Hindawi
Journal of Advanced Transportation
Volume 2017, Article ID 3127398, 12 pages
https://doi.org/10.1155/2017/3127398

Research Article

A Road Pricing Model for Congested Highways Based on Link Densities

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Received 7 June 2017; Revised 27 July 2017; Accepted 9 August 2017; Published 17 October 2017

Academic Editor: Sara Moridpour

A road pricing model is presented that determines tolls for congested highways. The main contribution of this paper is to include density explicitly in the pricing scheme and not just flow and time. The methodology solves a nonlinear constrained optimization problem whose objective function maximizes toll revenue or highway use (2 scenarios). The results show that the optimal tolls depend on highway design and the level of congestion. The model parameters are estimated from a Chile's highway data. Significant differences were found between the highway's observed tolls and the optimal toll levels for the two scenarios. The proposed approach could be applied to either planned highway concessions with recovery of capital costs or the extension or retendering of existing concessions.

1. Introduction

This study presents an analytic road pricing model based on macroscopic traffic models to determine the tolls for congested highways. The main motivation and contribution of this paper are to study fare design on urban highways, but considering the link density instead the link flow. The main advantage of this use of density data is that it captures the relationship between flow and cost (trip time or speed) on each link more accurately [1]. We develop two scenarios for our model, one in which the revenue is maximized and another in which the use of the road is maximized; however, other scenarios or criteria can be developed within the framework we propose. In both cases, the proposed formulation posits a constrained optimization problem. In the maximum revenue scenario (MR) the objective function maximizes toll revenue while in the maximum infrastructure use scenario (MIU) it maximizes highway use. The constraints for both scenarios are derived from the fundamental traffic equation relating flow, speed, and density on each highway segment. The parameters of the equation depend on highway design. The model's optimality conditions give optimal tolls for the MR and MIU scenarios, for each segment and operating period.

The model was implemented using data on the main controlled-access highway in the Santiago region of Chile. The parameters of the model were estimated using regression analysis, taking particular care to address possible problems of collinearity and endogeneity of tolls, traffic speed, and traffic density. The results of this application are that the optimal tolls in the MR scenario are significantly higher than those for the MIU scenario. They also indicate that once a highway's infrastructure capital costs have been recouped, the MIU optimal tolls will be significantly different from those designed to include capital cost recovery and should be modified accordingly. In some time periods (and highway segments, if tolls vary by segment) this will mean a major decrease in the tolls; in others it may mean an increase.

Two extensions to the proposed model are also presented. One adds a toll ceiling constraint to the MR scenario while the other incorporates financing constraints in the MIU scenario to recover the capital costs.

The remainder of this article is organized into five sections. Section 2 reviews the literature on optimal highway tolls under traffic congestion. Section 3 sets out our proposed optimization model for the MR and MIU optimal scenarios and its main properties. Section 4 applies the model to an urban highway in Chile, discusses the regression model used to estimate the parameters, and compares the results for the two scenarios with each other and with the observed situation reflected in the highway data. Section 5 develops two extensions to the optimization model incorporating a toll ceiling constraint for the MR scenario and financing constraints for the MIU scenario. Finally, Section 6 summarizes and discusses the main conclusions.

2. Literature Review

Road pricing is implemented for two main reasons [2]. The first is to mitigate road network congestion by increasing the cost of trips along certain routes in order to shift traffic flows from peak to off-peak periods, from congested to less congested routes, or from private vehicles to public transport. The second reason is to recover the cost of building and maintain a road network when the cost is financed totally or partially by private investment.

However, according to the same author the greater part of the published research has in fact been concerned with the congestion pricing issue. Indeed, it has been widely recognized as an issue of primary importance from both theoretical and a public policy standpoints. Successful application of congestion charges in cities such as London, Singapore, and Stockholm has stimulated interest among researchers around the world.

The origins of congestion pricing theory go back to Pigou [3] and Knight [4], who used the example of a congested highway to address the issue of externalities and optimal congestion costs. Since then, the notion of marginal cost pricing, that is, imposing a charge equal to the difference between the private cost and the social cost of a road in order to maximize net social benefit, has been extended to road networks in general [5–8]. Yang and Huang [9] review the marginal cost principle in the presence of queues and delays. Wu and Huang [10], considering body congestion in carriage and vehicle congestion in bottleneck queues in a competitive highway/transit system, investigate the departure patterns of commuters through analysing the equilibria under three road-use pricing strategies. An up-to-date overview of road pricing may be found in Rouhani [11].

In a specifically urban context, de Grange and Troncoso [12] survey the literature emphasizing the relevant conceptual and practical considerations for achieving a successful design, evaluation, and implementation of road pricing measures. They note, for example, that evaluating the social welfare implications of a Pigovian tax on commuter trips must take into account the charge's effect on not only congestion but also the labour market, already distorted by preexisting income taxes. In a study of the redistributive impact of road pricing, Foster [13] and Arnott et al. [14] show that it is regressive, an important finding for gauging the political cost of implementing such a policy. Other interesting works that take account of road pricing's redistributive effects are Gehlert et al. [15] and Linn et al. [16]. The cases of Sweden and Norway are explained in detail in the work of Börjesson et al. [17] and Ieromonachou et al. [18], respectively.

For multiple reasons, first-best optimal marginal cost prices cannot be implemented in practice [12]. To begin with, such measures tend to encounter strong political and public resistance. Also, the large additional administrative, personnel, and equipment expenditures involved in collecting tolls across an entire road network are typically not fully covered by the revenue generated. Many studies have therefore sought alternative or second-best pricing solutions that are technically inferior but can be feasibly implemented.

One such second-best or suboptimal solution is a bilevel optimization model suggested by Yan and Lam [19] that assumes fixed demand between origin-destination pairs. The upper level minimizes the road network's total trip time while the lower level defines network user behaviour (traffic assignment or route choice model). In a similar vein, Yang and Zhang [20] propose bilevel models that explicitly incorporate social and spatial equity conditions for various classes of vehicle drivers with different time valuations.

Zhang and Yang [21] study cordon-based road pricing models for delimited areas within a road network. Zhang and van Wee [22] propose the maximization of road network reserve capacity as the toll design criterion. Other works consider the time dimension of optimal congestion charges [14, 23–28]. Liu et al. [29] address the toll pricing framework for the first-best pricing with logit-based stochastic user equilibrium (SUE) constraints. Daganzo and Lehe [30] introduce trip-time heterogeneity and propose a usage-based, time-varying toll that alleviates congestion while prioritizing shorter trips.

Another approach aimed at avoiding the complexities of optimal toll schemes that are theoretically efficient but difficult to implement is based on the origins and destinations (O-D) of road network users [31]. In this system, users pay as a function of their destination. Thus, the same toll is levied on all routes connecting a given O-D pair regardless of their length.

Ahn [32] incorporates into optimal toll design the interactive effects of a decline in private vehicle traffic on congestion and a possible consequent improvement in bus services sharing the same roads. They conclude that road pricing has the potential to improve private welfare when there is congestion even without considering toll revenue use. Ekström et al. [33] evaluate the performance of a surrogate-based optimization method, when the number of pricing schemes are limited to between 20 and 40. A static traffic assignment model of Stockholm is used for evaluating a large number of different configurations of the surrogate-based optimization method. Their results show that the surrogate-based optimization method can indeed be used for designing a congestion charging scheme, which return a high social surplus.

On the issue of tolls as an instrument for financing road construction, optimal road pricing studies have also considered schemes that involve the building, operation, and transfer of road infrastructure (known as the BOT framework). Though this research is less abundant, recent decades have witnessed an increase worldwide in the supply of private toll highways in both developed and developing countries.

This public policy option has made it possible to undertake multiple large infrastructure projects simultaneously without putting pressure on public funds. For example, as of 2007 about one-third of the Western European highway network was under concession [34]. In Chile, private concessions have been let for Route 5 between La Serena and Puerto Montt, the country's main north-south highway, and various urban highways in greater Santiago, the nation's capital.

In this context, Verhoef and Rouwendal [35] study the relationship between optimal pricing, capacity choice, and road network financing. The authors report that optimal pricing and capacity choice policies may result in user charges for moderately congested areas of an already-built network that are only slightly less than those for highly congested areas. They also conclude that a fixed charge per kilometre may produce optimally efficient results.

Both Verhoef [34] and Ubbels and Verhoef [36] study the capacity and toll level choices of private bidders in a government-organized highway concession auction, considering and comparing different criteria for the award of the concession.

The involvement of the private sector in building and running highways may have advantages in terms of lower costs, greater innovation, and availability of funds. However, private objectives do not necessarily coincide with the maximization of social welfare [37, 38]. It is important, therefore, that regulatory authorities have the proper tools to design appropriate concession contracts.

With this caveat in mind, Verhoef [34] examines how concession award criteria impact route capacity and tolls. The author concludes that the socially optimal toll and capacity values are achieved when the award is based on the level of use. However, when second-best aspects are taken into account, maximizing level of use is no longer the optimal social solution but remains a second-best alternative.

Woensel and Cruz (2007) use queueing theory tools to incorporate dynamic and stochastic aspects of traffic behaviour into the calculation of marginal congestion cost and the consequent design of optimal congestion charges by the relevant authorities. In this way, Zhu and Ukkusuri [39] propose a dynamic tolling model based on distance and accounts for uncertain traffic demand and supply conditions. Their results show that the total travel time of tolling links reduces by 25% over simulation runs. Verhoef [40] presents dynamic extension of the economic model of traffic congestion, which predicts the average cost function for trips in stationary states.

Ferrari [41] proposes a method of calculating tolls that partitions the cost burden between motorists and public financing in such a way as to optimize social welfare. The approach assumes deterministic road networks. The author applies the model to a real case and concludes that the optimal toll for a road segment is independent of its fixed costs but strongly depends on the marginal cost of public funds and motorists' willingness to pay. Mun and Ahn [42] present a model of a transport system with two road links in a series that describes traffic patterns under various pricing regimes; this serial link approach is similar to our model representation.

Álvarez et al. [43] estimate optimal charges for the use of highways in Spain based on vehicle type. They compare these charges to the tolls actually imposed, finding that the latter are generally above marginal cost. In making their estimates they emphasize the difference between externalities on urban and interurban highways. Whereas the main external effect in the urban case is congestion, in the interurban case it may be accidents along the route.

On the other hand, there is extensive literature regarding the acceptance of payment by the road (recent analyses are presented in Hensher and Li [44], Hysing [45], and Grisolía et al. [46]). While it is beyond the scope of this article, this is a fundamental issue in the design of road pricing policies.

3. Optimal Toll Model

The model we develop in what follows incorporates various characteristics similar to those discussed in the studies reviewed above but differs in that we make particular use of the flow-speed relation given by macroscopic traffic models. In this context, a given flow can occur with high or low speeds and the optimal price will be different in each case. The proposed formulation is easy to implement in practice and generates price recommendations for changing highway traffic conditions.

The use of link densities has a number of advantages over the traditional flow-based approach that allow it to achieve a higher degree of realism. It recognizes that link maximum flow is not fixed but rather is a function of density levels. These maximum flows are determined as a function of the speed and density on each link as given by the fundamental traffic equation. Finally, the density-based approach identifies whether a reduced flow level on a given link is due to low latent demand for its use (e.g., low density) or, on the contrary, to heavy congestion (e.g., high density) reducing the flow that can use the link, thereby generating traffic queues and longer delays.

In this context, Ohta [47] shows how flow can become a misleading variable if it is interpreted as a direct policy variable. This is because both equilibrium flow and optimal flow keep increasing with demand only up to a certain critical point, beyond which they start to decline. However, from equilibrium point of view (which is outside the scope of our model), there is an interesting debate with Verhoef [48]. Liu et al. [49], using a macroscopic approach and microsimulation, show that the incorrect use of performance curves to estimate demand can thus seriously underestimate equilibrium traffic levels and the costs of congestion.

We begin by letting d_a^t be the density of vehicle flow on link or segment a of a highway in period t. As will be shown later, we consider density as a good proxy for demand that depends on the price or toll p_a^t as well as temporal and spatial control variables x_{ka}^t. Now consider the following relation:

$$d_a^t = \theta_0 + \sum_{k=1}^{K} \theta_k x_{ka}^t + \theta_p p_a^t \quad \forall a, t, \tag{1}$$

where θ_p is the parameter of toll p_a^t for segment a in period t and θ_k are the parameters of the K control variables (e.g., dummies for time of day, day of the week, geographical zone) and θ_0 is the model intercept. The sign of θ_p is negative

($\theta_p < 0$) and thus consistent with microeconomic theory, meaning that the higher the charge, the lower the demand and therefore the density. For simplicity's sake we define $\theta_a^t = \theta_0 + \sum_{k=1}^{K} \theta_k x_{ka}^t$.

With these basic elements we can now present our two optimization model variants for generating optimal tolls in MR and MIU scenarios. The MR version is set out in Section 3.1 and the MIU version in Section 3.2.

3.1. MR Optimal Toll Model. The optimality criterion for the MR scenario is the maximization of highway toll revenue. The optimization problem is then as follows:

$$\max_{\{p_a^t\}} \quad Z_1 = \sum_{a,t} p_a^t f_a^t \tag{2}$$

$$\text{s.t.:} \quad f_a^t = v_a^t \cdot d_a^t \quad \forall a, t, \tag{3}$$

where p_a^t is the toll, f_a^t is the traffic flow, and v_a^t is the traffic speed along segment a in period t. If the highway operating cost is constant, it will not affect the above model's optimality conditions; if, on the other hand, it is directly proportional to flow, p_a^t can simply be defined as the difference between revenue and operating cost per unit of flow. Either way, then, maintenance/operating cost does not need to be explicitly incorporated into the model [50].

The speed along a segment is a decreasing function of density (a good survey of such relations is found in Wang et al. [51]) and is defined in the following manner:

$$v_a^t = \beta_0 + \sum_{k'=1}^{K'} \beta_{k'} x_{k'a}^t + \beta_d d_a^t \quad \forall a, t, \tag{4}$$

where β_0 is the baseline parameter representing the speed of free-flowing traffic under normal highway conditions, $\beta_{k'}$ are the parameters for the K' control variables (e.g., dummies for time of day, day of the week, geographical zone, and incidents), and β_d is the density parameter for segment a in period t. The sign of β_d consistent with traffic theory is $\beta_d < 0$. For simplicity we define $\beta_a^t = \beta_0 + \sum_{k'=1}^{K'} \beta_{k'} x_{k'a}^t$.

The optimality condition for objective function (2) above is

$$\frac{\partial Z_1}{\partial p_a^t} = f_a^t + p_a^t \frac{\partial f_a^t}{\partial p_a^t} = 0. \tag{5}$$

Given constraint (3), that is, $f_a^t = v_a^t \cdot d_a^t$,

$$\frac{\partial f_a^t}{\partial p_a^t} = \frac{\partial v_a^t}{\partial p_a^t} d_a^t + \frac{\partial d_a^t}{\partial p_a^t} v_a^t. \tag{6}$$

Substituting (6) into (5), we have

$$\begin{gathered} p_a^t \frac{df_a^t}{dp_a^t} = -f_a^t \longrightarrow \\ p_a^t = \frac{-f_a^t}{\left(\left(\partial v_a^t / \partial p_a^t\right) d_a^t + \left(\partial d_a^t / \partial p_a^t\right) v_a^t\right)}. \end{gathered} \tag{7}$$

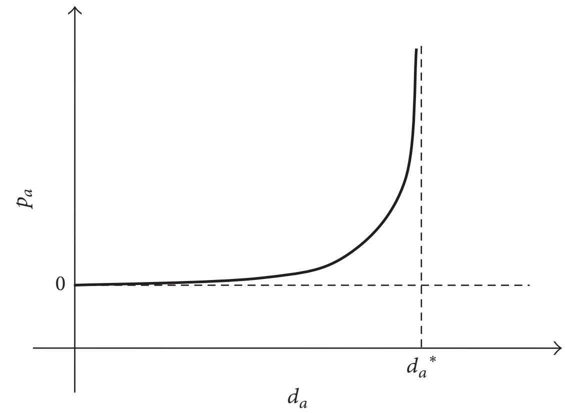

Figure 1: Relationship between MR optimal toll and vehicle traffic density.

This expression gives the optimal value of the toll for segment a in period t in the private benefit scenario.

Differentiating (1) and (4) we get

$$\begin{gathered} \frac{\partial d_a^t}{\partial p_a^t} = \theta_p < 0, \\ \frac{\partial v_a^t}{\partial p_a^t} = \frac{\partial v_a^t}{\partial d_a^t} \frac{\partial d_a^t}{\partial p_a^t} = \beta_d \cdot \theta_p > 0. \end{gathered} \tag{8}$$

Plugging (8) into (7), we obtain

$$\begin{gathered} p_a^t \frac{df_a^t}{dp_a^t} = -f_a^t \longrightarrow \\ p_a^t = \frac{-f_a^t}{\left(\theta_p \beta_d d_a^t + \theta_p v_a^t\right)}. \end{gathered} \tag{9}$$

Finally, given that $\theta_p < 0$ and that $(\beta_d d_a^t + v_a^t) > 0 \rightarrow v_a^t > -\beta_d d_a^t$, we arrive at

$$p_a^t = \frac{-v_a^t d_a^t}{\theta_p \left(\beta_d d_a^t + v_a^t\right)} > 0. \tag{10}$$

This expression gives the optimal toll for segment a in period t in the private benefit scenario. Since, by (1), density d_a^t depends on the toll p_a^t and, by (4), v_a^t depends on density d_a^t, (10) can be easily represented as a nonlinear function in p_a^t. More specifically, it is a quadratic function and therefore easy to solve.

The relationship between MP optimal toll and density is shown in Figure 1 for a simple baseline case of a highway 1 kilometre long under the following predefined relations: $v_a^t = \beta_a^t + \beta_d d_a^t$ and $d_a^t = \theta_a^t + \theta_p p_a^t$.

In this case, $d_a^{t*} = -\beta_a^t / 2\beta_d$. The first conclusion to be drawn from (10) and the figure is that, as density approaches the maximum flow level (or highway capacity), the private optimal toll tends towards infinity. Note that the maximum

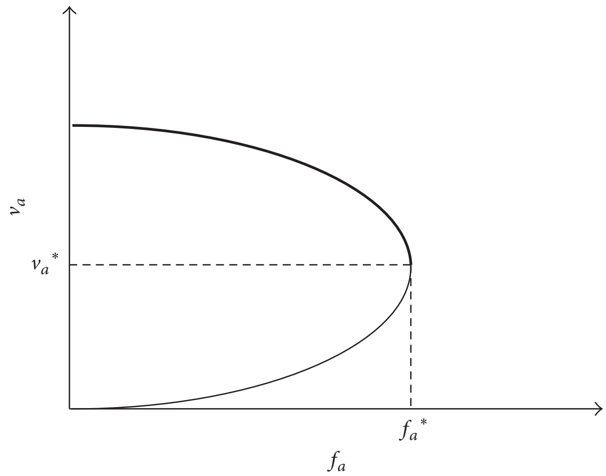

Figure 2: Relationship between vehicle traffic speed and flow.

flow conditions on a segment have been reached when the following are satisfied:

$$\begin{gathered}\frac{df_a^t}{dp_a^t} = \frac{\partial v_a^t}{\partial p_a^t} d_a^t + \frac{\partial d_a^t}{\partial p_a^t} v_a^t = 0, \\ \theta_p \left(\beta_d d_a^t + v_a^t \right) = 0 \longrightarrow \\ v_a^t = -\beta_d d_a^t. \end{gathered} \tag{11}$$

Thus, (10) indicates that the optimal private flow along a highway always falls within the flow interval marked by a thick line in Figure 2. This result is consistent with Li [52], which also incorporates the relationships between flow and speed to derive optimal congestion charges.

Another interesting, if intuitive, conclusion arising from (10) is that as the absolute value of θ_p is reduced, the toll increases. In the limit, if demand is perfectly inelastic, that is, independent of the toll value ($\theta_p = 0$), the optimal private toll will tend to infinity, consistent with what microeconomic theory would predict.

The relationship between MR optimal toll and speed, where $v_a^{t*} = -\beta_d d_a^t$, is shown in Figure 3. This clearly shows that, as traffic speed falls, the optimal toll tends to rise. The reason for this behaviour is consistent with Figure 2: as traffic levels approach highway capacity, speed falls and in the private scenario there will be an incentive to increase the toll and thus stimulate a greater flow so that more revenue will be generated.

Also, if there is no congestion ($\beta_d = 0$), the optimal toll according to (10) is given by

$$p_a^t = -\frac{d_a^t}{\theta_p}. \tag{12}$$

From this expression we deduce that, from a private perspective, as traffic density increases so will the toll, but in this case proportionally, that is, linearly, as opposed to (10) which is obviously nonlinear. Thus, (12) confirms that density is a good proxy for demand or drivers' willingness to pay.

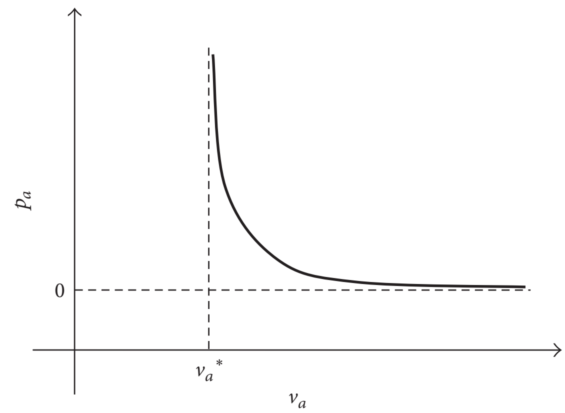

Figure 3: Relationship between MR optimal toll and vehicle traffic flow.

We may therefore conclude that the optimal MP toll will always increase with a rise in density, but in the presence of congestion its marginal increase with respect to density is greater than in its absence.

3.2. MIU Optimal Toll Model. The optimality criterion for the MIU scenario is assumed following Verhoef [34] to be the maximization of highway use (i.e., maximization of the use of the already-built resource). The optimization problem is then as follows:

$$\begin{aligned} \max_{\{p_a^t\}} \quad & Z_2 = \sum_{a,t} f_a^t \\ \text{s.t.:} \quad & f_a^t = v_a^t \cdot d_a^t \quad \forall a, t. \end{aligned} \tag{13}$$

Under the proposed approach, the effect of a vehicle entering or leaving the highway on the vehicles currently using the local street network is irrelevant or negligible (the same assumption is implicit in other works such as Verhoef and Rouwendal [35], Verhoef [34], and Woensel and Cruz (2007)).

Infrastructure cost is also excluded from consideration, the supposition being that the highway already exists (this was also assumed for the MR scenario in Section 3.1). In any case, as long as the infrastructure cost is a fixed value it would have no effect on the problem optimum. In Section 5.2 we will nevertheless consider an extension in which the highway capital cost is incorporated as a budget constraint that could impact future toll charges.

Problem (13) can be rewritten as

$$\max_{\{p_a^t\}} \quad Z_2 = \sum_{a,t} v_a^t \cdot d_a^t. \tag{14}$$

The optimality conditions of (14) are

$$\frac{dZ_2}{dp_a^t} = \frac{df_a^t}{dp_a^t} = \frac{\partial v_a^t}{\partial p_a^t} d_a^t + \frac{\partial d_a^t}{\partial p_a^t} v_a^t = 0, \tag{15}$$

$$\begin{gathered} \theta_p \left(\beta_d d_a^t + v_a^t \right) = 0 \longrightarrow \\ \theta_p \beta_d d_a^t = -\theta_p v_a^t. \end{gathered} \tag{16}$$

If there is no congestion ($\partial v_a^t/\partial p_a^t = 0$), optimality condition (15) indicates zero tolls given that the maximum value of Z_2 is obtained when $p_a^t = 0$.

In the presence of congestion, however, we obtain

$$\frac{d_a^t}{v_a^t} = -\frac{\partial d_a^t/\partial p_a^t}{\partial v_a^t/\partial p_a^t} = -\frac{\theta_p}{\theta_p \beta_d} = -\frac{1}{\beta_d}. \tag{17}$$

Thus, optimality condition (17) indicates that the theoretical MIU optimum is reached when the highway is operating at maximum flow.

Substituting (1) and (4) into (17) and recalling that

$$\begin{aligned}\theta_a^t &= \theta_0 + \sum_{k=1}^{K} \theta_k x_{ka}^t,\\ \beta_a^t &= \beta_0 + \sum_{k'=1}^{K'} \beta_{k'} x_{k'a}^t,\end{aligned} \tag{18}$$

we obtain

$$\begin{aligned}\frac{\theta_a^t + \theta_p p_a^t}{\beta_a^t + \beta_d d_a^t} &= -\frac{1}{\beta_d},\\ \beta_d\left(\theta_a^t + \theta_p p_a^t\right) &= -\left(\beta_a^t + \beta_d d_a^t\right),\\ p_a^t &= \frac{-\beta_a^t - \beta_d d_a^t - \beta_d \theta_a^t}{\beta_d \theta_p}.\end{aligned} \tag{19}$$

This last result states that the MIU optimal toll grows proportionally with increasing density. To ensure $p_a^t \geq 0$ and given that $\beta_d < 0$, it must be the case that

$$-\beta_a^t - \beta_d d_a^t - \beta_d \theta_a^t \geq 0, \tag{20}$$

$$d_a^t \geq \frac{\beta_a^t + \beta_d \theta_a^t}{-\beta_d}. \tag{21}$$

Thus, if density is such that (21) is not satisfied, the MIU optimal toll is zero (there are no negative tolls or economic incentives for using the highway) and the MIU optimal toll structure is

$$\begin{aligned}p_a^{t*} &= \begin{cases} \dfrac{-\beta_a^t - \beta_d d_a^t - \beta_d \theta_a^t}{\beta_d \theta_p}, & \forall d_a^t > \dfrac{\beta_a^t + \beta_d \theta_a^t}{-\beta_d} \\ 0, & \forall d_a^t \leq \dfrac{\beta_a^t + \beta_d \theta_a^t}{-\beta_d} \end{cases}\\ p_a^{t*} &= \max\left[\left(\frac{-\beta_a^t - \beta_d d_a^t - \beta_d \theta_a^t}{\beta_d \theta_p}\right); 0\right].\end{aligned} \tag{22}$$

Interestingly, the greater the free-flowing traffic speed is (i.e., the value of β_a^t, which includes parameter β_0), the lower the MIU optimal toll is. Similarly, the greater the potential demand for highway use is (i.e., the values of θ_a^t, which includes parameter θ_0), the greater the MIU optimal toll is.

Table 1: Summary of data for estimating parameters (N = 4,680 observations).

Variable	Mean	Std. dev.	Min.	Max.
Toll (CLP$)	410.4	124.3	222.0	580.0
Speed (Km/h)	70.8	16.1	15.3	95.7
Density (Veh/Km)	26.4	8.7	6.2	53.8
Flow (Veh/h)	3,501.1	672.0	611.0	5,194.0

4. Analysis of an Application

To apply our toll models we estimate the parameters in (1) and (4), which represent the design characteristics of the highway. With these estimates we can then derive the values of the remaining analytic equations obtained from our optimization models. The necessary data were taken from the real case of an urban motorway/freeway traversing the urban region of Santiago, Chile. It is the most important highway of its kind in the country, with daily flows in the two directions totalling more than 140 thousand vehicles. The data used were for the peak morning period from January 2012 to September 2013 and consisted of the averages of readings taken every time unit (half-hour) for vehicle flows and speeds over three contiguous toll segments (i.e., segments with different applicable tolls) plus the toll in real (inflation-adjusted) terms for each segment. Average density data, also for every half-hour, were constructed by dividing flow by speed.

The identification of the marginal effect of the toll on density (see (1)) is complicated by the fact that the tolls on the Santiago highway are adjusted only once a year in nominal terms. Using real values introduces a certain amount of variation, albeit rather small. A more serious source of variation is the time of day. We therefore confined our analysis to the morning peak period (8 am to 10 am), during which congestion is present and a toll increase occurs (at 9 am). Obviously, this increase is endogenous and is due precisely to the expected rise in densities beginning at approximately that hour. The endogeneity thus introduced was eliminated by including dichotomous variables for each time unit. These variables would be almost perfectly collinear with the toll if there were only one toll segment, but, with more than one (e.g., two contiguous ones), another source of variation arises. Here, we used the tolls for the three contiguous segments that are most heavily used.

Descriptive statistics on the data used for estimating the model parameters are summarized in Table 1. The estimates for the parameter θ_p in (1) and the parameter β_d in (4) are shown in Table 2 and were derived using multiple linear regression (two-way fixed effects model with composite error) that included temporal and spatial dichotomous control variables for month, year, time unit, and toll segment. Statistical tests, also reported in Table 2, found significant negative relationships between real toll and density (consistent with economic theory) as well as between speed and density. The control dummy variable and constant estimates are omitted.

In the regression model for estimating θ_p the dependent variable was density and the explanatory variables were

TABLE 2: Estimates of parameters θ_p and β_d in (1) and (4).

Parameter	Coeff.	Std. err.	t	$P > \|t\|$	[95% conf. interval]	
θ_p	−0.0194195	0.006253	−3.11	0.002	−0.0316784	−0.0071606
β_d	−1.701152	0.0222983	−76.29	0.000	−1.744867	−1.657437

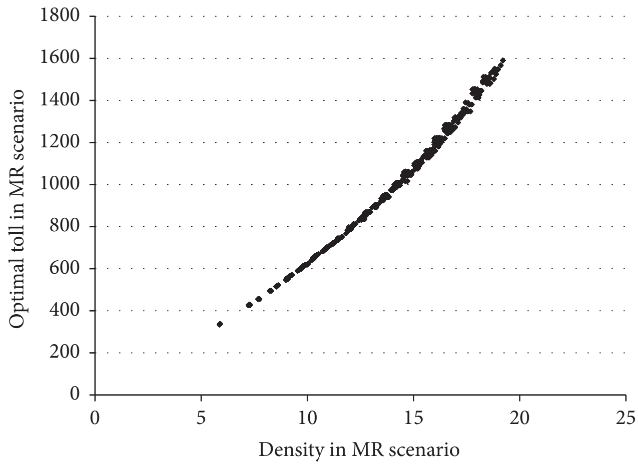

FIGURE 4: Relationship between optimal toll and density in MR scenario.

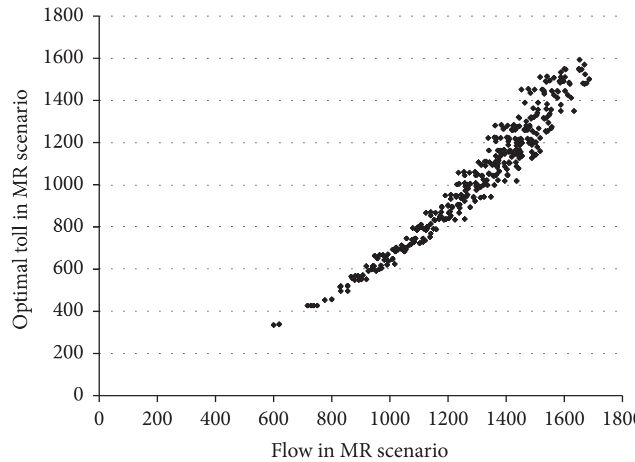

FIGURE 5: Relationship between optimal toll and flow in MR scenario.

the real toll plus the above-mentioned explanatory control variables. To estimate β_d, the dependent variable was speed and the explanatory variables were density and the controls.

With these estimates, we derived the values of the other terms in (1) and (4) residually for each time unit and toll segment. Thus, in (1) we estimated

$$\widehat{\theta_0 + \sum_{k=1}^{K} \theta_k x_{ka}^t} = d_a^t - \widehat{\theta}_p p_a^t, \tag{23}$$

analogously for (4). This allowed us to estimate the effects of other observable and nonobservable factors on density in (1) or on speed in (2) without having to identify each parameter in the equation. Furthermore, with these results we were able to estimate the MR and MIU tolls for each time unit and toll segment, thus reflecting the conditions of the moment.

Thus, with the estimates in Table 2 we can directly derive estimates of the optimal tolls for each segment a as a function of the demand and quality of service levels during any period t of interest.

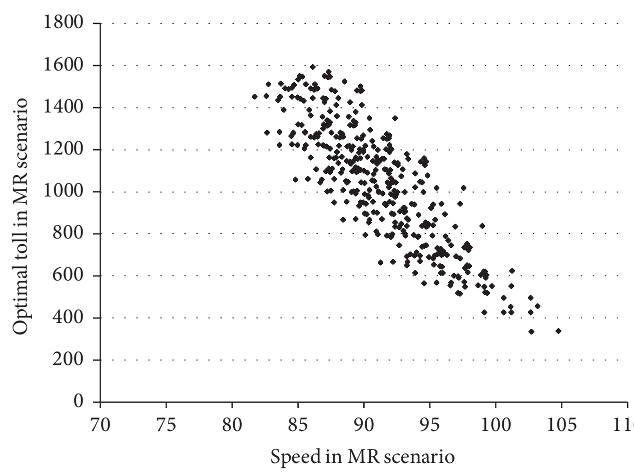

FIGURE 6: Relationship between optimal toll and speed in MR scenario.

4.1. Analysis of the Results for the MR Scenario. Once the MR optimal toll has been estimated from (10) for each segment a and period t, we also obtain the respective levels of density, speed, and flow. The relationships between these variables for the private scenario are illustrated in Figures 4, 5, and 6.

The first of these relationships, shown in Figure 4, plots the optimal toll against density. As was already shown in Figure 1, with rising density the optimal toll tends to increase more than proportionally. This is so because greater demand for highway use reflects a greater willingness to pay for it, and this effect is amplified by the increased congestion resulting from that stronger demand. The latter in turn reduces speed and therefore flow (ceteris paribus), stimulating the private owner to raise the toll more than in proportion to the density increase.

Figure 5 presents a complementary graph showing the relationship between MR optimal toll and flow while Figure 6 depicts the relationship between MR optimal toll and speed.

As we saw earlier in Figure 3, with falling speed (due to rising demand) the optimal toll increases to compensate the flow decline generated by the speed reduction.

Table 3: Average values of model variables.

Variable	Observed**	MR	MIU	Δ%***
Toll (CLP$)	410.4	1,033.8	181.7	17.6%
Speed (Km/h)	70.8	91.4	63.2	69.2%
Density (Veh/Km)	26.4	14.3	30.9	215.5%
Flow* (Veh/h)	1,750.6	1,297.9	1,911.9	147.3%

*Flow data are for every half-hour, the time unit used in specifying the regression models for estimating the parameters. **The tolls currently in force were defined during the tender process for the highway construction and take into account the infrastructure capital cost. ***The difference between the MIU optimum and the MR optimum, expressed as the percentage the former is of the latter.

4.2. Comparison of Results for the MR Scenario, MIU Scenario, and the Current Situation. The vector of tolls, speeds, densities, and flows for the MR scenario differs necessarily from those for both the MIU scenario and the actual observed situation, the lattermost being the source of the data for estimating our model. The reason is simply that the different tolls in the three cases induce different values for the other three variables. The average values for all four variables are summarized in Table 3.

From these results we see that on average, the MIU optimal toll for the different toll segments and time units is lower than the MR optimum (i.e., only 17.6% of the latter) and that the speed in the MIU optimal scenario is also lower. By contrast, the density and flow variables, both associated with demand levels, are higher in the MIU optimal scenario than the MR one.

These results are consistent with the predictions of economic theory: the available infrastructure is more intensely used in the MIU scenario because in the MP scenario the tolls are raised to the point where marginal revenue is zero whereas in the MIU scenario highway use is maximized so tolls must be lower. Indeed, in the MR scenario the highway never operates at full capacity as this would imply an infinitely high toll; the optimal flow in this case is therefore always less than that maximum.

Thus, in the MIU scenario the highway operates over the various toll segments and time units as close as possible to capacity. This is reflected in Figure 7 where the MIU optimal toll at many points is zero, particularly at lower demand levels. The figure also reveals how in all cases in the sample (regardless of segment or time slot) the MP optimal toll was higher than the MIU one. (Note that in this and Figures 7, 8, 9, and 10, the upward sloping line in grey is at a 45° angle and thus represents the set of points at which the quantities on the two axes are equal.)

The above results are further illustrated here in a series of graphs. Figure 8 compares the speeds of the MR and MIU optima while Figure 9 compares their densities and Figure 10 their flows. The percentage difference between the MIU optimal toll and the observed toll is plotted in Figure 11, where the vertical scale is the difference between the two expressed as the former divided by the latter in percentage terms. Thus, percentages greater than 0 are cases where the

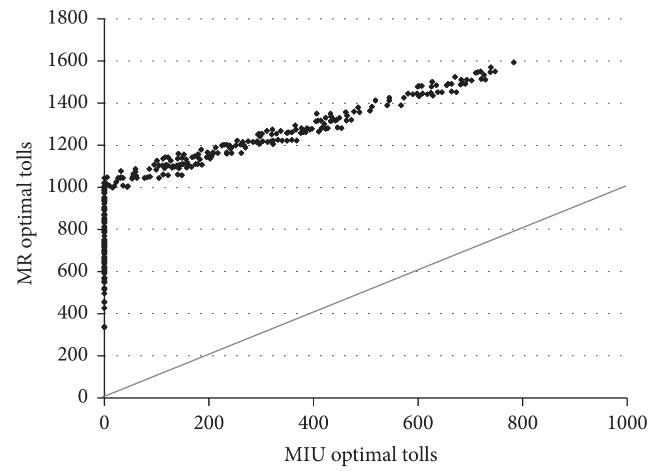

Figure 7: Relationship between MP optimal and MIU optimal tolls.

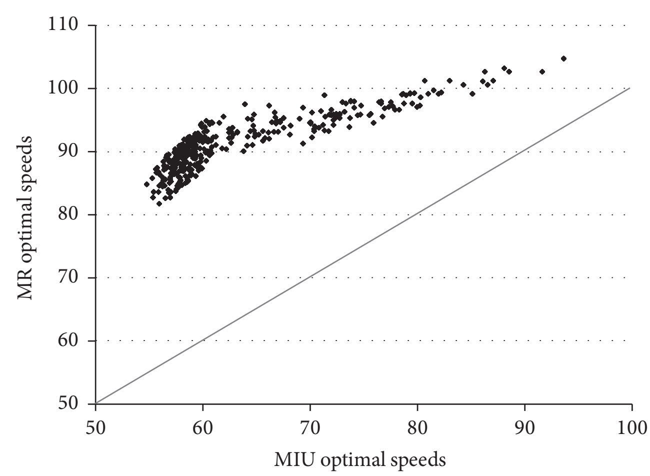

Figure 8: Relationship between MP optimal and MIU optimal speeds.

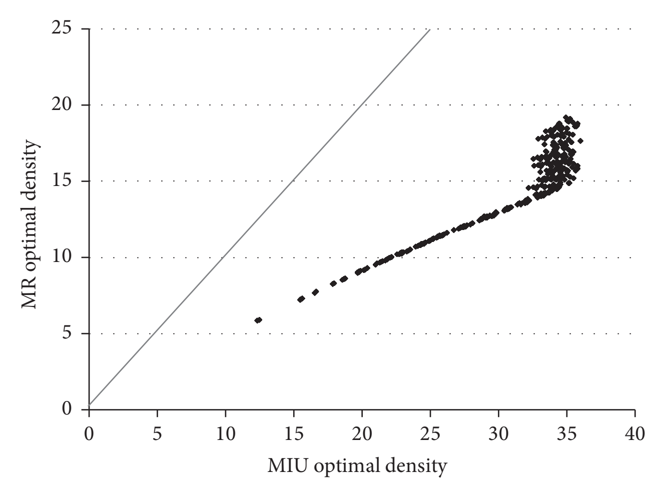

Figure 9: Relationship between MP optimal and MIU optimal densities.

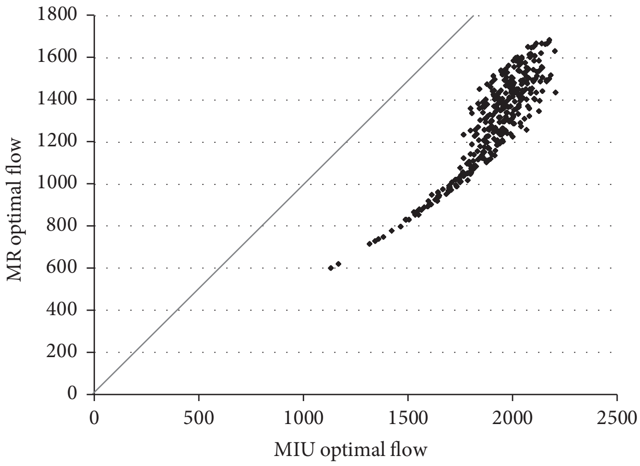

Figure 10: Relationship between MP optimal and MIU optimal flows.

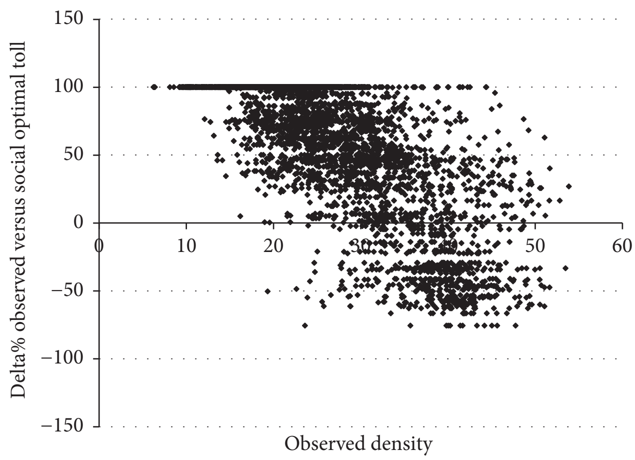

Figure 11: Percentage difference between MIU optimal and observed tolls.

MIU optimal toll is higher than the observed toll while percentages less than 0 are cases where the contrary is true.

As the graph shows, in most cases (86%, to be exact) the MIU optimal toll is below the observed level. There are, however, 14% of cases when the opposite holds, generally on high-demand (i.e., high-density) segments where congestion tends to be higher, as indicated by the curve fit to the data in Figure 11.

Finally, Figure 12 compares the MIU optimal and observed traffic flows, revealing that in the former case more use is made of the highway. This is consistent with the result just noted above that the MIU optimal toll is lower than the observed toll in 86% of cases. However, in 26% of cases the MIU optimal flow is less than the observed flow for reasons analogous to the explanation just given above in relation to Figure 11.

What finally emerges from all these results of the application of our model is that in a MIU optimal scenario the current highway toll structure should be redefined to induce

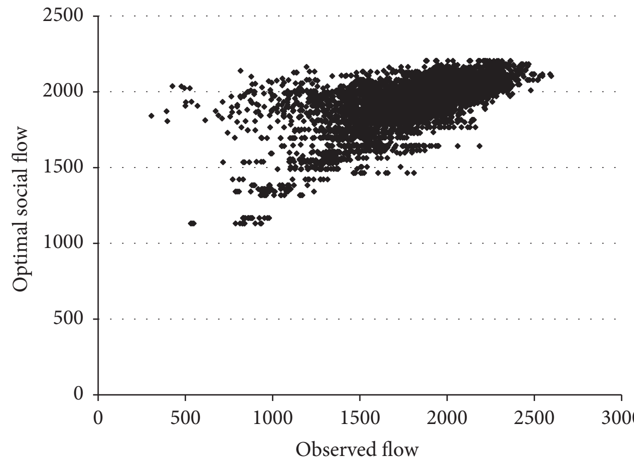

Figure 12: Relationship between MIU optimal and observed flows.

shifts in demand levels from heavily congested segments and periods to less congested ones.

5. Extensions

5.1. Maximum Toll Constraint (for MR Optimum). Consider the addition of constraints $p_a^t \leq k_a^t$ to the MR scenario specified by optimization problem (2)-(3). This restriction could represent a toll ceiling imposed by the government, for example. Letting λ_a^t be the Lagrange multipliers of the new constraints, the optimality conditions are then as follows:

$$f_a^t + p_a^t \frac{df_a^t}{dp_a^t} - \lambda_a^t = 0,$$

$$p_a^t \frac{df_a^t}{dp_a^t} = -f_a^t + \lambda_a^t \longrightarrow \tag{24}$$

$$p_a^t = \frac{-v_a^t d_a^t + \lambda_a^t}{\theta_p \left(\beta_d d_a^t + v_a^t\right)}.$$

In the absence of congestion, the optimal toll is

$$p_a^t = \frac{-v_a^t d_a^t + \lambda_a^t}{\theta_p v_a^t} = -\frac{1}{\theta_p}\left(d_a^t - \frac{\lambda_a^t}{v_a^t}\right). \tag{25}$$

This expression differs from (12) due to the presence of the λ_a^t / v_a^t term on the right hand side.

Since $p_a^t = k_a^t$ when $\lambda_a^t > 0$, we directly obtain

$$k_a^t = -\frac{1}{\theta_p}\left(d_a^t + \frac{\lambda_a^t}{v_a^t}\right) \longrightarrow$$
$$\lambda_a^t = v_a^t\left(d_a^t + k_a^t\theta_p\right). \tag{26}$$

In the presence of congestion, the optimality conditions are

$$k_a^t = \frac{-v_a^t d_a^t + \lambda_a^t}{\theta_p \left(\beta_d d_a^t + v_a^t\right)} \longrightarrow$$
$$\lambda_a^t = k_a^t \theta_p \left(\beta_d d_a^t + v_a^t\right) + v_a^t d_a^t. \tag{27}$$

5.2. Financing Constraint (for MIU Optimum). Consider now the MIU scenario specified by problem (13) with the addition of a highway financing constraint $\sum_{a,t} p_a^t f_a^t \geq C$, where C is the portion of the capital cost attributable to segment a in period t. Letting η be the constraint's Lagrange multiplier, the optimality conditions with respect to p_a^t are

$$\frac{df_a^t}{dp_a^t} + \eta \left(f_a^t + p_a^t \frac{df_a^t}{dp_a^t} \right) = 0, \tag{28}$$

$$\begin{aligned} p_a^t &= -\frac{f_a^t}{df_a^t/dp_a^t} - \frac{1}{\eta} \\ &= -\frac{f_a^t}{\left(\partial v_a^t/\partial p_a^t\right) d_a^t + \left(\partial d_a^t/\partial p_a^t\right) v_a^t} - \frac{1}{\eta}. \end{aligned} \tag{29}$$

Recalling (9) and substituting $df_a^t/dp_a^t = \theta_p(\beta_d d_a^t + v_a^t)$ into (29), we have

$$p_a^t = -\frac{v_a^t d_a^t}{\theta_p \left(\beta_d d_a^t + v_a^t\right)} - \frac{1}{\eta}. \tag{30}$$

This expression is very similar to (10), the unconstrained MR optimum, the sole difference being the $1/\eta$ term appearing in (30). The implication is that, with the incorporation of financing constraints, the MIU optimal toll is equivalent to the MR optimum but with the addition of a corrective term.

Also, if capital cost (C) in the budget constraint increases, the toll will increase compared to the unconstrained MIU optimum and flows will decline. Note, too, that the sign of parameter η must be negative ($\eta < 0$) so that the sum of the terms in (30) is always positive. This means that the MIU optimal toll must be higher in the presence of financing constraints than in their absence.

6. Conclusions

An analytic model was presented which estimates optimal toll levels for a congested highway in two different scenarios: a private one that maximizes toll revenue and another that maximizes highway use. The model consists in solving an optimization problem subject to demand constraints derived from the fundamental traffic equation relating flow, speed, and density and is therefore a macroscopic formulation.

Considering that density of traffic flow is a good proxy for demand and that demand depends on the toll, the optimal relationship between the two variables for a highway can be derived for different toll segments and operating periods. The proposed model can be used to estimate optimal congestion tolls for planned routes or for already-existing ones whose concession term is up for extension or retender.

Two extensions to the model were also developed. One incorporates the capital costs in order to design tolls for (postconstruction) financing of future highway infrastructure projects while the other imposes constraints on the toll structure such as a ceiling level.

The model was calibrated with high frequency data from the main highway serving Santiago, Chile. Applied to this real case, the formulation showed that the tolls maximizing private revenue benefits were significantly higher than those that would maximize infrastructure use. On average, the MR optimal toll was more than five times the MIU optimal toll.

This result is consistent with the fact that highway use in the MIU scenario will always be greater than in the MR scenario, implying that in the former case the traffic density and flow will also be greater but the speed will be lower. This reflects the MIU scenario optimization criterion of maximizing use; in the private case, by contrast, the highway will always operate at less than full capacity.

Also in the MIU scenario, the optimal toll will generally be lower than those currently observed on the Santiago highway, but when demand is high the MIU optimum will tend to be higher. These findings will be particularly relevant when a highway concession terminates and is to be extended or retendered.

A final conclusion of note is that when an infrastructure financing constraint is added, the MIU optimal toll tends to approach the MR optimum except for a corrective term associated with the constraint's shadow price.

An interesting further extension of the proposed model would be to incorporate, in the MIU scenario, the impact on the local street network of a reduction in network flow as a consequence of increased highway use (and vice versa). Note, however, that this would require a network approach, which would be difficult to model properly while retaining the fundamental traffic equation relating flow, density, and speed, one of the attractive qualities of our formulation.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This research was supported by FONDECYT Grant no. 1160251.

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