provides a secure wage.

$$-\frac{V_{LL}}{V_{eL}} < -\frac{V_{eL}}{V_{ee}} < 0 \quad (2.9)$$

and alternatively, if $V_{eL} > 0$, from eq. (2.6) we infer that

$$-\frac{V_{LL}}{V_{eL}} > -\frac{V_{eL}}{V_{ee}} > 0 \quad (2.10)$$

Now, totally differentiating eq. (2.2) we get $dV_e = V_{ee}de + V_{eL}dL$ and from this equation we can define the curve ee with slope $\frac{de}{dL} = -\frac{V_{eL}}{V_{ee}}$. Similarly, totally differentiating eq. (2.3) we obtain $dV_L = V_{LL}dL + V_{eL}de$ and from this equation we define the curve LL with slope $\frac{de}{dL} = -\frac{V_{LL}}{V_{Le}}$. The intersection of both curves represents the optimal allocation of (e^*, L^*) as shown in figure 1.

Note that equations (9) and (10) can be rewritten in the following way:

If $V_{eL} < 0$ (the curves ee and LL are downward sloping), that is $\left.\frac{de}{dL}\right|_{LL} < \left.\frac{de}{dL}\right|_{ee}$ (equation 9). Also, if $V_{eL} > 0$ (the curves ee and LL are upward sloping), that is $\left.\frac{de}{dL}\right|_{LL} > \left.\frac{de}{dL}\right|_{ee}$ (equation 10). We know that the sign of the cross-partial derivative V_{eL} determines whether the two occupations are substitutes ($V_{eL} < 0$) or complements ($V_{eL} > 0$). Figure 2.1 illustrates from left to right the cases shown in eq. (2.9) and eq. (2.10) respectively. Consequently, precautionary behavior in entrepreneurial time allocation is

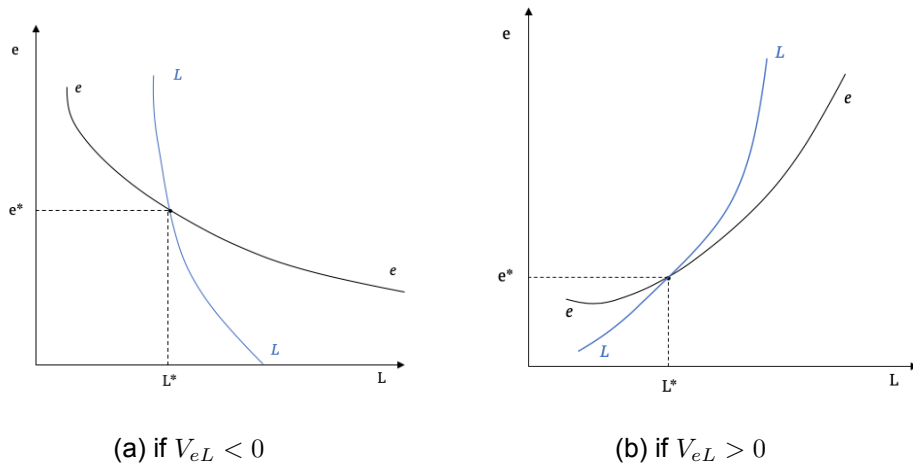


Figure 2.1: The LL curve and ee curve

mediated by how both activities interact. If they are substitutes, the precautionary decision to allocate more time to entrepreneurship implies the decision to deallocate time from paid employment (see fig. 2.2). If we focus on figure 2a, we see that a sufficient conditions that guarantee an increase in e and a decrease in L – when we go from a no risk to a risky entrepreneurship – is when curve LL moves to the left, curve ee moves to the right, or both curves move at the same time in those directions⁶.

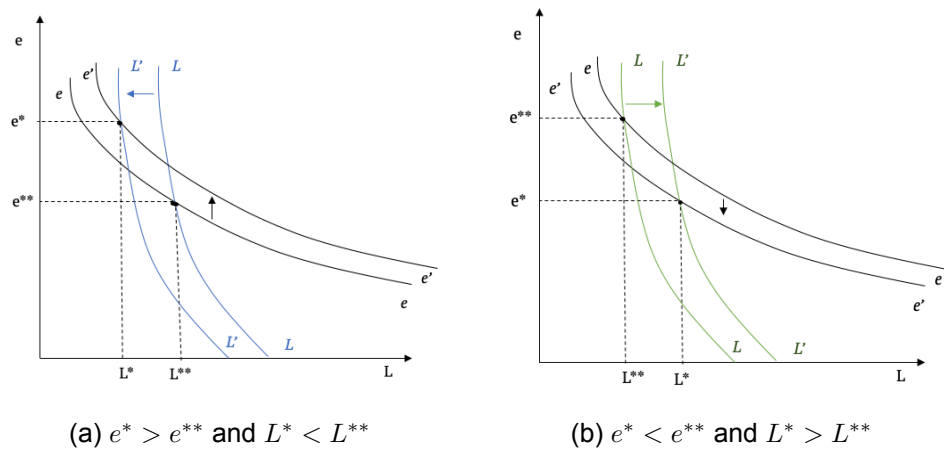


Figure 2.2: Precautionary entrepreneurial and labor time allocation (deallocation) when $V_{eL} < 0$

Conversely, when these occupations interact as complements ($V_{eL} > 0$), the precautionary decision to allocate more time to entrepreneurship implies the allocation of more time to paid employment (see fig. 2.3), which is when curve LL moves to the right, curve ee moves to the left, or both curves move at the same time in those directions.

As we will see in what follows, we used a behavioral approach to show that hybrid entrepreneurs can diversify risk by choosing time allocation between entrepreneurship and labor. Our results are based on preferences for harm disaggregation (and the strength of downside risk aversion⁷). We will develop a particular set of lotteries to

⁶In the proof of proposition 1 we will provide the mathematical conditions required for these comparative static exercise.

⁷The term strength of downside risk aversion was coined by Menezes and Wang (2004) and represents the disutility attached to a downside risk increase. We give this term a more intuitive interpretation using the risk apportionment approach.

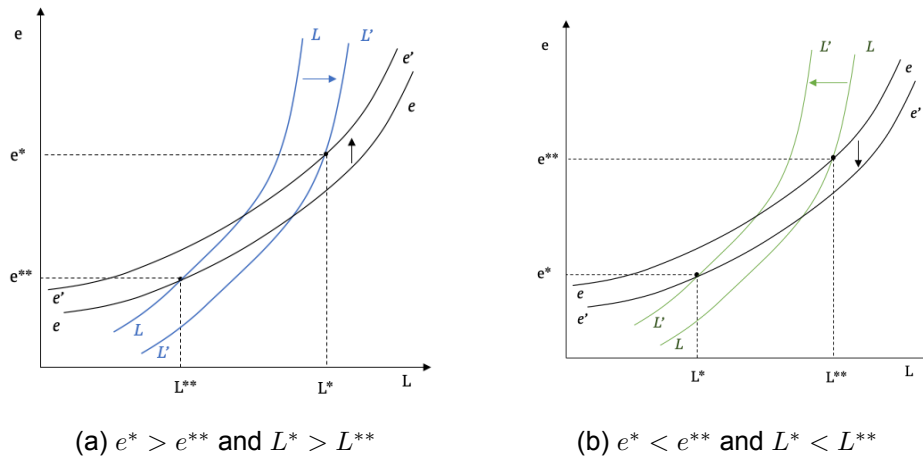


Figure 2.3: Precautionary entrepreneurial and labor time allocation (deallocation) when $V_{eL} > 0$

characterize an increase in risk and to analyze the precautionary behavior in this context.

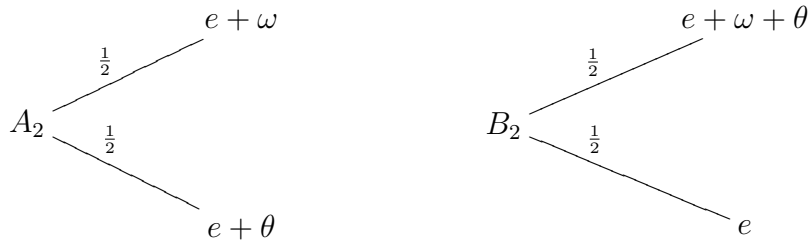
2.6.2 Risk aversion and downside risk aversion in the behavioral approach

Since we already argued that the behavioral approach enables us to analyze entrepreneurship predominantly as an application of decision theory under risk by employing lottery pairs that are particularly amenable to experimentation, here we will introduce our approach building from the basics of risk aversion. In this subsection we will introduce the concepts of risk aversion and downside risk aversion via 50-50 lottery pairs using a one argument utility function. Later we will extend such concepts to more complex pairs of lotteries which will help us to explain precautionary effects in the time allocation problem of hybrid entrepreneurs.

Risk aversion

In the expected utility space, utility function $u(c)$ is risk averse if its second derivative is negative ($u_{cc} < 0$), and the intensity of risk aversion is measured by $A = -\frac{u_{cc}}{u_c}$ (Arrow, 1965; Pratt, 1964).

An alternative way to define risk aversion is using pairs of lotteries. Let us initially assume that e is individual's initial wealth, $\theta, \omega > 0$ are fixed monetary amounts and $\theta > \omega$. Consider two pairs of lottery options A_2 versus B_2 .



A decision-maker is risk-averse if he prefers A_2 to B_2 for arbitrary parameter values e , ω and θ . This means that he prefers to disaggregate unavoidable gains ω and θ across states of nature. ⁸ In the expected utility framework, $A_2 \succeq B_2$ if and only if

$$Eu(A_2) = \frac{1}{2}u(e + \omega) + \frac{1}{2}u(e + \theta) \geq \frac{1}{2}u(e + \omega + \theta) + \frac{1}{2}u(e) = Eu(B_2) \quad (2.11)$$

where $SR = Eu(A_2) - Eu(B_2)$ is the disutility associated to the mean-preserving spreads and represents the strength of risk aversion. Condition (11) can be written as:

$$[u(e + \theta) - u(e)] - [u(e + \omega + \theta) - u(e + \omega)] \geq 0$$

For a small θ , we have:

$$\theta[u_c(e) - u_c(e + \omega)] \geq 0$$

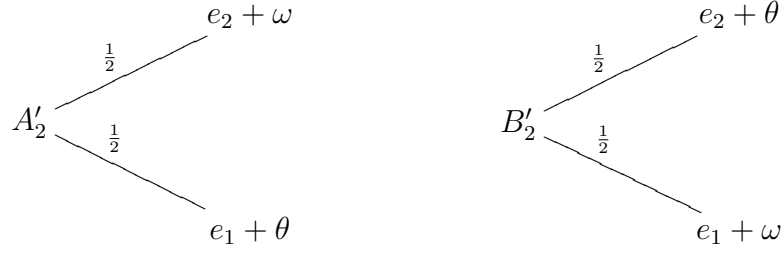
and then for small ω we get:

$$-\theta\omega u_{cc}(e) \geq 0$$

⁸An alternative way to represent risk aversion is through the 50 – 50 lottery pair: $A_2 = [e - \omega, e - \theta]$ and $B_2 = [e, e - \theta - \omega]$. Thus, a decision-maker prefers to disaggregate unavoidable losses $-\omega$ and $-\theta$ across states of nature.

which is true if and only if $u_{cc} < 0$.

There is still another way to represent risk aversion, which we present below. Consider now the following pair of lotteries where $e_2 > e_1$ and $\theta > \omega$.



With this pair of lotteries, a decision-maker is risk-averse if he prefers A'_2 to B'_2 . Note that the lottery A'_2 combines good with bad, since e_1 is attached to the best state of nature, while e_2 is attached to the worst state of nature. The opposite case occurs with the B'_2 lottery in which good is combined with good and bad with bad. Then e_1 has more one-degree risk than e_2 (e_2 dominates e_1 via first-order stochastic dominance) and ω has also more one-degree risk than θ (θ dominates ω via first-order stochastic dominance). Based on the corollary of [Eeckhoudt et al. \(2009\)](#), then the lottery B'_2 displays more second-degree risk than A'_2 or B'_2 is a mean-preserving increase in risk of A'_2 . In the expected utility framework, $A'_2 \succeq B'_2$ if and only if

$$Eu(A'_2) = \frac{1}{2}u(e_2 + \omega) + \frac{1}{2}u(e_1 + \theta) \geq \frac{1}{2}u(e_2 + \theta) + \frac{1}{2}u(e_1 + \omega) = Eu(B'_2) \quad (2.12)$$

where $SR = Eu(A'_2) - Eu(B'_2)$ is the disutility associated to the mean-preserving spread and represents the strength of risk aversion. This condition can be expressed equivalently as:

$$[u(e_1 + \theta) - u(e_1 + \omega)] - [u(e_2 + \theta) - u(e_2 + \omega)] \geq 0 \quad (2.13)$$

Defining $\Gamma(e, \omega, \theta) = u(e + \theta) - u(e + \omega)$ and denoting the partial derivative of Γ respect to e by Γ_e , (13) can be further written as $\Gamma(e_1, \omega, \theta) - \Gamma(e_2, \omega, \theta) \geq 0$, which is clearly true if and only if $\Gamma_e \leq 0$. Because $\Gamma_e = u_c(e + \theta) - u_c(e + \omega)$, then $\Gamma_e \leq 0$ is equivalent $u_c(e + \omega) \geq u_c(e + \theta)$ which is true if and only if $u_{cc} < 0$, i.e., if the decision-maker is risk-averse.

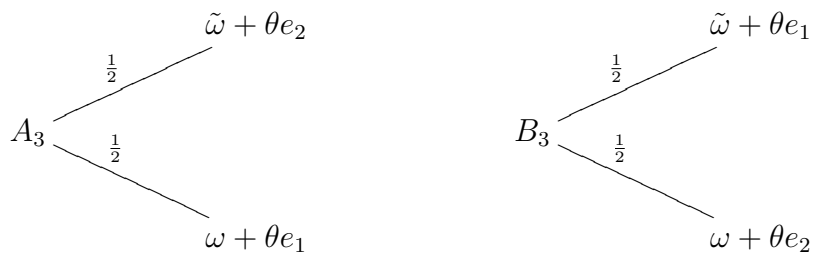
Then, when we have pairs of 50-50 lotteries, the following lemma characterizes a risk-averse decision maker.

Lemma 1. *In the Expected Utility framework, assuming $e_2 > e_1$ and $\theta > \omega$, then $A'_2 \succeq B'_2$ if and only if $u_{ccc} < 0$.*

Downside risk aversion

In the expected utility space, downside risk aversion (or prudence) is defined by a positive third derivative of the utility function ($u_{ccc} > 0$). This concept was coined by [Kimball \(1990\)](#) in the context of the precautionary demand for saving. Also, $u_{ccc} > 0$ implies that a decision-maker has a convex marginal utility function and therefore, he dislike transferring risk from a richer to a poorer state.

Let us assume that the positive constant ω in the definition for risk aversion is replaced by a random variable $\tilde{\omega}$ with mean $E(\tilde{\omega}) = \omega$. Let A_3 be the 50–50 lottery $[\tilde{\omega} + \theta e_2, \omega + \theta e_1]$ and B_3 be the 50 – 50 lottery $[\tilde{\omega} + \theta e_1, \omega + \theta e_2]$ and let us recall that in our model $e_2 > e_1$ and $\theta > \omega$. As we see θ is just a constant that multiplies e_1 and e_2 through the states of nature. Thus θ can take any non negative value and in particular may be chosen such that $\theta = 1$. These pair of lotteries can be expressed as:



A decision-maker is downside risk averse if he prefers A_3 to B_3 . Note that the lottery A_3 combines good with bad, since e_2 is attached to the state in which the random variable ($\tilde{\omega}$) appears, while e_1 is attached to the state in which the expected value of the random variable (ω) appears. The opposite case occurs with the B_3 lottery in which

good is combined with good and bad with bad. Thus e_1 has more one-degree risk than e_2 (e_2 dominates e_1 via first-order stochastic dominance) and $\tilde{\omega}$ has more two-degree risk than ω ($\tilde{\omega}$ is a mean-preserving spread of ω). If again we use the corollary of [Eeckhoudt et al. \(2009\)](#), then the lottery B_3 displays more third-degree risk than A_3 or B_3 has more downside risk than A_3 . In the expected utility framework, $A_3 \succeq B_3$ if and only if

$$Eu(A_3) = \frac{1}{2}Eu(\tilde{\omega} + \theta e_2) + \frac{1}{2}u(\omega + \theta e_1) \geq \frac{1}{2}Eu(\tilde{\omega} + \theta e_1) + \frac{1}{2}u(\omega + \theta e_2) = Eu(B_3) \quad (2.14)$$

where $SD = Eu(A_3) - Eu(B_3)$ is the disutility associated to the increase in downside risk and represents the strength of downside risk aversion. Condition (14) can be expressed equivalently as:

$$[u(\omega + \theta e_1) - Eu(\tilde{\omega} + \theta e_1)] - [u(\omega + \theta e_2) - Eu(\tilde{\omega} + \theta e_2)] \geq 0 \quad (2.15)$$

Let $\Gamma(e, \omega, \tilde{\omega}, \theta) = u(\omega + \theta e) - Eu(\tilde{\omega} + \theta e)$ be defined. Differentiating Γ with respect to e we obtain $\Gamma_e = u_c \theta - Eu_c \theta$. Observe that (15) can be expressed as $\Gamma(e_1, \omega, \tilde{\omega}, \theta) - \Gamma(e_2, \omega, \tilde{\omega}, \theta) \geq 0$ if and only if $\Gamma_e \leq 0$, which is equivalent to $Eu_c \geq u_c$ because θ is non random. By Jensen's inequality $g(\omega) = u_c$ is a convex function when $g_{\omega\omega} = u_{ccc} > 0$, which is the prudence condition.

Given the previous analysis on pairs of 50-50 lotteries, the following lemma characterizes a downside risk-averse decision maker.

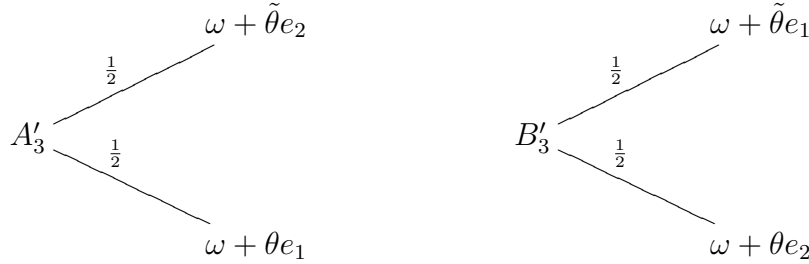
Lemma 2. *In the Expected Utility framework, assuming $e_2 > e_1$, $\omega > 0$ and $\tilde{\omega}$ being a mean-preserving spread of ω , then $A_3 \succeq B_3$ if and only if $u_{ccc} > 0$.*

The definition of downside risk aversion given above is based on an additive risk. In this context, what the literature calls the income effect arises. Thus a prudent decision maker prefers suffer risk in better states of nature and the income effect pushes him to increase his working hours (and therefore his expected wealth) devoted to the risky activity as a way to face increased risk.

As we will show below, in the case of a multiplicative risk, we have two opposing effects: an income effect induced by prudence and a substitution effect induced by risk aversion⁹. These two effects are embedded in an index of partial relative prudence which was introduced by Choi et al. (2001).

Partial relative prudence

Now consider the pair of lotteries A'_3 and B'_3 , where $\omega > 0$, $e_2 > e_1$ and $\tilde{\theta}$ is a random variable with $E(\tilde{\theta}) = \theta$.



In this case, ω is just a constant that is added to all states nature. As we see e_1 has more one-degree risk than e_2 (e_2 dominates e_1 via first-order stochastic dominance) and $\tilde{\theta}$ has more two-degree risk than θ ($\tilde{\theta}$ is a mean-preserving spread of θ). These two degrees of risk multiply each other giving rise to a behavior that combines risk aversion and downside risk aversion. In the expected utility framework, $A'_3 \succeq B'_3$ if and only if

$$Eu(A'_3) = \frac{1}{2}Eu(\omega + \tilde{\theta}e_2) + \frac{1}{2}u(\omega + \theta e_1) \geq \frac{1}{2}Eu(\omega + \tilde{\theta}e_1) + \frac{1}{2}u(\omega + \theta e_2) = Eu(B'_3) \quad (2.16)$$

where $SRD = Eu(A'_3) - Eu(B'_3)$ is the disutility associated to the increase in multiplicative risk and represents the strength risk aversion and downside risk aversion. Condition (16) can be rewritten as:

$$[u(\omega + \theta e_1) - Eu(\omega + \tilde{\theta}e_1)] - [u(\omega + \theta e_2) - Eu(\omega + \tilde{\theta}e_2)] \geq 0 \quad (2.17)$$

⁹See, for instance, [Bonilla and Vergara \(2022\)](#) who analyzed the problem of precautionary saving under linear and nonlinear risks.

Following the same procedure described above, let's define $\Gamma(e, \omega, \theta, \tilde{\theta}) = u(\omega + \theta e) - Eu(\omega + \tilde{\theta}e)$, where the derivative of Γ with respect to e is equal to $\Gamma_e = u_c\theta - Eu_c\tilde{\theta}$. Condition (17) can be rewritten as $\Gamma(e_1, \omega, \theta, \tilde{\theta}) - \Gamma(e_2, \omega, \theta, \tilde{\theta}) \geq 0$, which is true if and only if $\Gamma_e \leq 0$. By Jensen's inequality $Eu_c\tilde{\theta} \geq u_c\theta$ whenever $g(\theta) = u_c\theta$ is a convex function. Observe that $g_{\theta\theta} = [\theta eu_{ccc} + 2u_{cc}]e$. An income effect is associated to the term θeu_{ccc} , which includes prudence, while a substitution effect is associated to the term $2u_{cc}$, which include risk aversion. These two forces operate in opposite directions.

Then, based on pairs of 50-50 lotteries, the following lemma characterizes a risk-averse and downside risk-averse decision maker who faces a multiplicative risk.

Lemma 3. *In the Expected Utility framework, assuming $u_{cc} < 0$ and $u_{ccc}, \omega > 0$, $\tilde{\theta}$ is a mean-preserving spread of θ , and $e_2 > e_1$, then $A'_3 \succeq B'_3$ if and only if $-\theta e \frac{u_{ccc}}{u_{cc}} > 2$.*

A clear example of additive and multiplicative risks is the intertemporal savings model under risk, in which e is savings, $\tilde{\omega}$ is a labor income risk¹⁰ and $\tilde{\theta}$ corresponds to an interest-rate risk¹¹. We assume that a consumer has a separable utility function $u(c_1) + \beta Eu(c_2)$ where $c_1 = a - e$ and $c_2 = \tilde{\omega} + \tilde{\theta}e$ corresponds to consumption in the period $t = 1$ and $t = 2$, respectively. In this example a is the initial wealth and β is the discount factor¹². Assume that labor income is risky ($\tilde{\omega}$) and the interest-rate risk is replaced by its expected value ($E(\tilde{\theta}) = \theta$). From the pair of lotteries A_3 and B_3 it follows that a prudent consumer will increase his savings, that is, he will reallocate wealth from the present to the future to face risk. Thus, the disutility generated by future risk is reduced with precautionary savings. In this case, only the income effect arises, which pushes the consumer to increase his savings.

Now assume that labor income risk is replaced by its expected value ($E(\tilde{\omega}) = \omega$)

¹⁰Losing a job is the biggest blow to income because the majority of consumers rely on labor, and the likelihood of future unemployment is increased with financial, economic and political slumps (Lugilde et al., 2019). Also, empirical evidence has shown a significant increase in wage dispersion in last years due to globalization and digitization process (Berlingieri et al., 2017).

¹¹See, for instance, Eeckhoudt and Schlesinger (2008).

¹²Vereshchagina and Hopenhayn (2009) utilized this model to explain why self-financed entrepreneurs can think it's best to invest in risky ventures with no risk premium.

and an interest-rate risky ($\tilde{\theta}$). From the pair of lotteries A'_3 and B'_3 it follows that the income effect pushes the prudent consumer to increase savings to increase the wealth in the period where he faces risk. The substitution effect, on the other hand, induces the risk-averse consumer to save less. This occurs when the consumer's exposure to risk increases as a result of saving and investing in assets with risky interest rates. Thus, interest-rate risk is a multiplicative risk because it multiplies the decision variable, i.e. savings, while labor income risk is an additive risk given that it does not multiply the decision variable.

Our model presents characteristics similar to the model of intertemporal consumption under risk. The hybrid entrepreneur faces risk in the entrepreneurial sector and in the labor sector, which can be additive or multiplicative and he must decide how much time to invest in each sector in order to maximize his expected utility, and consequently income and substitution effects will emerge in both sectors.

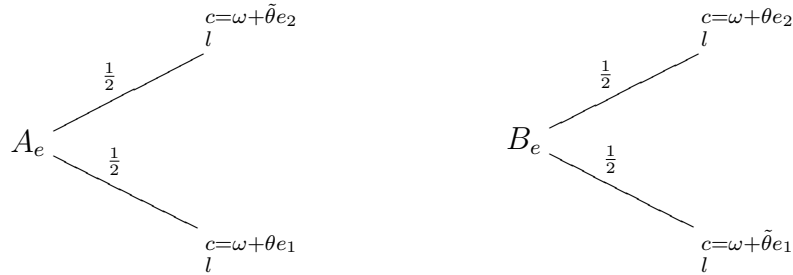
Next, we will use concepts analyzed in this subsection to represent pairs of lotteries that will help us understand hybrid entrepreneur's precautionary behavior. To do this we will use a utility function with two arguments, that is, $u(c, l)$, where c is consumption and l is leisure.

2.6.3 Harm disaggregation in consumption in the behavioral approach

In this subsection, we will introduce the strength of downside risk in entrepreneurship and labor. These strengths represent preferences for harm disaggregation in consumption by changes in time allocated between entrepreneurship (e) and paid employment (L).

Strength of risk aversion and downside risk aversion on entrepreneurship

Consider first the traditional case when the source of income risk comes only from the entrepreneurial sector, where $\tilde{\theta}$ is a multiplicative risk. Let us define the strength of risk averse and downside risk aversion in entrepreneurship (SRD_e) as the difference (in expected utility terms) between 50-50 lotteries A_e and B_e :



where $\omega = wL$, e_i for $i = 1, 2$ are the time allocated to the entrepreneurial activity and $e_2 > e_1$. Also, $\tilde{\theta}$ and $E[\tilde{\theta}] = \theta$ are the risky return on entrepreneurship and the expected return on entrepreneurship respectively. These pair of lotteries are essentially the same of lotteries A'_3 and B'_3 , in which time allocated to labor and leisure remain constant through states of nature.

Thus, in expected utility framework $A_e \succeq B_e$ if and only if

$$SRD_e \equiv Eu(A_e) - Eu(B_e) \geq 0 \quad (2.18)$$

then by Lemma 3 we know that

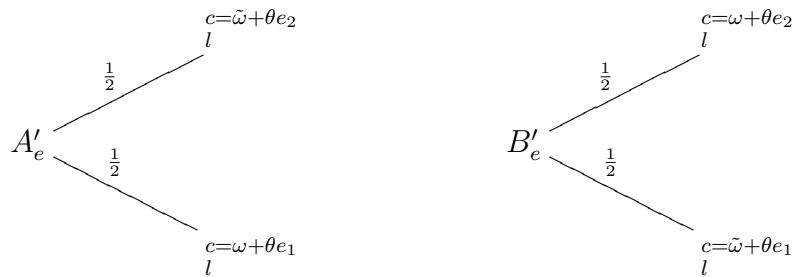
$$SRD_e \geq 0 \text{ iff } -\theta e \frac{u_{ccc}}{u_{cc}} > 2 \quad (2.19)$$

For the SRD_e to be positive, prudence must be strong enough in the sense that the index $-\theta e \frac{u_{ccc}}{u_{cc}}$ is greater than 2. In this problem we have two opposing forces. On the one hand, risk aversion leads to a decrease in risk-taking and therefore, hybrid entrepreneurs will want to reduce their risk exposure by decreasing the time allocated to entrepreneurial activities. On the other hand, prudence implies an aversion to downside

risk, which induces a precautionary time allocation effect on entrepreneurship, with an opposite effect. We can call this index $(-\theta e \frac{u_{ccc}}{u_{cc}})$ the partial relative prudence index in entrepreneurship as a parallel concept to the partial relative prudence index.

Since $e_2/e_1 > 1$, with lottery A_e the entrepreneur would be bearing greater risk when he is richer ($\tilde{\theta}$ is a mean-preserving spread of θ). As a result, if he prefers to bear a greater degree of risk as he becomes wealthier, the decision between A_e and B_e will be dictated by the respective strengths of two opposing effects. The first is the substitution effect which makes A_e less attractive because the entrepreneur is risk averse so he wants to escape from risk. The second is the income effect which makes A_e more attractive because the entrepreneur is prudent, which means that he prefers to bear a greater risk when he is richer, i.e., with A_e the harm of a greater risk and a lower wealth are disaggregated. Thus the substitution effect is dominated by the income effect if and only if the partial relative prudence index is large than 2.

Now consider the uncommon case where the source of income uncertainty comes only from the paid-employment sector, where $\tilde{\omega}$ is an additive risk. Let us define the strength of downside risk aversion in entrepreneurship (SD_e) as the difference (in expected utility terms) between 50-50 lotteries A'_e and B'_e :



where $\tilde{\omega} = \tilde{w}L$ and $E(\tilde{\omega}) = wL$, are the labor income risk and its expectation respectively. This pair of lotteries is essentially the same pair of lotteries A_3 and B_3 , in which time allocated to labor and leisure remain constant through states of nature.

Thus, in expected utility framework $A'_e \succeq B'_e$ if and only if

$$SD_e \equiv Eu(A'_e) - Eu(B'_e) \geq 0 \tag{2.20}$$

then by Lemma 2 we know that

$$SD_e \geq 0 \text{ if } u_{ccc} > 0 \quad (2.21)$$

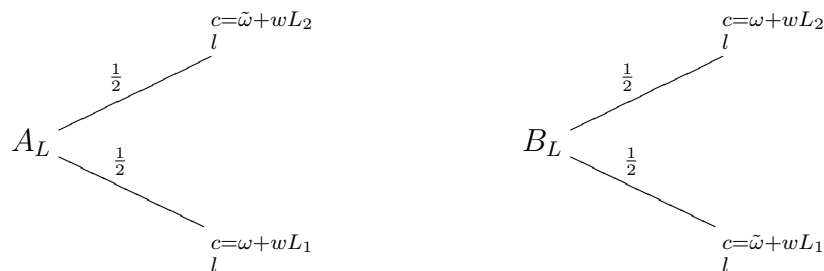
Therefore, in the presence of an additive risk, the income effect prevails (prudent behavior since $u_{ccc} > 0$). This means that a hybrid entrepreneur prefers to attach a risk to the state with the greater wealth, that is, to the state in which he allocates the most time to entrepreneurial activity (θe_2).

In conclusion, regardless of the source of income risk, we have shown that the strength of downside risk aversion in entrepreneurship (SD_e) and strength of risk aversion and downside risk aversion in entrepreneurship (SRD_e) entails a preference for combining risk with higher levels of income produced by a concordant increased time allocation.

Strength of risk aversion and downside risk aversion on labor

Now we will analyze the effect of risk on the allocation of time into labor (or paid employment). The results are similar to those obtained in the previous section. However, we will see that the risk in the entrepreneurial sector becomes an additive risk, while the risk in the paid employment sector becomes a multiplicative risk.

Consider again the traditional case in which the source of income risk comes only from the entrepreneurial sector. Let us define the strength of downside risk aversion in labor (SD_L) as the difference (in expected utility terms) between the 50-50 lotteries A_L and B_L :



where L_i for $i = 1, 2$ are the time allocated to paid employment and $L_2 > L_1$. Also, $\tilde{\theta}$ and $E[\tilde{\theta}] = \theta$ are the risky return on entrepreneurship and its expectation. Let $\tilde{\omega} = \tilde{\theta}e$ and $E(\tilde{\omega}) = \theta e$ be the time allocated to entrepreneurship (in this exercise it does not change), then $\tilde{\omega}$ becomes an additive risk.

Thus, in expected utility framework $A_L \succeq B_L$, if and only if

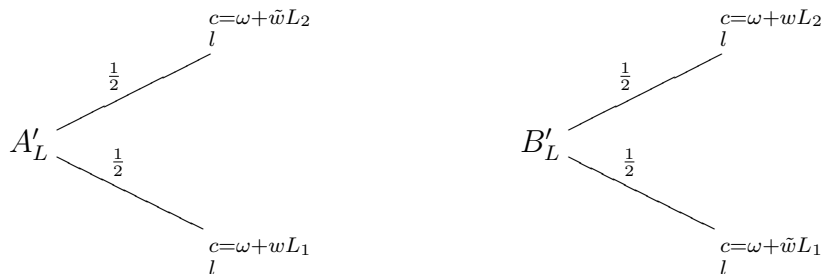
$$SD_L \equiv Eu(A_L) - Eu(B_L) \geq 0 \quad (2.22)$$

then by Lemma 2, we have

$$SD_L \geq 0 \text{ iff } u_{ccc} > 0 \quad (2.23)$$

In consequence, the strength of downside risk aversion in labor (paid employment) is positive only when the individual is prudent. The intuition mimics the previous case: the individual prefers to allocate risk in the good state, which combines risk with higher levels of income that in this case is produced by an increased time allocated to non-random paid employment (wL_2). In this case, only arises income effect.

Now consider that the source of income risk comes only from the paid employment sector. In this case, the risk is multiplicative because the time allocated to labor changes through states of nature. So both variables interact with each other. Let us define the strength of risk aversion and downside risk aversion in labor (SRD_L) as the difference in the following 50-50 lotteries A'_L and B'_L :



Thus, in expected utility framework, $A'_L \succeq B'_L$ if and only if

$$SRD_L \equiv Eu(A'_L) - Eu(B'_L) \geq 0 \quad (2.24)$$

and by Lemma 3 we know that

$$SRD_L \geq 0 \text{ if } -wL \frac{u_{ccc}}{u_{cc}} > 2 \quad (2.25)$$

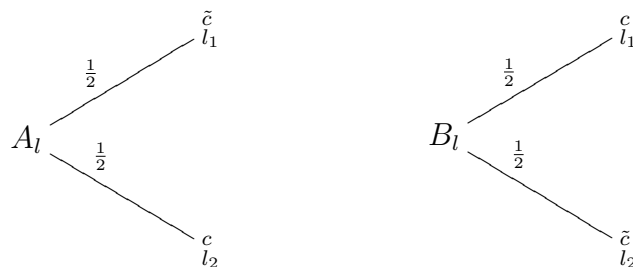
This result tells us that for SRD_L to be positive, the partial prudence index in labor ($-wL \frac{u_{ccc}}{u_{cc}}$) must be greater than 2. Again, this is very similar to the condition found in the previous section, and both conditions are in line with traditional results from the risk literature.

The income effect pushed the hybrid entrepreneurs to increase the time allocated to labor to increase its wealth. The substitution effect, on the other hand, arises only in the presence of a multiplicative risk and pushes the hybrid entrepreneurs to reduce his level of time allocated to paid employment. This is because he allocates time into the paid employment sector with risky return, which increases his exposure to risk. And, in the end, we have a positive net precautionary effect on the time dedicated to paid-employment when income effect dominates the substitution effect.

2.6.4 Harm disaggregation in leisure in the behavioral approach

In this section we address the strength of downside risk aversion on leisure. To do this, we take the comparative statics analysis to the second argument of the utility function and keep constant the first argument of the utility function (consumption).

Consider the pair of lotteries A_l and B_l , where $l_1 > l_2$ and \tilde{c} is random consumption with $E(\tilde{c}) = c$.



where l_i for $i = 1, 2$ are the time allocated to leisure.

In this pair of lotteries, consumption is random because the hybrid entrepreneur faces risk in the entrepreneurial sector or in paid employment sector. On the other hand, $l_1 > l_2$ when $e_2 > e_1$ or $L_2 > L_1$.

Next, we will derive the condition behind the definition of strength of downside risk aversion on leisure that encompasses the different combinations of risk and leisure described in the previous paragraph.

Note that lottery A_l combines good with bad, since l_1 is attached to the state in which the random consumption (\tilde{c}) appears, while l_2 is attached to the state in which its expected value (c) appears. The opposite case occurs with the B_l lottery in which good is combined with good and bad with bad. Thus l_2 has more one-degree risk than l_1 (l_1 dominates l_2 via first-order stochastic dominance) which is reflected in the second argument of the utility function, while \tilde{c} has more two-degree risk than c (\tilde{c} is a mean-preserving spread of c) which is reflected in the first argument. In the expected utility framework, $A_l \succeq B_l$ if and only if

$$Eu(A_l) = \frac{1}{2}Eu(\tilde{c}, l_1) + \frac{1}{2}u(c, l_2) \geq \frac{1}{2}u(c, l_1) + \frac{1}{2}Eu(\tilde{c}, l_2) = Eu(B_l) \quad (2.26)$$

where $SD_l = Eu(A_l) - Eu(B_l)$ is the disutility associated to the one-degree increase in risk (leisure) and mean-preserving increase in risk (consumption) and represents the strength of downside risk aversion on leisure. Condition (26) can be rewritten as:

$$[Eu(\tilde{c}, l_1) - u(c, l_1)] - [Eu(\tilde{c}, l_2) - u(c, l_2)] \geq 0 \quad (2.27)$$

Let $\Gamma(\tilde{c}, c, l) = Eu(\tilde{c}, l) - u(c, l)$ be denoted. Differentiating Γ with respect to l we get $\Gamma_l = Eu_l - u_l$. Condition (27) can be rewritten as $\Gamma(\tilde{c}, c, l_1) - \Gamma(\tilde{c}, c, l_2) \geq 0$ if and only if $\Gamma_l \geq 0$ which is equivalent to $Eu_l \geq u_l$. By Jensen's inequality $g(c) = u_l$ is convex function whenever $g_{cc} = u_{lcc} > 0$.

The results above shows that regardless of the source of income uncertainty, for the strength of downside risk aversion on leisure (SD_l) to be positive, then u_{lcc} has

to be positive. Following [Eeckhoudt et al. \(2007\)](#) and [Liu2019a](#), we can interpret the condition $u_{lcc} > 0$ as “cross-prudence in leisure”. Therefore, higher levels of leisure help entrepreneurs to mitigate the detrimental effects on consumption created by the presence of risky returns on entrepreneurship or in the paid employment sector (with risky wages). Hybrid entrepreneurs can raise their levels of leisure by decreasing the time devoted to any of these two alternatives.

The SD_l entails a strength towards the preference for combining bad outcomes from income risk affecting consumption (c) with higher levels of leisure (l) by reducing the time allocated to entrepreneurship or the time allocated to paid employment.

Based on pairs of lotteries 50-50, the following lemma characterizes a cross-prudence decision maker.

Lemma 4. *In the Expected Utility framework, assuming $l_1 > l_2$, $c > 0$ and \tilde{c} is a mean-preserving spread of c , then $A_l \succeq B_l$ if and only if $u_{lcc} > 0$.*

2.7 Optimal time allocations

Based on the definitions and the preliminary results developed in the previous section, we now analyze how individuals allocate their time between occupations in three scenarios: first, under risky returns on entrepreneurship (the usual way according to the occupational choice literature), under risky labor (earning a random wage), and under risky returns on both occupations, entrepreneurship and paid employment. In each scenario, we analyze cases where these two occupational choices interact as substitutes or complements. Also, keep in mind that e^* and L^* denote the optimal solutions under uncertainty (problem (1a)) and e^{**} and L^{**} denote the optimal solutions under certainty (problem (8)).

2.7.1 Risky returns on entrepreneurship

Proposition 1. *Let e and L be substitute occupational choices ($V_{eL} < 0$), if the hybrid entrepreneur is risk-averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship that exceeds 2, ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$). Then, under risky entrepreneurship, when $SRD_e > SD_l > SD_L$, the hybrid entrepreneur will decide to make a precautionary entrepreneurial time allocation and a precautionary labor time deallocation. The opposite occurs when $SRD_e < SD_l < SD_L$, i.e., the hybrid entrepreneur will decide to make a precautionary entrepreneurial time deallocation and a precautionary labor time allocation.*

Proof. See the Appendix A. □

Proposition 1 includes four conditions: $SRD_e > 0$, $SD_L > 0$, $SD_l > 0$ and $SRD_e > SD_l > SD_L$. In this scenario, the hybrid entrepreneur faces a multiplicative risk in the entrepreneurship sector. So condition $SRD_e > 0$ means that the income effect prevails over the substitution effect ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$), emerging a precautionary effect in time allocated to entrepreneurship. On the other hand, condition $SD_l > 0$ tells us the entrepreneur is cross-prudent ($u_{lcc} > 0$) which pushes to increase the level of leisure and, therefore, reduce the time allocated to entrepreneurship. However, condition $SRD_e > SD_l$ prevents the time precautionary for leisure from reversing the precautionary effect on the time allocated to entrepreneurship ($e^* > e^{**}$). In this same scenario, risky returns on entrepreneurship become a additive risk in the labor sector. So, condition $SD_L > 0$ tells us that the hybrid entrepreneur is prudent ($u_{ccc} > 0$) which pushes to increase the level time allocated to labor. Nevertheless, condition $SD_l > SD_L$ causes the time precautionary for leisure to reverse the precautionary effect on the time allocated to labor ($L^* < L^{**}$).

A similar argument can be given for the condition $SRD_e < SD_l < SD_L$. As the strength of downside risk aversion on leisure is higher than the strength of risk averse and downside risk aversion on entrepreneurship ($SRD_e < SD_l$), then the time precau-

tionary to leisure prevails over precautionary effect on time allocated to entrepreneurship, which causes a reduction to time allocated to entrepreneurship ($e^* < e^{**}$). On the other hand, as the strength of downside risk aversion on labor is higher than the strength of downside risk aversion on leisure, then precautionary effect on time allocated to labor dominates the force of precautionary time allocation to leisure ($L^* > L^{**}$).

Proposition 2. *Let e and L be complement occupational choices ($V_{eL} > 0$), if the hybrid entrepreneur is risk-averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship that exceeds 2, ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$). Then, under risky entrepreneurship when $SRD_e > SD_l$ and $SD_L > SD_l$, the hybrid entrepreneur decides to make a precautionary entrepreneurial time allocation and a precautionary labor time allocation as well. The opposite occurs when $SRD_e < SD_l$ and $SD_L < SD_l$, i.e., the hybrid entrepreneur will decide to make a precautionary entrepreneurial time deallocation and a precautionary labor time deallocation.*

Proof. See the Appendix B. □

Proposition 2 includes five conditions: $SRD_e > 0$, $SD_L > 0$, $SD_l > 0$, $SRD_e > SD_l$ and $SD_L > SD_l$. The interpretation of the first three conditions is the same that we did to give intuition to proposition 1. Conditions $SRD_e > SD_l$ and $SD_L > SD_l$ prevents that the time precautionary allocated to leisure to reverse the precautionary effect on the time allocated to entrepreneurship ($e^* > e^{**}$) and labor ($L^* > L^{**}$), respectively. The opposite occurs when $SRD_e < SD_l$ and $SD_L < SD_l$, i.e., the hybrid entrepreneur decide to make a precautionary leisure time allocation, reducing the time allocated to entrepreneurship ($e^* < e^{**}$) and labor ($L^* < L^{**}$).

Thus the results from Proposition 2 show us that when occupational choices interact as complements, the decision to allocate more time to the entrepreneurship does not imply the decision to deallocate time from labor but rather to allocate more time to that occupational choice as long as the strengths of risk aversion and downside risk aversion

on entrepreneurship and strengths of downside risk aversion on labor are both higher than the strength of downside risk aversion on leisure.

Proposition 1 and Proposition 2 show that precautionary entrepreneurial and labor time allocation are mediated by the sign of V_{Le} and depend on the comparison between the relative magnitudes of the effects of the strength of downside risk aversion in entrepreneurship, labor and leisure. Therefore, the usual sufficient conditions of risk aversion and DARA used in entrepreneurship literature (Kihlstrom and Laffont, 1979; Cressy, 2000; Hvide and Panos, 2014; Bonilla and Vergara, 2021) are not enough when the entrepreneur is a hybrid entrepreneur. In this case, the strength of downside risk aversion becomes the key factor in inducing precautionary behavior.

2.7.2 Risky returns on labor

We will now study what happens with the optimal time allocations in each occupation when the source of risk comes from the paid-employment sector. In this case the hybrid entrepreneur's optimization problem is

$$\max_{L,e} Eu(\tilde{w}L + \theta e, T - L - e) \quad (2.28)$$

where \tilde{w} is a positive random variable describing the random wage and θ is a non-random return on entrepreneurship.

The analysis is analogous to the previous case where the source of risk comes from risky entrepreneurship $\tilde{\theta}$. Therefore, the following two propositions mirror Proposition 1 and 2 in the previous section. However, in this case, e^* and L^* are the optimal choices in problem (38), while e^{**} and L^{**} are the optimal choice of the following:

$$\max_{L,e} u(E[\tilde{w}]L + \theta e, T - L - e) \quad (2.29)$$

Proposition 3. *Let e and L be substitute occupational choices ($V_{eL} < 0$), if the hybrid entrepreneur is risk-averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in labor that exceeds 2,*

$(-wL \frac{u_{ccc}}{u_{cc}} > 2)$. Then, in risky labor when $SD_e > SD_l > SRD_L$, the hybrid entrepreneur decides to make a precautionary entrepreneurial time allocation and a precautionary labor time deallocation. The opposite occurs when $SD_e < SD_l < SRD_L$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time deallocation and a precautionary labor time allocation.

Proof. See the Appendix C. □

Proposition 4. Let e and L be complement occupational choices ($V_{eL} > 0$), if the hybrid entrepreneur is risk-averse ($u_{cc} < 0$), cross-prudent ($u_{lcc} > 0$) in leisure, prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in labor that exceeds 2, $(-wL \frac{u_{ccc}}{u_{cc}} > 2)$. Then, in risky labor when $SD_e > SD_l$ and $SRD_L > SD_l$, the hybrid entrepreneur will decide to make both a precautionary entrepreneurial time allocation and a precautionary labor time allocation. The opposite occurs when $SD_e < SD_l$ and $SRD_L < SD_l$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time deallocation and a precautionary labor time deallocation.

Proof. See the Appendix D. □

The results of Propositions 3 and 4 confirm our results from Propositions 1 and 2 by providing the sufficient conditions for precautionary entrepreneurial and labor time allocation (or deallocation). As said earlier, this conditions depend on the relative magnitude of the strength of downside risk aversion in entrepreneurship, leisure, and labor. However, despite the source of income risk (whether entrepreneurship or labor), the results remain the same. We know that occupations can interact as substitutes or as complements and that hybrid entrepreneurs can mitigate the effects of risky income from either labor or entrepreneurship through precautionary behavior. Our results show that a hybrid entrepreneur will always prefer to allocate more time to entrepreneurship in the substitute case, or to both entrepreneurship and labor, in the complement case if it will helps them better apportion the detrimental effects of risk. Therefore, hybrid

entrepreneurs will self-insure to increase the expected value of their random income, and in the case of complements, to increase the deterministic part of their income.

2.7.3 Risky returns on both entrepreneurship and labor

Finally, taking into account the recent experience with the pandemic that affected both products and labor markets, we analyze the case of random returns on both occupations. We assume that these two risks are independent. Our case is where entrepreneurship is associated with the economic cycle and the level of intrinsic competition within the industry while the risk associated with the returns on labor is connected with specific individual characteristic like hierarchical position in the organization and years of experience.

In this case, the maximization problem of a hybrid entrepreneur becomes

$$\max_{L,e} Eu(\tilde{w}L + \tilde{\theta}e, T - L - e) \quad (2.30)$$

The optimal decision (e^*, L^*) is obtained when the first order conditions of problem (2.30) are satisfied and the second order conditions hold.

Moreover, similarly to the previous sections, to study precautionary entrepreneurial and labor time allocation or deallocation by comparing the optimal choices in (2.30) to those in the following problem:

$$\max_{L,e} u(E[\tilde{w}]L + E[\tilde{\theta}]e, T - L - e) \quad (2.31)$$

In the next propositions let us call e^* and L^* the optimal choices in (2.30) and e^{**} and L^{**} the optimal choices in (2.31).

Proposition 5. *Let e and L be substitute occupational choices ($V_{eL} < 0$), if the hybrid entrepreneur is risk-averse ($u_{cc} < 0$), cross-prudent in leisure ($u_{lcc} > 0$), prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship and labor that exceeds 2, ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$ and $-wL \frac{u_{ccc}}{u_{cc}} > 2$). Then, in simultaneously risky*

entrepreneurship and labor, when $SRD_e > SD_l > SRD_L$, the hybrid entrepreneur will decide to make a precautionary entrepreneurial time allocation and a precautionary labor time deallocation. The opposite occurs when $SRD_e < SD_l < SRD_L$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time deallocation and a precautionary labor time allocation.

Proof. See the Appendix E. □

Proposition 6. Let e and L be complement occupational choices ($V_{eL} > 0$), if the hybrid entrepreneur is risk-averse ($u_{cc} < 0$), cross-prudent in leisure ($u_{lcc} > 0$), prudent in consumption ($u_{ccc} > 0$), and has a partial prudence index in entrepreneurship and labor that exceeds 2, ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$ and $-wL \frac{u_{ccc}}{u_{cc}} > 2$). Then, in simultaneously risky entrepreneurship and labor, when $SRD_e > SD_l$ and $SRD_L > SD_l$, the hybrid entrepreneur will decide to make both a precautionary entrepreneurial time allocation and a precautionary labor time allocation. The opposite occurs when $SRD_e < SD_l$ and $SRD_L < SD_l$, i.e., the hybrid entrepreneur decides to make a precautionary entrepreneurial time deallocation and a precautionary labor time deallocation.

Proof. See the Appendix E. □

Propositions 5 and 6 demonstrate that the precautionary time allocation behavior of hybrid entrepreneurs depends first on how the occupations interact with each other and second, on the relative magnitude of the strength of downside risk in entrepreneurship, labor and leisure. The analysis of this particular case (independent risks in both sectors at the same time) does not differ from the previous ones, where precautionary time allocation to sectors is a way to self-insure against risk. In particular, higher downside risk aversions in entrepreneurship tend to make a hybrid entrepreneur more willing to put additional hours into entrepreneurship as a way to avoid, at all costs, bad states of nature from that activity and to increase the expected payment from entrepreneurship. This means that, if both activities are substitutes, then time allocation to labor

will decrease while, if they are complements, time allocation to labor will increase. A summary of the results is presented in the following table:

Table 2.1: e^* and L^* , under labor income risk

Source of risk	V_{eL}	e	L	Sufficient condition	Risk preferences
Entrepreneurial sector	< 0	$e^* > e^{**}$	$L^* < L^{**}$	$SRD_e > SD_l > SD_L$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-\theta e \frac{u_{ccc}}{u_{cc}} > 2$
	> 0	$e^* > e^{**}$	$L^* > L^{**}$	$SRD_e > SD_l$ and $SD_L > SD_l$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-\theta e \frac{u_{ccc}}{u_{cc}} > 2$
Paid-employment sector	< 0	$e^* > e^{**}$	$L^* < L^{**}$	$SD_e > SD_l > SRD_L$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2$
	> 0	$e^* > e^{**}$	$L^* > L^{**}$	$SD_e > SD_l$ and $SRD_L > SD_l$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2$
Both sectors	< 0	$e^* > e^{**}$	$L^* < L^{**}$	$SRD_e > SD_l > SRD_L$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0, -\theta e \frac{u_{ccc}}{u_{cc}} > 2$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2$
	> 0	$e^* > e^{**}$	$L^* > L^{**}$	$SRD_e > SD_l$ and $SRD_L > SD_l$	$u_{cc} < 0, u_{lcc} > 0, u_{ccc} > 0, -\theta e \frac{u_{ccc}}{u_{cc}} > 2$, and $-wL \frac{u_{ccc}}{u_{cc}} > 2, u_{cc} < 0$,

2.8 Conclusion and implications

This study aims to shed more light on hybrid entrepreneurship, a trend that will become more prevalent in the near future. Technological innovation and labor contract flexibility provide a recipe that will catapult hybrid entrepreneurship to new heights.

Traditional literature on entrepreneurship studying the transition from labor to self-employment has been based on the occupational choice discrete model. However, some recent literature (see [Folta et al., 2010](#)) as well as the disruptive effect of the pandemic that made employment relationship more flexible and granted more freedom in the use of working time through remote technologies, highlights the importance of studying the intermediate case of hybrid entrepreneurship in order to better understand how the productive sector and the labor market interact in the post-Covid 19 economy. This is specially important when the economy gets hit by a negative and unprecedented shock and policy-makers need to make policy decisions to help the economy bounce quickly back from the economic downturn and regain a healthy economic growth.

We developed a bi-variate choice model in which entrepreneurship and paid-employment can interact as substitutes or complements, provided the conditions exist for precautionary time allocation to entrepreneurship in the face of increased risk. We also studied

different cases, such as when the risk comes solely from the entrepreneurial sector (the classical case), when risk comes solely from the labor sector (rare, blackboard experimental thought) and when risk comes from both sectors, which is the new and more realistic perspective of hybrid entrepreneurship.

One of the key findings in our bi-variate model of hybrid entrepreneurship is that many of the results rest on conditions of downside risk aversion. In particular, when the strength of downside risk aversion in entrepreneurship is high enough, when risk increases, a hybrid entrepreneur will allocate more time to entrepreneurship (precautionary behavior) as a way to avoid, at all costs, bad entrepreneurship results.

Our findings have implications for entrepreneurship scholars and policymakers. First, we connect an entrepreneurial economic model with the risk apportionment approach based on decision theory and demonstrate that hybrid entrepreneurs can self-insure in response to income risks by allocating more time to entrepreneurship. Our results therefore contribute to the long-standing puzzle identified by [Parker et al. \(2005\)](#) about why self-employed individuals work longer hours for lower average compensation than their employed counterparts do.

Second, our findings show that hybrid entrepreneurs who have a precautionary time allocation behavior are willing to allocate more time to entrepreneurship in scenarios of increased risk. Therefore, boosting hybrid entrepreneurship during recessions would be a viable economic policy on economic recovery through entrepreneurial innovation, especially considering the counter-cyclical dynamic between self-employment and the business cycle (see [Fossen, 2021](#)).

Third, our findings have a very practical application for organizations that employ hybrid entrepreneurs. It is critical for these organizations to understand how these risks affect their employees' time devoted to both occupations, especially if they believe that this type of worker is more imaginative, motivated, and entrepreneurial, positively impacting the work of their businesses through emerging opportunities for innovation ([Marshall et al., 2019](#); [Sessions et al., 2021](#); [Urbig et al., 2021](#)).

Our analysis, however, has certain drawbacks. First, we do not account for entrepreneurial inclinations; that is, some people have a natural proclivity for entrepreneurship and hence begin in paid work just as a temporary activity while "getting things right" in order to transition into entrepreneurship. As a result, these types of personalities are not adequately represented in the rational marginal analysis that we have just constructed. Another limitation is the exclusive utilization of 50-50 lotteries, which restricts the analysis to several alternative potential distributions.

Future research should empirically explore the hybrid entrepreneur's precautionary time allocation behavior with data. For example, increased risk can be measure using the EPU index as a way to test the implications of our model in the real world. Also, since entrepreneurial choice theory is primarily focused on 50-50 lotteries; a far more technically complex study involving various types of lotteries is left for future research.

The new economy has incorporated significant new flexibility in the labor market that must be considered in any serious economic policy that aims to boost the entrepreneurial sector, and our paper provides an initial analytical conceptualization of this issue.

2.9 Appendix

Appendix A

Proof of Proposition 1.

Proof. We want to show that under the previous assumptions, when $V_{eL} < 0$ and the returns on entrepreneurship are risky, then, a sufficient condition for $e^* > e^{**}$ and $L^* < L^{**}$ is $SRD_e > SD_l > SD_L$. This is exactly what is represented in figure 2a, which is the case in which we replace $E[\tilde{\theta}]$ by $\tilde{\theta}$ to incorporate risky entrepreneurship and the curve LL moves to the left while the curve ee moves to the right. We will now determine the conditions for such movements.

Let us suppose that $e^* > e^{**}$. Then, evaluating e^{**} into the FOC of eq. (2) implies:

$$E\tilde{\theta}u_c(wL + \tilde{\theta}e^{**}, T - L - e^{**}) - Eu_l(wL + \tilde{\theta}e^{**}, T - L - e^{**}) > 0 \quad (2.32)$$

while FOC of problem (8) guarantees that

$$E[\tilde{\theta}]u_c(wL + E[\tilde{\theta}]e^{**}, T - L - e^{**}) - u_l(wL + E[\tilde{\theta}]e^{**}, T - L - e^{**}) = 0 \quad (2.33)$$

Using eq. (2.32) and eq. (2.33) together we see that

$$E\tilde{\theta}u_c - E[\tilde{\theta}]u_c > Eu_l - u_l \quad (2.34)$$

Given eq. (2.19) and lemma 4 this occurs if $SRD_e > SD_l$, which is the first part of the sufficient condition of the proof.

Now, let us focus on L . We know that $L^* < L^{**}$ is satisfied when:

$$wEu_c(wL^{**} + \tilde{\theta}e, T - L^{**} - e) - Eu_l(wL^{**} + \tilde{\theta}e, T - L^{**} - e) < 0 \quad (2.35)$$

while the FOC of problem eq. (2.8) guarantees that

$$wu_c(wL^{**} + E[\tilde{\theta}]e, T - L^{**} - e) - u_l(wL^{**} + E[\tilde{\theta}]e, T - L^{**} - e) = 0 \quad (2.36)$$

Using eq. (2.35) and eq. (2.36) together we see that

$$wEu_c - wu_c < Eu_l - u_l \quad (2.37)$$

Given eq. (2.23) and lemma 4 this occurs if $SD_L < SD_l$.

Summarizing, under risky entrepreneurship and when $V_{eL} < 0$, precautionary entrepreneurial time allocation and precautionary labor time deallocation occurs when:

$$SRD_e > SD_l > SD_L \quad (2.38)$$

Following the same procedure, we show straightforwardly that precautionary entrepreneurial time deallocation and precautionary labor time allocation occurs when $SRD_e < SD_l < SD_L$ which is case illustrated in figure 2b. \square

Appendix B

Proof of Proposition 2.

Proof. We want to show that under the previous assumptions, when $V_{eL} > 0$ and the returns on entrepreneurship are risky, then, a sufficient condition for $e^* > e^{**}$ and $L^* > L^{**}$ is $SRD_e > SD_l$ and $SD_L > SD_l$. This is exactly what is represented in Figure 3a, which is the case where we replace $E[\tilde{\theta}]$ by $\tilde{\theta}$ to incorporate risky entrepreneurship and the curve LL moves to the right while the curve ee moves to the left. We will now determine the conditions for such movements, and since the proof is homologous with the previous one, we will present it very succinctly. So, when $e^* > e^{**}$ and $L^* > L^{**}$ we know that

$$E\tilde{\theta}u_c - E[\tilde{\theta}]u_c > Eu_l - u_l \quad (2.39)$$

$$wEu_c - wu_c > Eu_l - u_l \quad (2.40)$$

Similar to the proof of Proposition 1 and in accordance with eqs. (2.19) and (2.23) and lemma 4, this occurs as long as

$$SRD_e > SD_l \text{ and } SD_L > SD_l \quad (2.41)$$

Straightforwardly calculations show that precautionary entrepreneurial time deallocation and precautionary labor time deallocation occurs when $SRD_e < SD_l$ and $SD_L < SD_l$ which is case illustrated in figure 3b. \square

Appendix C

Proof of Proposition 3.

Proof. We want to show that under the previous assumptions that, when $V_{eL} < 0$ and the return on labor is risky, then, a sufficient condition for $e^* > e^{**}$ and $L^* < L^{**}$ is $SD_e > SD_l > SRD_L$. Please, note that for $e^* > e^{**}$ and $L^* < L^{**}$ to be satisfied we only need that

$$\theta Eu_c - \theta u_c > Eu_l - u_l \quad (2.42)$$

$$E\tilde{w}u_c - E[\tilde{w}]u_c < Eu_l - u_l \quad (2.43)$$

In accordance with eq. (2.21) and lemma 4, we know that eq. (2.42) occurs when $SD_e > SD_l$ and from eq. (2.25) and lemma 4, we know that eq. (2.43) occurs if $SRD_L < SD_l$. To summarize in the same way as for Proposition 1 proof, this occurs as long as $SD_e > SD_l > SRD_L$.

In this scenario, the hybrid entrepreneur faces an additive risk in the entrepreneurship sector. So, condition $SD_e > 0$ tells us the entrepreneur is prudent ($u_{ccc} > 0$) which pushes to increase the time allocated to entrepreneurship. Therefore, condition $SD_e > SD_l$ means precautionary entrepreneurship effect is stronger than precautionary leisure effect and as results $e^* > e^{**}$. Otherwise, as the entrepreneur faces a multiplicative risk in labor sector, then condition $SRD_L > 0$ tells us that the income effect prevails over substitution effect, that is, $-wL\frac{u_{ccc}}{u_{cc}} > 2$. Then, condition $SD_l > SRD_L$ means precautionary leisure effect is stronger than precautionary labor effect and as results $L^* < L^{**}$.

A similar argument we can to give to explain the condition $SD_e < SD_l < SRD_L$, where $SD_e < SD_l$ implies $e^* < e^{**}$ while $SD_l < SRD_L$ implies $L^* > L^{**}$. \square

Appendix D

Proof of Proposition 4.

Proof. The proof and interpretation of the results is similar to proposition 2, except that we use conditions eqs. (2.21) and (2.25) and lemma 4. So condition $SD_e > (<)SD_l$ implies $e^* > (<)e^{**}$ while condition $SRD_L > (<)SD_l$ implies $L^* > (<)L^{**}$. \square

Appendix E

Proof of Proposition 5.

Proof. We want to show that under the above assumptions, when $V_{eL} < 0$ and the return on both labor and entrepreneurship is risky, then a sufficient condition for $e^* > e^{**}$ and

$L^* < L^{**}$ is $SRD_e > SD_l > SRD_L$. Note that for $e^* > e^{**}$ and $L^* < L^{**}$ to be satisfied we only need that

$$E\tilde{\theta}u_c - E[\tilde{\theta}]u_c > Eu_l - u_l \quad (2.44)$$

$$E\tilde{w}u_c - E[\tilde{w}]u_c < Eu_l - u_l \quad (2.45)$$

Condition (2.44) was already proved in proposition 1 while condition (2.45) was proved in proposition 3. Then, from both conditions is deduced that $SRD_e > SD_l > SRD_L$.

In this new scenario, the hybrid entrepreneur faces a multiplicative risk in both the entrepreneurship sector as in labor sector. So conditions $SRD_e > 0$ tells us that the income effect prevails over the substitution effect when increase in risk occurs in the entrepreneurial sector ($-\theta e \frac{u_{ccc}}{u_{cc}} > 2$) while $SRD_L > 0$ means that the income effect prevails over the substitution effect when increase in risk occurs in the labor sector ($-wL \frac{u_{ccc}}{u_{cc}} > 2$). As we saw before, $SRD_e > SD_l$ implies $e^* > e^{**}$, i.e., precautionary effect in time allocated to entrepreneurship is stronger than precautionary effect in time allocated to leisure. Also, $SD_l > SRD_L$ implies $L^* < L^{**}$, i.e., precautionary effect in time allocated to leisure is stronger than precautionary effect in time allocated to labor. The opposite occurs whenever $SRD_e < SD_l < SRD_L$. □

Appendix F

Proof of Proposition 6.

Proof. Analogous to the proofs of propositions 2 and 4. □

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Chapter 3

Uncertainty and export behavior: The SMEs case

3.1 Abstract

This study examines the influence of background risk on small and medium-sized enterprises (SME) production. We accomplish this by constructing a model that exhibits the decision-making process of an entrepreneur who must determine the allocation of sales between the domestic and foreign markets. Entrepreneurs mitigate background risk by strategically distributing their sales between two markets, thus self-insuring against eventual damages.

We employ a compensation method to break down the consequences of background risk shock. Our research focuses on the substitution and income effects arising in both markets due to an increase in nonlinear risk. We give the conditions under which an uninsurable risk (background risk) has a precautionary effect on the optimal sales allocated to each market. These conditions depend on the respective magnitudes of the income and substitution effects and whether the two markets function as complements or substitutes for one another.

Keywords: Exports, Background risk, Nonlinear risk, Entrepreneurship.

JEL Classification: D81; D21; F31; M21

3.2 Introduction

Small and medium-sized enterprises (henceforth SMEs) are critical to a country's development, employment growth, innovation capability, and global competitiveness in a globalized environment ([Paul et al., 2017](#); [Catanzaro and Teyssier, 2021](#)), as well as their role in boosting economic recovery from the global crisis through the export-led recovery strategy ([Mansion and Bausch, 2020](#); [Haddoud et al., 2021](#); [Hessels and](#)

Parker, 2013). Although SMEs doing business abroad tend to obtain better growth rates than those operating only in domestic markets, their proportion of global trade remains relatively small (Garcia-Martinez et al., 2023). Only 26% of SMEs across the 27 member states in the European Union engage in exporting (European Parliament, 2021). SMEs face significant challenges known as the liability of smallness when they expand internationally. Compared to large firms, SMEs often lack the necessary resources, such as finance and detailed foreign market information, for their survival and growth (Buckley, 1989; Kahiya and Dean, 2016; Knight, 2001; OECD, 2019 as cited in Child et al., 2022). Despite their varying resources, they share similar concerns. PwC's Global Treasury Survey (2023) reveals that financial risk management is a top priority, with foreign exchange risk remaining the top economic risk for corporate treasuries.

Internationalized SMEs encounter distinct obstacles and novel sources of uncertainty compared to those operating in domestic markets. These include foreign exchange risk, political risk, unfamiliarity with operating in a new environment, labor constraints, various cultures, and infrastructure issues (Johanson and Vahlne, 2009; Johanson and Vahlne, 1977; Mascarenhas, 1982). The existing body of literature on international entrepreneurship has identified uncertainty as a significant factor in the SME's internationalization context. This uncertainty can be influenced by both the foreign countries where the SME is expanding its operations and the domestic economy of the SME (Child et al., 2022), as well as by environmental factors that shape export behavior (Martineau and Pastoriza, 2016). In next decade, due to the deteriorating global outlook, risk and uncertainty levels will likely increase, resulting in SMEs confronting increased risks regarding the price of goods and services, along with the overall competitive environment, as outlined in the Global Risks Report (2024). This report also addresses both the complex and nonlinear dimensions of global risks, which will inevitably influence the precautionary actions of SMEs.

Unlike large international firms, smaller firms as SMEs must practice risk management significantly more, given their resource limitation and additional challenges (Lu and Beamish, 2001; Paul et al., 2017). While SMEs, like international firms, may

have access to hedging strategies and derivative instruments, such as futures and options, which provide managers with the means to manage risk, the use of derivatives in small and medium-sized enterprises (SMEs) is not very common ([Pennings and Garcia, 2004](#)). Exporting can serve as a strategy to manage risk by distributing sales over multiple markets, allowing SMEs to mitigate the impact of negative shocks, such as a decrease in demand in the domestic market, by replacing sales at home with international sales ([Wagner, 2012](#); [Wagner, 2013](#)). Furthermore, diversification improves the survival chances of firms by reducing the risk of exit, as adverse conditions in one market can be offset by more favorable conditions in other markets ([Holmes and Schmitz, 1990](#); [Parker, 2004](#); [Esteve-Perez et al., 2008](#)). For SMEs doing business overseas, diversifying exports across markets provides a valuable opportunity to manage risk in the absence of access to financial hedging in global markets. Therefore, it is crucial to comprehend the decision-making export behavior of SMEs and how they manage risk when conventional hedging strategies are not feasible. This understanding is relevant for shaping and improving economic policies that enable sustainable growth in increasingly uncertain global markets. With this aim, we examine the impact of background risk on the sales of SMEs in both domestic and foreign markets.

The impact of risk and uncertainty have been extensively studied across various economic subfields, including precautionary saving theory ([Baiardi et al., 2020](#); [Lugilde et al., 2019](#)), optimal labor supply ([Chiu and Eeckhoudt, 2010](#)), competitive firm ([Lippman and McCall, 1981](#); [Bonilla et al., 2024](#)), optimal effort ([Crainich et al., 2016](#); [Wang and Li, 2015](#)), self-employment ([Benitez et al., 2023](#); [Bonilla and Vergara, 2021](#)), health risk ([Gallagher et al., 2020](#)), and pandemic risk ([Stiglitz and Guzman, 2021](#)). Researchers in these subfields have identified intuitive conditions that motivate decision-makers to respond by engaging in precautionary behavior in the face of increased uncertainty. Decision-makers frequently engage in self-insurance by using a variable under their control to buffer against risk. Consequently, we assert that SME entrepreneurs mitigate background risk by strategically distributing their sales between two markets, thus self-insuring against eventual damages.

Therefore, this paper examines the impact of background risk on the sales of an SME in both domestic and foreign markets and seeks to determine the conditions in which the SME will exhibit precautionary behavior in the distribution of its sales. Furthermore, we assume that the risk is both uninsurable and non-linear. [Gutierrez Cubillos and Pastén \(2023\)](#) shown that, in contrast to linear risk models, which tend to oversimplify the decision-making process by failing to capture the nuances of risk attitudes and preferences under varying conditions, nonlinear risks reflect the complexity of real-world scenarios in which agents face uncertain outcomes. Thus, assuming non-linearity in risk, our research enables the study of the export behavior of SMEs within a more realistic context.

SME's export behavior is conceptualized as two distinct dimensions, the decision of export entry (export propensity) and the export decisions of existing exporters (export intensity) ([Martineau and Pastoriza, 2016](#); [Gashi et al., 2014](#)). Entry and post-entry export decisions are determined by numerous factors ([Giarratana and Torrisi, 2010](#)). While existing entrepreneurial literature has extensively examined SMEs' decisions to enter export markets (export propensity) from a variety of theoretical perspectives (e.g. self-selection hypothesis, sunk cost approach, the network approach, resource-based view, stages, Uppsala model, and institutional theory among others), far less attention has been paid to export intensity, especially under conditions of uncertainty. [Gashi et al. \(2014\)](#) emphasized the lack of a comprehensive theory on the behavior of SMEs and international trade. Moreover, most studies rely on linear risk models, which fail to capture the complexity of decision-making processes under nonlinear risks. Our study addresses this gap by introducing a theoretical model that explores how individual (risk preferences) and environmental (background risk) factors interact to influence SME export intensity. To that end, our research contributes to understanding better SME precautionary export behavior in an increasingly uncertain global environment, through the theory of decision-making under risk and uncertainty.

In addition, employing a compensation method, we analyzed the impact of background risk on SME sales, separating it into a substitution effect and an income effect

occurring in both markets. This allowed us to present the complex outcomes of non-linearity in risks in a comprehensive manner regarding income and substitution effects. Essentially, our findings suggest that we are dealing with two opposing forces. At first, if the SMEs exhibit risk-averse behavior, an increase in risk negatively impacts their expected utility. Consequently, their immediate response is to reduce sales in order to avoid experiencing higher levels of pain (substitution effect). However, when faced with increasing uncertainty, a prudent SME owner is more likely to increase sales as a form of self-insurance against the new risk circumstances (income effect). This behavior is observed in both markets. Therefore, the relative importance of these forces will determine the impact of risk on the optimal allocation of sales in both domestic and foreign markets. Our findings also reveal that the interplay between the two markets, whether acting as substitutes or complements, influences the level of precautionary behavior.

In simple terms, the behavior of SMEs described above can be understood as combining a good with a bad. The good is increased revenue from sales, but the bad is a risk. The precautionary effect, or the additional sales generated by uncertainty, acts as a buffer against the negative effects of an uncertainty shock. This implies that the owners of SMEs are able to mitigate background risk and maximize expected utility by utilizing sales reallocation as a control variable. They essentially combine the good and the bad to improve their well-being.

This precautionary effect is similar to the concept of resilience, defined as the ability to respond to unpredictable events to avoid negative consequences, which [Bak et al., \(2023\)](#) explore in the context of SMEs and supply chains. We believe our work contributes to this literature by demonstrating that in scenarios of greater and more complex risks, the risk attitudes of SME owner-managers influence their precautionary behavior and resilience.

This research further examines how background risk affects an SME's sales in domestic and foreign markets using the (μ, σ) -approach, which requires linear interactions among all random variables in the choice set. Assuming linear risk, we relate how a first-order increase in risk affects sales in both domestic and foreign markets to the con-

cept of risk aversion elasticity with respect to μ and the partial relative aversion index. Similarly, we related a second-order increase in risk to the notion of risk aversion elasticity with respect to σ and the partial relative prudence index. Expanding upon these findings, we extend the analysis of [Broll and Mukherjee \(2017\)](#), who argued that uninsured exchange rate risk exclusively impacts foreign sales, focusing only on the foreign market. However, according to their assumptions, we demonstrate that exchange rate risk acts as a multiplicative risk in the foreign sector and it acts as an additive risk in the domestic sectors. This distinction is essential, as their analysis is incomplete since it does not recognize that the elasticity of risk aversion is undefined for additive risk.

The rest of this article is organized as follows. Section 2 presents an exporting model of SMEs doing business overseas while serving the domestic market, facing background risk. Section 3 introduces the risk changes, particularly the concepts of first- and second-degree risk increase, and presents the main results of this paper by the utilization of the compensation approach. Section 4 shows some applications of the linear and multiplicative risk case. Section 5 presents the exporting model of SMEs based on the (μ, σ) -approach. Finally, Section 6 presents some conclusions

3.3 The model

The SME exporting behavior related to the export intensity dimension is characterized as follows. We assume that an internationalized SME is producing a single product in its home country while serving both domestic and foreign markets.

The revenues of the SME are denoted as $R(x, d, e)$, where x represents the sales in the foreign market and d represents the sales in the domestic market. The background risk is denoted by a stochastic variable e . It is assumed that the first derivatives R_x and R_d are positive, whereas the second derivatives R_{xx} and R_{dd} are negative. Additionally, the cross-derivative $R_{xd} = R_{dx}$ is positive. The function revenues is also increasing and concave in e ($R_e > 0$ and $R_{ee} < 0$), which means that a positive shock of the random

variable is beneficial to the SME at a decreasing rate. Finally, both R_x as well as R_d are increasing and concave in e ($R_{xe} > 0$, $R_{de} \geq 0$, $R_{xee} \leq 0$, $R_{dee} \leq 0$), which implies that the random variable raises the incentive to sold, both domestic and foreign markets, at a decreasing rate. It is important that any one of these assumptions regarding the curvature of R guarantees a concave and tractable program and makes sense from an economic perspective. Furthermore, in both markets, we can determine the direction of the income effect and the substitution effect with the use of these assumptions.

The production cost is characterized by a fixed cost FC and a known variable cost function $C(x + d)$, which is twice continuously differentiable, where $C(0) = 0$, $C_x > 0$, $C_d > 0$, $C_{xx} > 0$, $C_{dd} > 0$, and $C_{xd} > 0$. Consequently, the profit function is represented as:

$$y(e, x, d) = R(x, d, e) - C(x + d) - FC \quad (3.1)$$

We consider an entrepreneur whose preferences are characterized by von Neumann-Morgenstern utility function over profit function $u(y)$, that is differentiable at least 3 times. The utility function satisfies non-satiation and strict risk aversion, $u_{yy} < 0 < u_y$, as well as prudence ($u_{yyy} > 0$) (Kimball, 1990).

Therefore, SMEs will choose the level of exports (x^*) and sales in the domestic markets (d^*) that maximizes their expected utility of profits. Let $F(e, \tau)$ denote the cumulative distribution function of a background risk e that effects the profit function, defined over a support within the closed interval $[a, b]$, where τ is a parameter whose shift represents changes in risk of first- or second-order.

We have used the terminology of Diamond and Stiglitz (1974) to describe the issue faced by the owner of the SME. Therefore, in order to analyze the possible consequences of increased risk, we examine an agent who seeks to maximize their utility. This utility depends on by a random variable, e , as well as two control variables, x and d . The implicit function theorem will be employed to examine the effects of an increase in risk (as indicated by a change in the parameter τ) on the optimal allocations x and d .

The entrepreneur maximizes the SME's expected utility profit according to the following objective:

$$(x^*(\tau), d^*(\tau)) = \arg \max_{x,d} \{EU(x, d, e) = \int_a^b U(x, d, e) dF(e, \tau)\} \quad (3.2)$$

$$U(x, d, e) = u(R(x, d, e) - C(x + d) - FC) \quad (3.3)$$

The optimal levels of both foreign and domestic sales that solve the problem of SMEs are represented as $x^*(\tau)$ and $d^*(\tau)$.

The first-order necessary condition for the entrepreneur's maximization problem can be expressed as $dEU = 0$. In the present scenario with two variables, the total differentiation is as follows:

$$dEU = EU_x dx + EU_d dd = 0 \quad (3.4)$$

Condition (3.4) is satisfied if and only if both partial derivatives EU_x and EU_d are equal to zero simultaneously. Hence, the deducible form of the first-order condition can be expressed as $EU_x = EU_d = 0$, or in other words:

$$EU_x = Eu_y(R_x - C_x) = 0 \quad (3.5)$$

$$EU_d = Eu_y(R_d - C_d) = 0 \quad (3.6)$$

To establish the second-order sufficient condition for a maximum, we employ the method of total differentiation on the variable dEU . Therefore, we get the following equation:

$$d^2EU = EU_{xx}dx^2 + 2EU_{xd}dxdd + EU_{dd}dd^2 < 0 \quad (3.7)$$

where $dx^2 \equiv (dx)^2$ and $dd^2 \equiv (dd)^2$. This condition is satisfied if $EU_{xx} < 0$, $EU_{dd} < 0$ and $EU_{xx}EU_{dd} > (EU_{xd})^2$.

The sign condition of the second-order derivatives is satisfied, based on the assumptions of the model. The expression of these derivatives can be stated as:

$$EU_{xx} = Eu_{yy}(R_x - C_x)^2 + Eu_y(R_{xx} - C_{xx}) < 0 \quad (3.8)$$

$$EU_{dd} = Eu_{yy}(R_d - C_d)^2 + Eu_y(R_{dd} - C_{dd}) < 0 \quad (3.9)$$

$$EU_{xd} = Eu_{yy}(R_x - C_x)(R_d - C_d) + Eu_y(R_{xd} - C_{xd}) < (>)0 \quad (3.10)$$

The first- and second-order conditions for this problem are satisfied for all τ . Furthermore, it is essential to understand that EU_{xd} denotes whether the two markets are substitutes ($EU_{xd} < 0$) or complements ($EU_{xd} > 0$). This distinction will play a significant role in the findings presented in this paper.

In addition, from eq. (3.3) it is straightforward to demonstrate the following second and third order derivatives:

$$U_{xe} = u_{yy}(R_x - C_x)R_e + u_y R_{xe} \quad (3.11)$$

$$U_{de} = u_{yy}(R_d - C_d)R_e + u_y R_{de} \quad (3.12)$$

$$U_{xee} = u_{yyy}(R_x - C_x)R_e^2 + u_{yy}(2R_{xe}R_e + (R_x - C_x)R_{ee}) + u_y R_{xee} \quad (3.13)$$

$$U_{dee} = u_{yyy}(R_d - C_d)R_e^2 + u_{yy}(2R_{de}R_e + (R_d - C_d)R_{ee}) + u_y R_{dee} \quad (3.14)$$

The conditions U_{xe} and U_{de} enable us to identify the direction of $x(\tau)^*$ and $d(\tau)^*$ when there is a first-order shift in risk. Meanwhile, conditions U_{xee} and U_{dee} enable us to determine the direction of $x(\tau)^*$ and $d(\tau)^*$ when faced with a second-order shift in risk.

3.4 Risk changes

We are specifically interested in comprehending the reaction of $x^*(\tau)$ and $d^*(\tau)$ to a change in the risk level, whether it is first-order or second-order. Therefore, to examine the export behavior of SMEs in the presence of background risk, we analyze two distinct types of changes in the parameter τ that modify the distribution $F(e, \tau)$.

3.4.1 First-order risk increase

By using the definition of [Diamond and Stiglitz \(1974\)](#), we define an increase in τ as a first-order risk increase if

$$F_{\tau}(e, \tau) \geq 0 \quad (3.15)$$

and $F_{\tau}(a, \tau) = F_{\tau}(b, \tau) = 0$.

The discrete version of eq. (3.15), considers $F(e, \tau_1)$ and $F(e, \tau_2)$ to denote two cumulative distribution functions of the random variable e . If the random variables e_1 and e_2 have the distribution functions $F(e, \tau_1)$ and $F(e, \tau_2)$, respectively, we will say that e_1 dominates e_2 via first-order stochastic dominance or e_2 is a first-order increase in risk of e_1 if $F(e, \tau_2) \geq F(e, \tau_1)$.

The following proposition analyses the effect of a first-order risk increase on optimal export $x^*(\tau)$ and domestic sales $d^*(\tau)$ levels.

Proposition 1. *Let $x^*(\tau)$ and $d^*(\tau)$ be the optimal levels of both foreign and domestic sales that maximizes (2). If a shift in τ represents a first-order risk increase, then we have the following two results:*

(i) $x^*(\tau)$ increases (decreases) if $U_{xe} < (>)0$, $EU_{xd} < 0$ and $U_{de} > (<)0$ or $U_{xe} < (>)0$, $EU_{xd} > 0$ and $U_{de} < (>)0$.

(ii) $d^*(\tau)$ increases (decreases) if $U_{de} < (>)0$, $EU_{xd} < 0$ and $U_{xe} > (<)0$ or $U_{de} < (>)0$, $EU_{xd} > 0$ and $U_{xe} < (>)0$.

Proof. Implicitly differentiating the first-order conditions, we obtain:

$$EU_{xx}dx^* + EU_{xd}dd^* + \int_a^b U_x dF_{\tau}d\tau = 0 \quad (3.16)$$

$$EU_{xd}dx^* + U_{dd}dd^* + \int_a^b U_d dF_{\tau}d\tau = 0 \quad (3.17)$$

which rearranged eqs. (3.16) and (3.17), become:

$$\begin{bmatrix} EU_{xx} & EU_{xd} \\ EU_{xd} & EU_{dd} \end{bmatrix} \begin{pmatrix} \frac{dx^*}{d\tau} \\ \frac{dd^*}{d\tau} \end{pmatrix} = - \begin{pmatrix} \int_a^b U_x dF_\tau \\ \int_a^b U_d dF_\tau \end{pmatrix} \quad (3.18)$$

Now, applying integration by parts once to the right-hand side of eq. (3.18), given that $F_\tau(a, \tau) = F_\tau(b, \tau) = 0$, we get

$$\int_a^b U_x dF_\tau = - \int_a^b U_{xe} F_\tau de \quad (3.19)$$

$$\int_a^b U_d dF_\tau = - \int_a^b U_{de} F_\tau de \quad (3.20)$$

then:

$$\frac{dx^*}{d\tau} = \frac{EU_{dd} \int_a^b U_{xe} F_\tau de - EU_{xd} \int_a^b U_{de} F_\tau de}{EU_{xx} EU_{dd} - (EU_{xd})^2} \quad (3.21)$$

$$\frac{dd^*}{d\tau} = \frac{EU_{xx} \int_a^b U_{de} F_\tau de - EU_{xd} \int_a^b U_{xe} F_\tau de}{EU_{xx} EU_{dd} - (EU_{xd})^2} \quad (3.22)$$

where the denominator of eqs. (3.21) and (3.22) is positive, given the second-order sufficient condition. The sign of $\frac{dx^*}{d\tau}$ and $\frac{dd^*}{d\tau}$ in the numerator depends on the sign of EU_{xd} , indicating whether the two markets are substitutes or complements. Moreover, the direction of optimal levels of sales in both markets similarly depends on the signs of U_{xe} and U_{de} . \square

The equation $U_{xe} = u_{yy}(R_x - C_x)R_e + u_y R_{xe}$ represents two opposing forces. The expression $u_{yy}(R_x - C_x)R_e$ is associated with risk aversion, whereas the expression $u_y R_{xe}$ is associated with non-satiation. When the magnitude of the first term prevails over the magnitude of the second term, then $U_{xe} < 0$. Conversely, when the force exerted by the second term dominates the force exerted by the first term, then $U_{xe} > 0$. Therefore, the condition $U_{xe} < (>)0$ is equivalent to

$$A(y) \left[\frac{(R_x - C_x)R_e}{R_{xe}} \right] > (<)1 \quad (3.23)$$

where $A(y) = -\frac{u_{yy}}{u_y}$ is the absolute aversion coefficient. Thus, the expression (3.23) on the left-hand side is a partial relative aversion index for a non-linear risk.

Analogous analysis may be conducted for $U_{de} = u_{yy}(R_d - C_d)R_e + u_y R_{de}$. This implies that $U_{de} < (>)0$ if and only if the following condition is satisfied:

$$A(y) \left[\frac{(R_d - C_d)R_e}{R_{de}} \right] > (<)1 \quad (3.24)$$

3.4.2 Second-order risk increase

In this section, we utilize two definitions of the second-order risk increase: the mean preserving spread (MPS) definition by [Diamond and Stiglitz \(1974\)](#) and the definition by [Sandmo\(1970; 1971\)](#). This allows us to provide some insight into the results. In [Sandmo's](#) analysis of MPS, we analyze the impacts of an uncertainty shock on SME export behavior by examining the income and substitution effects. To do this, we employ [Davis's \(1989\)](#) compensation method.

D-S (Diamond and Stiglitz) definition of a MSP

Now, following the definition of [Diamond and Stiglitz \(1974\)](#), we define an increase in τ as a second-order risk increase (mean-preserving spread) if

$$\int_a^b F_\tau(e, \tau) de = 0 \quad (3.25)$$

and

$$T(e, \tau) = \int_a^e F_\tau(\theta, \tau) d\theta \geq 0 \quad \text{for all } a \leq \theta \leq b \quad (3.26)$$

and also, $T(a, \tau) = T(b, \tau) = 0$. Condition in (3.25) indicates that both distributions have the same mean, and condition in (3.26) is the single-crossing property of a mean-preserving spread.

As well as before in the case of a first-order increase in risk, it can be understood in its discrete version. Let $F(e, \tau_1)$ and $F(e, \tau_2)$ denote two cumulative distribution functions of the random variable. If the random variables e_1 and e_2 have the distribution functions $F(e, \tau_1)$ and $F(e, \tau_2)$, respectively, we will say that e_1 dominates e_2 via second-order

increase in risk or e_2 is a mean-preserving spread e_1 if $\int_a^b [F(e, \tau_2) - F(e, \tau_1)] de = 0$ and $\int_a^e [F(\theta, \tau_2) - F(\theta, \tau_1)] d\theta \geq 0$ for all $a \leq \theta \leq b$.

The following proposition analyses the effect of a second-order risk increase on optimal export $x^*(\tau)$ and domestic sales $d^*(\tau)$ levels.

Proposition 2. *Let $x^*(\tau)$ and $d^*(\tau)$ be the optimal levels of both foreign and domestic sales that maximizes (2). If a shift in τ represents a second-order risk increase, then we have the following two results:*

(i) $x^*(\tau)$ increases (decreases) if $U_{xee} > (<)0$, $EU_{xd} < 0$ and $U_{dee} < (>)0$ or $U_{xee} > (<)0$, $EU_{xd} > 0$ and $U_{dee} > (<)0$.

(ii) $d^*(\tau)$ increases (decreases) if $U_{dee} > (<)0$, $EU_{xd} < 0$ and $U_{xee} < (>)0$ or $U_{dee} > (<)0$, $EU_{xd} > 0$ and $U_{xee} > (<)0$.

Proof. Now, applying integration by parts twice to the right-hand side of eq. (3.18), and given that $F_\tau(a, \tau) = F_\tau(b, \tau) = T(a, \tau) = T(b, \tau) = 0$, the conditions (3.19) and (3.20) are transformed to:

$$\int_a^b U_x dF_\tau = \int_a^b U_{xee} T(e, \tau) de \quad (3.27)$$

$$\int_a^b U_d dF_\tau = \int_a^b U_{dee} T(e, \tau) de = 0 \quad (3.28)$$

Now, we have

$$\frac{dx^*}{d\tau} = \frac{-EU_{dd} \int_a^b U_{xee} T(e, \tau) de + EU_{xd} \int_a^b U_{dee} T(e, \tau) de}{EU_{xx} EU_{dd} - (EU_{xd})^2} \quad (3.29)$$

$$\frac{dd^*}{d\tau} = \frac{-EU_{xx} \int_a^b U_{dee} T(e, \tau) de + EU_{xd} \int_a^b U_{xee} T(e, \tau) de}{EU_{xx} EU_{dd} - (EU_{xd})^2} \quad (3.30)$$

Since the denominator of eqs. (3.29) and (3.30) is positive, the sign of $\frac{dx^*}{d\tau}$ and $\frac{dd^*}{d\tau}$ in the numerator is determined by the sign of EU_{xd} . This indicates whether the two markets are substitutes or complements. Furthermore, the direction of the optimal sales level in both markets is similarly dependent upon the signs of U_{xee} and U_{dee} . \square

The expression U_{xee} consists of three elements: the term $u_{yyy}(R_x - C_x)R_e^2$ is related to prudence, the term $u_{yy}(2R_{xe}R_e + (R_x - C_x)R_{ee})$ is linked to risk aversion, and the term $u_y R_{xee}$ is connected to non-satiation. A comparable analysis can be carried out on U_{dee} . The significance of each of these forces will influence the direction of the U_{xee} and U_{dee} functions.

Next, we will continue our examination of a mean-preserving spread, as proposed by [Sandmo\(1970; 1971\)](#). The forthcoming section's findings will enhance the economic understanding of how SMEs react to changes in non-linear risks.

Sandmo's definition of a MSP

Based on [Sandmo's](#) concept of the mean-preserving spread (MPS), the risk e can be expressed as:

$$\alpha + \gamma e \quad (3.31)$$

where γ is the multiplicative shift parameter, α is the additive one and $E(e) = \mu_e$. Following Sandmo, an increase of γ alone (from the point $\gamma = 1, \alpha = 0$) represents a mean-preserving increase in risk if $\mu_e d\gamma + d\alpha = 0$, i.e.,

$$\frac{d\alpha}{d\gamma} = -\mu_e \quad (3.32)$$

We can rewrite the SME's problem in (3.2) as:

$$(x^*(\alpha, \gamma), d^*(\alpha, \gamma)) = \arg \max_{x,d} \{EU(x, d, \alpha + \gamma e)\} \quad (3.33)$$

The first-order conditions of (3.33) are:

$$EU_x = 0 \quad (3.34)$$

$$EU_d = 0 \quad (3.35)$$

The solution to SME's problem in (3.33) is given by the optimum levels of foreign and domestic sales, denoted as $x^*(\alpha, \gamma)$ and $d^*(\alpha, \gamma)$, respectively.

Proposition 2 can be replicated by performing implicit differentiation on the first-order conditions in (3.34) and (3.35). By rearranging the resulting expression, we obtain:

$$\begin{bmatrix} EU_{xx} & EU_{xd} \\ EU_{xd} & EU_{dd} \end{bmatrix} \begin{pmatrix} \frac{dx^*}{d\gamma} \\ \frac{dd^*}{d\gamma} \end{pmatrix} = - \begin{pmatrix} EU_{x\gamma} \\ EU_{d\gamma} \end{pmatrix} \quad (3.36)$$

Note that $EU_{x\gamma} = Eu_{yy}(R_x - C_x)R_e(e - \mu_e) + Eu_y R_{xe}(e - \mu_e) = cov(U_{xe}, e)$ and $EU_{d\gamma} = Eu_{yy}(R_d - C_d)R_e(e - \mu_e) + Eu_y R_{de}(e - \mu_e) = cov(U_{de}, e)$.

Then, now we get

$$\frac{dx^*}{d\gamma} = \frac{-EU_{dd}cov(U_{xe}, e) + EU_{xd}cov(U_{de}, e)}{EU_{xx}EU_{dd} - (EU_{xd})^2} \quad (3.37)$$

$$\frac{dd^*}{d\gamma} = \frac{-EU_{xx}cov(U_{de}, e) + EU_{xd}cov(U_{xe}, e)}{EU_{xx}EU_{dd} - (EU_{xd})^2} \quad (3.38)$$

Based on the above findings, we can add the following proposition.

Proposition 3. *Let $x^*(\alpha, \gamma)$ and $d^*(\alpha, \gamma)$ be the optimal levels of both foreign and domestic sales that maximizes (33). If a shift in γ represents a mean-preserving spread, then we have the following two results:*

(i) $x^*(\tau)$ *increases (decreases) if $cov(U_{xe}, e) > (<)0$, $EU_{xd} < 0$ and $cov(U_{de}, e) < (>)0$ or $cov(U_{xe}, e) > (<)0$, $EU_{xd} > 0$ and $cov(U_{de}, e) > (<)0$.*

(ii) $d^*(\tau)$ *increases (decreases) if $cov(U_{de}, e) > (<)0$, $EU_{xd} < 0$ and $cov(U_{xe}, e) < (>)0$ or $cov(U_{de}, e) > (<)0$, $EU_{xd} > 0$ and $cov(U_{xe}, e) > (<)0$.*

With $i = \{x, d\}$, it is important to note that $cov(U_{ie}, e) = cov(u_{yy}(R_i - C_i)R_e, e) + cov(u_y R_{ie}, e)$ and $sgn(cov(U_{ie}, e)) = sgn(U_{iee})$. As a result, proposition 3 explains outcomes that are consistent with those stated in proposition 2. As a plus of Sandmo's definition of MPS is that it makes it possible to distinguish between the income effect and the substitution effect when analyzing the total effect of a second-order increase in risk. We are going to show that $cov(u_{yy}(R_i - C_i)R_e, e)$ represents the income effect and $cov(u_y R_{ie}, e)$ corresponds to the substitution effect using Davis's (1989) compensation method.

Income and substitution effects of a MSP

Income and substitution effects play a significant role in determining the outcomes of comparative static exercises in many economic models. These effects often have contrasting impacts, which can lead to potential indeterminacies.

The compensation approach employed in this article has its roots on [Davis \(1989\)](#)'s study, which builds upon Sandmo's concept that a mean-preserving spread can be separated into income and substitution effects. Thus, this paper aims to clarify the specific technique of compensation required to isolate the substitute effect within the context of SMEs.

Let's us assume that the fixed cost is $FC = \varphi(\gamma)$. In this case, the profits are given by:

$$y(e, x, d) = R(x, d, \alpha + \gamma e) - C(x + d) - \varphi(\gamma) \quad (3.39)$$

Following [Davis \(1989\)](#), the compensation rule is as follows:

$$\frac{\partial y}{\partial \gamma} = R_e \left(\frac{\partial \alpha}{\partial \gamma} + e \right) - \varphi_\gamma = 0 \quad (3.40)$$

or equivalently

$$\varphi_\gamma = R_e(e - \mu_e) \quad (3.41)$$

It should be noted that the entrepreneur will receive a compensation amount that is exactly enough for him to obtain the same value of y if the selected option of x and d remains as x^* and d^* . It also is evident that the level of compensation is random. The entrepreneur is unaware of the precise amount of compensation when making the decision regarding x and d .

In order to obtain the substitution effect, we plug $\varphi(\gamma)$ into the first-order conditions (3.34) and (3.35), then implicitly differentiating these first-order conditions and finally replacing in these both, the compensation rule in eq. (3.41) and the condition (3.32), we obtain the substitution effect:

$$\frac{dx^*}{d\gamma}|_{comp} = \frac{-EU_{dd}cov(u_y R_{xe}, e) + EU_{xd}cov(u_y R_{de}, e)}{|A|} \quad (3.42)$$

$$\frac{dd^*}{d\gamma}|_{comp} = \frac{-EU_{xx}cov(u_y R_{de}, e) + EU_{xd}cov(u_y R_{xe}, e)}{|A|} \quad (3.43)$$

where $|A| = EU_{xx}EU_{dd} - (EU_{xd})^2 > 0$.

Therefore, with these results, we can rewrite the results in eqs. (3.37) and (3.38), as follows:

$$\underbrace{\frac{dx^*}{d\gamma}}_{\text{Total effect}} = \underbrace{-\frac{EU_{dd}cov(u_{yy}(R_x - C_x)R_e, e)}{|A|}}_{\text{Income effect}} - \underbrace{\frac{EU_{dd}cov(u_y R_{xe}, e)}{|A|}}_{\text{Substitution effect}} + \underbrace{\frac{EU_{xd}cov(u_{yy}(R_d - C_d)R_e, e)}{|A|}}_{\text{Income effect}} + \underbrace{\frac{EU_{xd}cov(u_y R_{de}, e)}{|A|}}_{\text{Substitution effect}} \quad (3.44)$$

$$\underbrace{\frac{dd^*}{d\gamma}}_{\text{Total effect}} = \underbrace{-\frac{EU_{xx}cov(u_{yy}(R_d - C_d)R_e, e)}{|A|}}_{\text{Income effect}} - \underbrace{\frac{EU_{xx}cov(u_y R_{de}, e)}{|A|}}_{\text{Substitution effect}} + \underbrace{\frac{EU_{xd}cov(u_{yy}(R_x - C_x)R_e, e)}{|A|}}_{\text{Income effect}} + \underbrace{\frac{EU_{xd}cov(u_y R_{xe}, e)}{|A|}}_{\text{Substitution effect}} \quad (3.45)$$

From the preceding, it follows that the income effect depends of the sign of $cov(u_{yy}(R_i - C_i)R_e, e)$ with $i = \{x, d\}$, whereas the substitution effect is to the sign of $cov(u_y R_{ie}, e)$. By differentiating the subsequent expression, one can determine the sign of the substitution effect:

$$\frac{\partial u_y R_{ie}}{\partial e} = u_{yy}R_e R_{ie} + u_y R_{iee} \quad (3.46)$$

It follows from eq. (3.46) that the substitution effect is negative.

Now, we can determine the income effect's sign by differentiating the following expressions:

$$\frac{\partial u_{yy}(R_i - C_i)R_e}{\partial e} = u_{yyy}(R_i - C_i)R_e^2 + u_{yy}(R_e R_{ie} + R_{ee}(R_i - C_i)) \quad (3.47)$$

Notice, that if we add eq. (3.46) and eq. (3.47), we get:

$$U_{iee} = \underbrace{u_{yyy}(R_i - C_i)R_e^2 + u_{yy}(R_{ie}R_e + (R_i - C_i)R_{ee})}_{\text{Income effect sign}} + \underbrace{u_y R_{iee} + u_{yy} R_{ie} R_e}_{\text{Substitution effect sign}} \quad (3.48)$$

The substitution effect has a negative sign as a result of risk aversion ($u_{yy} < 0$). Additionally, the marginal revenue function, R_i , exhibits increasing behavior ($R_{ie} > 0$) and concavity ($R_{iee} < 0$) with respect to the variable e . SMEs reduce sales in the i market as a way to decrease their risk exposure. On the other hand, the income effect operates in the contrary direction. This suggests that it pushes the entrepreneur to increase their sales in the i market in order to confront uncertainty in a more advantageous scenario.

In addition, it is worth noting that $\text{sgn}(\text{cov}(U_{ie}, e)) = \text{sgn}(U_{iee})$ and that $\text{cov}(U_{ie}, e) = \text{cov}(u_{yy}(R_i - C_i)R_e, e) + \text{cov}(u_y R_{ie}, e)$. Considering this, $\frac{dx^*}{d\tau}$ and $\frac{dd^*}{d\tau}$ from Proposition 2 can be rewritten as follows:

$$\begin{aligned} \frac{dx^*}{d\tau} = & \frac{-EU_{dd} \int_a^b [u_{yyy}(R_x - C_x)R_e^2 + u_{yy}(R_{xe}R_e + (R_x - C_x)R_{ee})]T(e, \tau)de}{|A|} & (3.49) \\ & \underbrace{\hspace{10em}}_{\text{Income effect}} \\ & \frac{-EU_{dd} \int_a^b [u_y R_{xee} + u_{yy} R_{xe} R_e]T(e, \tau)de}{|A|} \\ & \underbrace{\hspace{10em}}_{\text{Substitution effect}} \\ & \frac{EU_{xd} \int_a^b [u_{yyy}(R_d - C_d)R_e^2 + u_{yy}(R_{de}R_e + (R_d - C_d)R_{ee})]T(e, \tau)de}{|A|} \\ & \underbrace{\hspace{10em}}_{\text{Income effect}} \\ & \frac{EU_{xd} \int_a^b [u_y R_{dee} + u_{yy} R_{de} R_e]T(e, \tau)de}{|A|} \\ & \underbrace{\hspace{10em}}_{\text{Substitution effect}} \end{aligned}$$

$$\begin{aligned}
\frac{dd^*}{d\tau} = & \underbrace{\frac{-EU_{xx} \int_a^b [u_{yyy}(R_d - C_d)R_e^2 + u_{yy}(R_{de}R_e + (R_d - C_d)R_{ee})T(e, \tau)de]}{|A|}}_{\text{Income effect}} \\
& \underbrace{\frac{-EU_{xx} \int_a^b [u_y R_{dee} + u_{yy} R_{de} R_e] T(e, \tau) de}{|A|}}_{\text{Substitution effect}} \\
& \underbrace{\frac{EU_{xd} \int_a^b [u_{yyy}(R_x - C_x)R_e^2 + u_{yy}(R_{xe}R_e + (R_x - C_x)R_{ee})T(e, \tau)de]}{|A|}}_{\text{Income effect}} \\
& \underbrace{\frac{EU_{xd} \int_a^b [u_y R_{xee} + u_{yy} R_{xe} R_e] T(e, \tau) de}{|A|}}_{\text{Substitution effect}}
\end{aligned} \tag{3.50}$$

The previous analysis demonstrates that the magnitude of the income and substitution effects generated in each market and the relationship between these markets, whether they function as substitutes or complements, determine the total impact of a risk increase that preserves the mean on sales in both the domestic and foreign markets.

Based on the foregoing research, it is evident that the definitions of mean-preserving increase in risk put forth by [Diamond and Stiglitz \(1974\)](#) and [Sandmo\(1970; 1971\)](#) are equivalent. These enable us to examine the impact of both linear and non-linear risks on the optimal decisions made by an entrepreneur of SMEs.

Nonlinear risk allows the conceptualization of novel risks that normally are not addressed in the economic field, such as the risk associated with COVID-19 and its significant global economic impacts. Propositions 1, 2, and 3 encompass both the conventional risk analysis found in existing research and for novel risks that have more complex functional impacts on export levels and domestic market sales.

3.5 Applications

This section will examine the linear and multiplicative risk case. We shall link our prior findings with the indices of partial relative aversion and partial relative prudence.

3.5.1 Linear and multiplicative risk

Let's assume that the revenue of the SME is represented by the function $R(e, x, d) = eR(x, d)$. The profit function can be expressed as $y(e, x, d) = eR(x, d) - C(x + d) - FC$. Therefore, equations eqs. (3.11) to (3.14) are simplified to:

$$U_{xe} = u_{yy}(eR_x - C_x)R + u_y R_x \quad (3.51)$$

$$U_{de} = u_{yy}(eR_d - C_d)R + u_y R_d \quad (3.52)$$

$$U_{xee} = u_{yyy}(eR_x - C_x)R^2 + 2u_{yy}R_x R \quad (3.53)$$

$$U_{dee} = u_{yyy}(eR_d - C_d)R^2 + 2u_{yy}R_d R \quad (3.54)$$

When there is a first-order increase in risk, proposition 1 can be simplified to the following corollary:

Corollary 1. *Let $x^*(\tau)$ and $d^*(\tau)$ be the optimal levels of both foreign and domestic sales that maximizes (2). If a shift in τ represents a first-order risk increase, then we have the following two results:*

(i) $x^*(\tau)$ *increases (decreases) if $A(y)(y - b_x) > (<)1$, $EU_{xd} < 0$ and $A(y)(y - b_d) < (>)1$ or $A(y)(y - b_x) > (<)1$, $EU_{xd} > 0$ and $A(y)(y - b_d) > (<)1$.*

(ii) $d^*(\tau)$ *increases (decreases) if $A(y)(y - b_d) > (<)1$, $EU_{xd} < 0$ and $A(y)(y - b_x) < (>)1$ or $A(y)(y - b_d) > (<)1$, $EU_{xd} > 0$ and $A(y)(y - b_x) > (<)1$.*

where $A(y) = -\frac{u_{yy}}{u_y}$, $b_i = \frac{C_i R}{R_i} - (C + FC)$ with $i \in \{x, d\}$.

The expression $A(y)(y - b_i)$ represents a specific instance of the index of partial relative risk aversion, which was first introduced by [Menezes and Hanson \(1970\)](#) and [Zeckhauser and Keeler \(1970\)](#).

When there is a second-order increase in risk, proposition 2 can be reduced to the following corollary:

Corollary 2. *Let $x^*(\tau)$ and $d^*(\tau)$ be the optimal levels of both foreign and domestic sales that maximizes (2). If a shift in τ represents a first-order risk increase, then we have the following two results:*

(i) $x^*(\tau)$ increases (decreases) if $P(y)(y - b_x) > (<)2$, $EU_{xd} < 0$ and $P(y)(y - b_d) < (>)2$ or $P(y)(y - b_x) > (<)2$, $EU_{xd} > 0$ and $P(y)(y - b_d) > (<)2$.

(ii) $d^*(\tau)$ increases (decreases) if $P(y)(y - b_d) > (<)2$, $EU_{xd} < 0$ and $P(y)(y - b_x) < (>)2$ or $P(y)(y - b_d) > (<)2$, $EU_{xd} > 0$ and $P(y)(y - b_x) > (<)2$.

where $P(y) = -\frac{u_{yyy}}{u_{yy}}$, $b_i = \frac{C_i R}{R_i} - (C + FC)$ with $i \in \{x, d\}$.

Similar to the introduction of the index of partial relative risk aversion, [Choi et al. \(2001\)](#) proposed an index of partial prudence denoted as $P(y)(y - b)$. According to corollary 2, the linear and multiplicative risk can be considered as a specific instance of the index defined by [Choi et al. \(2001\)](#).

The income effect and substitution effect, as described in eq. (3.48), can be expressed as follows for a linear and multiplicative risk:

$$U_{iee} = \underbrace{u_{yyy}(eR_i - C_i)R^2 + u_{yy}R_iR}_{\text{Income effect sign}} + \underbrace{u_{yy}R_iR}_{\text{Substitution effect sign}} \quad (3.55)$$

The second term in eq. (3.55) is negative because it represents the $cov(u_y R_i, e)$, and the sign of this covariance is negative. In order to have a positive precautionary effect,

the income effect must be positive, as indicated by the sign of $cov(u_{yy}(eR_i - C_i), e)$, and it must outweigh the substitution effect. The requirement $P(y)(y - b_i) > 2$ ensures a guaranteed positive precautionary effect. Subsequently, the equations referenced as eqs. (3.49) and (3.50) are simplified to get the result:

$$\begin{aligned} \frac{dx^*}{d\tau} = & \underbrace{\frac{-EU_{dd} \int_a^b [u_{yyy}(eR_x - C_x)R^2 + u_{yy}R_xR]T(e, \tau)de}{|A|}}_{\text{Income effect}} \underbrace{\frac{-EU_{dd} \int_a^b [u_{yy}R_xR]T(e, \tau)de}{|A|}}_{\text{Substitution effect}} \\ & + \underbrace{\frac{EU_{xd} \int_a^b [u_{yyy}(eR_d - C_d)R^2 + u_{yy}R_dR]T(e, \tau)de}{|A|}}_{\text{Income effect}} + \underbrace{\frac{EU_{xd} \int_a^b [u_{yy}R_dR]T(e, \tau)de}{|A|}}_{\text{Substitution effect}} \end{aligned} \quad (3.56)$$

$$\begin{aligned} \frac{dd^*}{d\tau} = & \underbrace{\frac{-EU_{xx} \int_a^b [u_{yyy}(eR_d - C_d)R^2 + u_{yy}R_dR]T(e, \tau)de}{|A|}}_{\text{Income effect}} \underbrace{\frac{-EU_{xx} \int_a^b [u_{yy}R_dR]T(e, \tau)de}{|A|}}_{\text{Substitution effect}} \\ & + \underbrace{\frac{EU_{xd} \int_a^b [u_{yyy}(eR_x - C_x)R^2 + u_{yy}R_xR]T(e, \tau)de}{|A|}}_{\text{Income effect}} + \underbrace{\frac{EU_{xd} \int_a^b [u_{yy}R_xR]T(e, \tau)de}{|A|}}_{\text{Substitution effect}} \end{aligned} \quad (3.57)$$

In the next section, we will analyze the problem of SMEs exporting using the (μ, σ) -approach. We will connect our findings to the concept of risk aversion's elasticity. We also will examine the International trade model developed by [Broll and Mukherjee \(2017\)](#).

3.6 (μ, σ) -approach

The firm's preferences are determined by a two-parameter utility function $V(\mu, \sigma)$, where $\mu = \mu_e R(x, d) - C(x + d) - FC$ and $\sigma = \sigma_e R(x, d)$ define the expected value and standard deviation of random profit y , respectively.¹ Hence, the expected utility from y for any entrepreneur with utility function $u(\cdot)$ can be written as $Eu(y) = \int_a^b u(\mu + \sigma\epsilon)dF(\epsilon) \equiv V(\mu, \sigma)$. The variable ϵ is derived from the normalization of y , namely as $\epsilon = \frac{y-\mu}{\sigma}$.

When using the (μ, σ) -approach, the entrepreneur faces the following problem:

$$(x^*(\mu_e, \sigma_e), d^*(\mu_e, \sigma_e)) = \arg \max_{x,d} \{\Phi(x, d)\} \quad (3.58)$$

$$\Phi(x, d) = V(\mu(x, d), \sigma(x, d)) \quad (3.59)$$

The solution to the problem faced by SMEs, which involves finding the optimal levels of foreign and domestic sales, can be expressed as $x^*(\mu_e, \sigma_e)$ and $d^*(\mu_e, \sigma_e)$.

The first-order conditions of the maximization problem are as follows:

$$\Phi_x = V_\mu \frac{\partial \mu}{\partial x} + V_\sigma \frac{\partial \sigma}{\partial x} = 0 \quad (3.60)$$

$$\Phi_d = V_\mu \frac{\partial \mu}{\partial d} + V_\sigma \frac{\partial \sigma}{\partial d} = 0 \quad (3.61)$$

By doing implicit differentiation on the first-order conditions with respect to μ_e , we obtain:

$$\Phi_{xx} dx^* + \Phi_{xd} dd^* + \Phi_{x\mu_e} d\mu_e = 0 \quad (3.62)$$

$$\Phi_{xd} dx^* + \Phi_{dd} dd^* + \Phi_{d\mu_e} d\mu_e = 0 \quad (3.63)$$

¹Sinn (1983) and Meyer (1987) established the required conditions for the equivalency between the expected utility framework, which is based on the von Neumann-Morgenstern utility functions, and (μ, σ) analysis. This property is determined by risks that satisfy the criteria of location and scale. The foundation for this condition is that all random variables in the choice set must exhibit a linear relationship with each other. Hence, the (μ, σ) -method is a perfect substitute for the expected utility approach.

which rearranged eqs. (3.62) and (3.63), become:

$$\begin{bmatrix} \Phi_{xx} & \Phi_{xd} \\ \Phi_{xd} & \Phi_{dd} \end{bmatrix} \begin{pmatrix} \frac{dx^*}{d\mu_e} \\ \frac{dd^*}{d\mu_e} \end{pmatrix} = - \begin{pmatrix} \Phi_{x\mu_e} \\ \Phi_{d\mu_e} \end{pmatrix} \quad (3.64)$$

By solving the system of equations (3.64), we obtain:

$$\frac{dx^*}{d\mu_e} = \frac{-\Phi_{dd}\Phi_{x\mu_e} + \Phi_{xd}\Phi_{d\mu_e}}{\Phi_{xx}\Phi_{dd} - (\Phi_{xd})^2} \quad (3.65)$$

$$\frac{dd^*}{d\mu_e} = \frac{-\Phi_{xx}\Phi_{d\mu_e} + \Phi_{xd}\Phi_{x\mu_e}}{\Phi_{xx}\Phi_{dd} - (\Phi_{xd})^2} \quad (3.66)$$

Assuming the second-order sufficient condition holds, the denominator of eqs. (3.65) and (3.66) is positive. The sign of the derivatives $\frac{dx^*}{d\mu_e}$ and $\frac{dd^*}{d\mu_e}$ in the numerator is determined by the sign of Φ_{xd} , which indicates whether the two markets are substitutes or complements. Furthermore, the direction of the optimal sales levels in both markets is also influenced by the indications of $\Phi_{x\mu_e}$ and $\Phi_{d\mu_e}$.

Taking the derivative of Φ_i with respect to μ_e , where $i \in \{x, d\}$, results in:

$$\Phi_{i\mu_e} = V_{\mu\mu} \frac{\partial \mu}{\partial i} \frac{\partial \mu}{\partial \mu_e} + V_{\mu} \frac{\partial^2 \mu}{\partial i \partial \mu_e} + V_{\sigma\mu} \frac{\partial \sigma}{\partial i} \frac{\partial \mu}{\partial \mu_e} \quad (3.67)$$

In order to establish a connection between our results in the (μ, σ) -space and the expected utility method, we follow the works of [Sinn \(1983\)](#) and [Meyer \(1987\)](#). We formulate the equivalences between the functions $u(\cdot)$ and $U(\cdot, \cdot)$ in the following manner:

$$V_{\mu}(\mu, \sigma) = \int_a^b u_y(\mu + \sigma\epsilon) dF(\epsilon) > 0 \quad (3.68)$$

$$V_{\mu\mu}(\mu, \sigma) = \int_a^b u_{yy}(\mu + \sigma\epsilon) dF(\epsilon) < 0 \quad (3.69)$$

$$V_{\mu\sigma}(\mu, \sigma) = \int_a^b u_{yy}(\mu + \sigma\epsilon)\epsilon dF(\epsilon) > 0 \quad (3.70)$$

$$V_{\sigma}(\mu, \sigma) = \int_a^b u_y(\mu + \sigma\epsilon)\epsilon dF(\epsilon) < 0 \quad (3.71)$$

$$V_{\sigma\sigma}(\mu, \sigma) = \int_a^b u_{yy}(\mu + \sigma\epsilon)\epsilon^2 dF(\epsilon) < 0 \quad (3.72)$$

Substituting eqs. (3.68) to (3.70) into eq. (3.67) yields

$$\Phi_{i\mu_e} = \int_a^b [u_{yy}(eR_i - C_i)R + u_y R_i] dF(\epsilon) = \int_a^b U_{ie} dF(\epsilon) \quad (3.73)$$

The equation U_{ie} , where i belongs to the set $\{x, d\}$, corresponds to the equations mentioned in eqs. (3.11) and (3.12) for special case of linear risks.

The following proposition examines the impact of a decrease in the mean of the random variable e (representing a first-order increase in risk) on the optimal levels of export $x^*(\mu_e, \sigma_e)$ and domestic sales $d^*(\mu_e, \sigma_e)$.

Proposition 4. *Let $x^*(\mu_e, \sigma_e)$ and $d^*(\mu_e, \sigma_e)$ be the optimal levels of both foreign and domestic sales that maximizes (58). If a shift in μ_e represents a first-order risk increase, then we have the following two results:*

(i) $x^*(\mu_e, \sigma_e)$ increases (decreases) if $\Phi_{x\mu_e} < (>)0$, $\Phi_{xd} < 0$ and $\Phi_{d\mu_e} > (<)0$ or $\Phi_{x\mu_e} < (>)0$, $\Phi_{xd} > 0$ and $\Phi_{d\mu_e} < (>)0$.

(ii) $d^*(\mu_e, \sigma_e)$ increases (decreases) if $\Phi_{d\mu_e} < (>)0$, $\Phi_{xd} < 0$ and $\Phi_{x\mu_e} > (<)0$ or $\Phi_{d\mu_e} < (>)0$, $\Phi_{xd} > 0$ and $\Phi_{x\mu_e} < (>)0$.

The previous analysis enables us to deduce that the inequality $\frac{di^*}{d\mu_e} < (>)0$ is equivalent with the inequality $\frac{di^*}{d\tau} > (<)0$, when considering a first-order increase in risk with $i \in \{x, d\}$ within the context of linear risks.

The impact of changes in μ_e on the optimal levels of export and domestic sales is examined using the elasticity of risk aversion associated with μ . The subsequent definition comes from [Broll and Mukherjee \(2017\)](#).

Definition 1 (Risk-aversion elasticity with respect to μ). *Let ρ_μ denote the elasticity of risk aversion in relation to the mean of profits. Then,*

$$\rho_\mu = \frac{\partial R}{\partial \mu} \frac{\mu}{R} \quad (3.74)$$

where $R = -\frac{V_\sigma}{V_\mu}$ corresponds to the marginal rate of substitution between μ and σ .

Using the substitution of $\frac{\partial \sigma}{\partial i}$ from either eq. (3.60) or eq. (3.61) into eq. (3.67), we proceed to rearrange the expression in the following way:

$$\Phi_{i\mu_e} = [V_{\mu\mu} + \frac{V_{\mu\sigma}}{R}] \frac{\partial \mu}{\partial i} \frac{\partial \mu}{\partial \mu_e} + V_{\mu} \frac{\partial^2 \mu}{\partial i \partial \mu_e} \quad (3.75)$$

To establish a connection between our findings and risk-aversion elasticity, we derive the value of R in respect to μ . The optimal value of the μ -relating elasticity of risk aversion is given by

$$\rho_{\mu} \equiv \frac{\partial R}{\partial \mu} \frac{\mu}{R} = -\frac{\mu}{V_{\mu}} [V_{\mu\mu} + \frac{V_{\mu\sigma}}{R}] \quad (3.76)$$

By replacing eq. (3.76) into eq. (3.75) and rearranging the expression, we obtain

$$\Phi_{i\mu_e} = V_{\mu} \left[-\frac{\rho_{\mu}}{\mu} \frac{\partial \mu}{\partial i} \frac{\partial \mu}{\partial \mu_e} + \frac{\partial^2 \mu}{\partial i \partial \mu_e} \right] \quad (3.77)$$

The condition $V_{\mu} > 0$ implies that $\Phi_{i\mu_e} < (>)0$ whenever the expression in the square brackets is negative (positive). Hence, $\Phi_{i\mu_e} < (>)0$ if and only if the following condition is satisfied:

$$\rho_{\mu} > (<) \mu \left[\frac{\partial^2 \mu}{\partial i \partial \mu_e} / \frac{\partial \mu}{\partial i} \frac{\partial \mu}{\partial \mu_e} \right] \quad (3.78)$$

Consequently, proposition 4 can be reformulated by considering the elasticity of risk aversion determined by condition eq. (3.78).

Now we examine the relationship between optimum levels of exports and domestic sales and a shift in the standard deviation of the random variable e .

We get the following by doing implicit differentiation on the first-order conditions with regard to σ_e :

$$\Phi_{xx} dx^* + \Phi_{xd} dd^* + \Phi_{x\sigma_e} d\sigma_e = 0 \quad (3.79)$$

$$\Phi_{xd}dx^* + \Phi_{dd}dd^* + \Phi_{d\sigma_e}d\sigma_e = 0 \quad (3.80)$$

which rearrangement of eqs. (3.79) and (3.80) results in:

$$\begin{bmatrix} \Phi_{xx} & \Phi_{xd} \\ \Phi_{xd} & \Phi_{dd} \end{bmatrix} \begin{pmatrix} \frac{dx^*}{d\sigma_e} \\ \frac{dd^*}{d\sigma_e} \end{pmatrix} = - \begin{pmatrix} \Phi_{x\sigma_e} \\ \Phi_{d\sigma_e} \end{pmatrix} \quad (3.81)$$

Through the resolution of the system of equations (3.81), we draw:

$$\frac{dx^*}{d\sigma_e} = \frac{-\Phi_{dd}\Phi_{x\sigma_e} + \Phi_{xd}\Phi_{d\sigma_e}}{\Phi_{xx}\Phi_{dd} - (\Phi_{xd})^2} \quad (3.82)$$

$$\frac{dd^*}{d\sigma_e} = \frac{-\Phi_{xx}\Phi_{d\sigma_e} + \Phi_{xd}\Phi_{x\sigma_e}}{\Phi_{xx}\Phi_{dd} - (\Phi_{xd})^2} \quad (3.83)$$

The determination of the sign of $\frac{dx^*}{d\sigma_e}$ and $\frac{dd^*}{d\sigma_e}$ is based on the direction of the Φ_{xd} together with of the sign both $\Phi_{x\sigma_e}$ and $\Phi_{d\sigma_e}$, given that the sign of the denominator is positive in eqs. (3.82) and (3.83).

Assessing the derivative of Φ_i in relation to σ_e yields:

$$\Phi_{i\sigma_e} = V_{\mu\sigma} \frac{\partial \mu}{\partial i} \frac{\partial \sigma}{\partial \sigma_e} + V_{\sigma} \frac{\partial^2 \sigma}{\partial i \partial \sigma_e} + V_{\sigma\sigma} \frac{\partial \sigma}{\partial i} \frac{\partial \sigma}{\partial \sigma_e} \quad (3.84)$$

then, substituting eqs. (3.70) to (3.72) into eq. (3.84) yields

$$\Phi_{i\sigma_e} = \int_a^b [u_{yy}(eR_i - C_i)R + u_y R_i] \epsilon dF(\epsilon) = cov(U_{ie}, \epsilon) \quad (3.85)$$

It should be noted that $cov(U_{ie}, \epsilon) > (<)0$ if and only if $U_{iee} > (<)0$. The expression U_{iee} conforms to the equations identified in eqs. (3.13) and (3.14) for the specific situation of linear risks.

Analysis of the effect of a second-order increase in risk, represented by an increase in the standard deviation of the random variable e , on the optimal levels of export $x^*(\mu_e, \sigma_e)$ and domestic sales $d^*(\mu_e, \sigma_e)$ is shown in the following proposition.

Proposition 5. *Let $x^*(\mu_e, \sigma_e)$ and $d^*(\mu_e, \sigma_e)$ be the optimal levels of both foreign and domestic sales that maximizes (58). If a shift in σ_e represents a second-order risk increase, then we have the following two results:*

(i) $x^*(\mu_e, \sigma_e)$ *increases (decreases)* if $\Phi_{x\sigma_e} > (<)0$, $\Phi_{xd} < 0$ and $\Phi_{d\sigma_e} < (>)0$ or $\Phi_{x\sigma_e} > (<)0$, $\Phi_{xd} > 0$ and $\Phi_{d\sigma_e} > (<)0$.

(ii) $d^*(\mu_e, \sigma_e)$ *increases (decreases)* if $\Phi_{d\sigma_e} > (<)0$, $\Phi_{xd} < 0$ and $\Phi_{x\sigma_e} < (>)0$ or $\Phi_{d\sigma_e} > (<)0$, $\Phi_{xd} > 0$ and $\Phi_{x\sigma_e} > (<)0$.

The preceding analysis allows us to infer that the inequality $\frac{di^*}{d\sigma_e} > (<)0$ is equally valid as the inequality $\frac{di^*}{d\tau} > (<)0$, when taking into account a second-order increase in risk in the setting of linear risks.

We analyse how shifts in σ_e affect the optimum levels of export and domestic sales by considering the elasticity of risk aversion linked to σ . The definition provided below is taken from [Broll and Mukherjee \(2017\)](#).

Definition 2 (Risk-aversion elasticity with respect to σ). *Let ρ_σ denote the elasticity of risk aversion in relation to the standar deviation of profits. Then,*

$$\rho_\sigma = \frac{\partial R}{\partial \sigma} \frac{\sigma}{R} \quad (3.86)$$

where $R = -\frac{V_\sigma}{V_\mu}$ corresponds to the marginal rate of substitution between μ and σ .

At its optimal value, the derivative of R with respect to σ can be expressed as:

$$\rho_\mu \equiv \frac{\partial R}{\partial \sigma} \frac{\sigma}{R} = \frac{\sigma}{V_\sigma} [V_{\sigma\sigma} + RV_{\mu\sigma}] \quad (3.87)$$

We get the following expression by substituting $\frac{\partial \mu}{\partial i}$ from either eq. (3.60) or eq. (3.61) into eq. (3.84) together with eq. (3.87):

$$\Phi_{i\mu_e} = V_\sigma \left[\frac{\rho_\sigma}{\sigma} \frac{\partial \sigma}{\partial i} \frac{\partial \sigma}{\partial \sigma_e} + \frac{\partial^2 \sigma}{\partial i \partial \sigma_e} \right] \quad (3.88)$$

The condition $V_\sigma < 0$ states that $\Phi_{i\sigma_e} > (<)0$ whenever the value expressed in the square brackets is negative (positive). Hence, $\Phi_{i\sigma_e} > (<)0$ if and only if the following condition is met:

$$\rho_\sigma < (>) - \sigma \left[\frac{\partial^2 \sigma}{\partial i \partial \sigma_e} / \frac{\partial \sigma}{\partial i} \frac{\partial \sigma}{\partial \sigma_e} \right] \quad (3.89)$$

As a result, proposition 5 can be restated by taking into account the elasticity of risk aversion established by condition eq. (3.89).

3.6.1 Broll and Mukherjee's International trade model

The authors [Broll and Mukherjee \(2017\)](#) assumed that the random variable e is indicative of uninsurable exchange rate risk. The same idea is also present in the study conducted by [Broll et al., \(2006\)](#). They postulated that the revenue function of the firm is defined as $R(e, x, d) = eR(x) + R(d)$. Hence, the profit function can be expressed as $y = eR(x) + R(d) - C(x + y)$.

This model is a special case of the issue discussed in subsection 4.2 since [Broll and Mukherjee](#) assume that risk solely affects the entrepreneur's income in the foreign sector. Equations eqs. (3.51) to (3.54) are therefore reduced to:

$$U_{xe} = u_{yy}(eR_x - C_x)R + u_y R_x \quad (3.90)$$

$$U_{de} = u_{yy}(R_d - C_d)R \quad (3.91)$$

$$U_{xee} = u_{yyy}(eR_x - C_x)R^2 + 2u_{yy}R_x R \quad (3.92)$$

$$U_{dee} = u_{yyy}(R_d - C_d)R^2 \quad (3.93)$$

It is important to understand that exchange rate risk functions as a multiplying risk in the foreign currency sector, but as an adding risk in the domestic sector. This implies that when there is a mean-preserving increase in risk, both the income effect and the substitution effect occur in the foreign sector. However, in the domestic sector, only the income effect is observed. The fact that [Broll and Mukherjee's](#) exclusive focus on the foreign sector, the last piece is absent from their analysis.

Based on the (μ, σ) -approach, [Broll and Mukherjee \(2017\)](#) examine the elasticity of risk aversion as a metric to evaluate the impact of a risk increase on the foreign sector. This concept is specifically specified for multiplicative risk. The calculation of the elasticity of aversion for exchange rate risk, which is an additive risk in the domestic sector, is not possible due to the lack of definition of this concept for this particular type of risk.

3.7 Conclusions

This paper has examined the role of background risk in how SME production is diversifying into domestic and export sales. We offer the conditions under which increasing first- and second-degree risk raise the precautionary export behavior of SMEs. By employing a compensation method, we decomposed the effects of risk into substitution and income effects, which allowed us to identify conditions under which precautionary export behavior emerges. Specifically, we show that an increase in risk leads to two opposing forces: SMEs tend to reduce sales due to the negative impact of risk on expected utility (substitution effect), while simultaneously increasing sales to buffer against risk (income effect). Our findings contribute to the literature by incorporating a more realistic assumption on non-linearity under background risk and its impact on SME precautionary export behavior.

To our knowledge, our paper contributions are threefold. First, we extended the existing literature by acknowledging that SMEs can use their decision on how to diversify their production into the domestic and foreign markets as an operational hedging strategy to manage their risk exposure against background risk. Moreover, this strategy depends on how SME production supply interacts across domestic and foreign markets. When they do as substitutes SMEs with a positive export behavior will choose to reallocate production from the domestic market towards abroad. While, if they interact as complementaries, SMEs will increase their supply to either the foreign and domestic

markets.

Second, our paper extends the existing literature by incorporating a nonlinear risk characterization, which allows the theorization of new risks that, commonly in the economic literature, are not considered. Thus, improving the realism of the decision problem that SMEs face. Third, we present our results as signs of covariances, which provide a more informative measure delivering the sign and magnitudes of the effects of uncertainty on the optimal decision problems. Furthermore, from these can be detached, the risk attitudes behind certain behaviors.

Finally, the findings in this paper can serve policymakers in charge of SME export promotion to understand how SMEs may respond to increases in background risk and, therefore, contribute to enhancing the effectiveness of public policies on the international performance of SMEs.

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Chapter 4

Conclusions

In contexts where risk and uncertainty are increasingly prevalent, understanding how individuals cope with and manage risk becomes essential, particularly for entrepreneurs, whose decision-making processes significantly impact economic growth and development. This thesis studies entrepreneurial behavior under risk and uncertainty through two research articles: hybrid entrepreneurship and SMEs' export behavior.

The first article develops a bivariate choice model in which entrepreneurship and paid employment can interact as substitutes or complements, demonstrating the condition assuring precautionary behavior in entrepreneurial time allocation under two sources of risk is related to the relative strength of downside risk aversion of entrepreneurship, labor, and leisure, where the precautionary behavior of hybrid entrepreneurs is associated with the idea of harm disaggregation recently developed in the risk literature and these strengths represent preferences for harm disaggregation in consumption by changes in the time allocated between entrepreneurship and paid employment. These findings show hybrid entrepreneurs with precautionary time allocation behavior are willing to allocate more time to entrepreneurship in increased risk scenarios. Therefore, boosting hybrid entrepreneurship during recessions would be a viable economic policy for economic recovery through entrepreneurial innovation, especially considering the countercyclical dynamic between self-employment and the business cycle (see Fossen, 2021).

The second article examines how SMEs adapt their export behavior in response to uncertainty characterized by background risk presence. It outlines the conditions in which first- and second-order nonlinear risk increases influence precautionary export behavior by reconciling two opposing forces, the substitution and income effects associated with risk. The findings indicate that SMEs employ export diversification as an operational hedging strategy, reallocating output between domestic and foreign markets based on the nature of their relationship, whether as substitutes or complements. It provides a deeper insight into how SMEs operating internationally, lacking access to hedging instruments, manage their risk by raising their export behavior. This insight

holds important implications for policymakers attempting to design export promotion programs that suit the risk management needs of SMEs.

This thesis demonstrates the significant influence of risk preferences on the risk management strategies and precautionary behavior of entrepreneurs, particularly in the context of hybrid entrepreneurship and exporting SMEs. This research integrates decision theory under risk with entrepreneurial decision-making, enhancing the understanding of how entrepreneurs navigate uncertainty and providing new insights into their adaptive responses.

These findings establish a foundation for policy development that highlights the necessity of considering the heterogeneity of entrepreneurs' risk preferences and the significant adaptability of the entrepreneurial sector in times of uncertainty. Utilizing these insights, policymakers can formulate strategies that improve entrepreneurial performance while contributing to economic growth and recovery efforts.