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# A Multiobjective Model for the Cutting Pattern Problem with Unclear Preferences

**Cristian D. Palma and Francisco P. Vergara**

**The cutting pattern problem has been traditionally approached using single objective optimization models, although the sawmill performance is usually measured using more than a single indicator. One of the shortcoming of using multiobjective approaches is that they need a preference relationship among the objectives, which is difficult to determine in practice, and solutions are very sensitive to these preferences. In this article, we consider different criteria in a sawmill decisionmaking context using a multiobjective linear optimization model and handle the unclear definition of the objective preferences by formulating a robust version of the model. Although the deterministic formulation assumes perfect information of the objective preferences, in the robust formulation we consider that preferences may be different from their estimate. We show that deterministic decisions are more balanced in terms of the different criteria than the traditional single objective models, although their quality is very sensitive to the objective preferences. We also show that robust decisions are also balanced but less sensitive to the preferences. We explore how the level of the different indicators and the cutting decisions are affected when the preferences are unclear.**

**Keywords:** multiobjective, uncertain preferences, robust optimization, sawmill planning

One of the most important decisions in a sawmill is how to cut different logs so as to obtain the products demanded by customers, and several optimization tools have been proposed for this purpose (see, for example, Maness and Adams 1991, Maturana et al. 2010). These tools optimize decisions based on a single objective, but unfortunately sawmill managers are usually required to meet various performance indicators simultaneously, such as low operating costs and high productivity, among others. This situation leads to unbalanced solutions that although very efficient in one indicator, present poor results in others. In addition, if different indicators were considered in the decisionmaking process, the preferences for each of them would seldom be well defined.

Different decisionmaking techniques can be applied when more than a single objective is to be considered, and many applications can be found in the forest management context (Diaz-Balteiro and Romero 2008). Regardless of the technique used, the relative importance of the objectives under consideration must be determined and translated into objective preferences. The correct determination of these preferences is crucial for a successful application of any multicriteria techniques but doing it in practice is still difficult (Steuer 1986, Cohon 2004).

In most of the multicriteria decision methods, the basis for determining the preferences is the pairwise comparison (David 1988), in which objectives (or decision alternatives in cases of a discrete decision space) are compared in pairs to judge which of each is preferred. Outranking methods (Brans et al. 1986) and the analytic hierarchy process (Saaty 1987) are common techniques used to elicit preferences, both of which are based on pairwise comparison. SMART methods are a set of techniques that determine the objective preferences through direct rating of the objectives rather than pairwise comparison. In all cases, the relative importance of the objectives is obtained through different approaches that process the scores obtained from the comparisons or rating, including matrix-based and optimization methods (see, for example, Siraj 2011). Interested readers are referred to Figueira et al. (2005) and Riabacke et al. (2012) for a recent review of these and other preference elicitation methods.

Regardless of how the preferences are determined, it has been shown that even small changes in the preferences used to solve a model may affect both the quality and the type of solutions obtained (Kangas 1994, 2006, Butler et al. 1997, Kurttila et al. 2009). Only in specific situations in which the decision space is reduced may optimal decisions remain unchanged if weights are modified (Kangas 1994). The high sensitivity of the solutions with respect to the preferences implies that traditional deterministic solutions may largely differ from solutions that do consider the uncertainty in these preferences. Ignoring this uncertainty may mislead decisions and

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**Affiliations:** *Cristian D. Palma (cristianpalma@ingenieros.udd.cl), Facultad de Ingeniería, Universidad del Desarrollo, Concepcion, Chile. Francisco P. Vergara (fvergara@ubiobio.cl), Faculty of Engineering, Department of Wood Engineering, Universidad del Bío-Bío.*

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produce suboptimal results (Pukkala and Miina 1997). In this context, either a good estimation of the objective preferences or a solution that is less sensitive to these preferences must be sought. In this study, we approach the cutting pattern problem from a multiobjective framework and consider that the objective preferences cannot be defined precisely. To do this, a traditional weighted sum approach is used to consider three performance indicators as the planning objectives (i.e., operating cost, amount of waste produced, and overproduction), and a robust version of this model that considers unclear definition of the objective preferences is formulated. The single objective model and the deterministic and robust multiobjective models are applied to a real problem and their solutions are compared. A robust model is a modified deterministic model that produces robust solutions. These solutions are, in the same sense as in Barrico and Antunes (2006), solutions that remain good even though the real objective preferences may differ from the ones used to generate the solution. We note that our goal is not to determine the objective preferences but to find robust solutions for a given set of preferences that may not be accurately elicited.

In the next sections, we briefly review the most relevant literature, describe the case study and the mathematical formulations, and show and discuss numerical results. Finally, we present the main conclusions of the study and ideas for future work.

## Literature Review

Sawmills transform logs of different diameter and length, known as log classes, into rectangular cross-section lumber, of standardized thickness and width, by applying a cutting pattern to each log. Different patterns can be applied to a single log class, each of them producing a number of lumber pieces of different dimensions, as well as waste material. Although for each log class there is an optimal cutting pattern that minimizes waste, the log availability seldom matches the lumber requirement and a log cannot be processed using its optimal cutting pattern, therefore leading to an increase in the waste production and to poor sawmill performance. Under these conditions, a good combination of cutting patterns applied to different log diameters largely affects the amount of raw material required to meet the customer demands.

Different optimization models have been built to deal with the cutting pattern problem as described. Although a few mixed integer programming formulations have been proposed to solve the problem (Maness and Adams 1991, Pradenas et al. 2004), linear programming has been the most common technique (Maness and Norton 2002, Singer and Donoso 2007, Caballero et al. 2009, Maturana et al. 2010, Alvarez and Vera 2014, Varas et al. 2014). Although the problem has been formulated in the literature in a similar way, in some cases it has been combined with bucking decisions (Maness and Adams 1991, Maness and Norton 2002) and timber transfer decisions among sawmills in a supply chain context (Singer and Donoso 2007). Multiperiod models have also been used with the possibility of handling inventory, of either logs or lumber, to add more flexibility (Maness and Norton 2002, Singer and Donoso 2007, Maturana et al. 2010). Only recently, has uncertainty for this type of problems been considered in the yield of the cutting patterns (Kazemi et al. 2010) and in the product demand and availability of logs (Varas et al. 2014), with the use of robust optimization in both articles.

In all cases, either the minimization of costs or the maximization of the profits or volume has been considered as the single objective, although the performance of sawmills is measured using different criteria. In forestry, the consideration of more than one objective in decisionmaking models dates back to the early 1970s (Field 1973) and has been applied to a broad range of forest planning problems (Diaz-Balteiro and Romero 2008). Nevertheless, to our knowledge no application to cutting planning problems has been reported. The most common approaches include multiobjective optimization and goal programming, both of which with a range of variations. In the first case, different objectives are optimized by combining them in a compounded objective function (Mendoza et al. 1987); in the second, a set of target values (goals) are defined for each objective, and the sum of all the differences between the observed value of each objective and its target is minimized (Field 1973). In both cases, the units of measure of the different objectives have to be standardized, and the objectives can be weighted differently to reflect the decisionmaker preferences of one objective over the others (Marler and Arora 2010).

These techniques have been mainly used in a deterministic setting, and most of the work that considers uncertainty in multiple criteria decisionmaking has been done in techniques that evaluate solutions previously identified (Butler et al. 1997, Kangas et al. 2006; for a review, see Durbach and Stewart 2012) rather than in techniques that generate the solutions. The explicit inclusion of uncertainty in forest planning models has grown as more computer capacity has become available. Strategic (Hoganson and Rose 1987, Palma and Nelson 2009) and tactical (Alonso-Ayuso et al. 2011, Palma and Nelson 2014, Varas et al. 2014) models that include uncertainty have been proposed, but most of them have dealt with a single objective. In a multiobjective setting, the uncertainties in the future state of the forests have been considered using multiobjective dynamic programming (Gong 1992) and the vagueness in the definition of the objectives has been modeled using fuzzy set theory (Ells et al. 1997). To our knowledge, the uncertainty in the preferences of the objectives has been only scarcely considered, using a combination of simulation and optimization (Pukkala and Miina 1997) and using robust optimization (Palma and Nelson 2010). In both of these articles, deterministic solutions are compared with robust solutions, those that remain good even if the uncertain parameters change. However, none of these works explores how the presence of uncertainty in the objective preferences determines the type of decisions that should be preferred and how the amount of the different objectives is affected.

In this study, we present a deterministic and a robust formulation of a multiobjective model for the cutting planning problem and explore how the unclear definition of the objective preferences affects the optimal decisions and the level of the different objectives. To do this, we consider three performance indicators of the sawmilling process as objectives and evaluate both deterministic and robust decisions under simulated objective preferences.

## Methods

### Study Case

The need for decisions that produce a good balance among different performance indicators motivated the production manager of a medium size sawmill located in a southern province of Chile to explore the use of a multiobjective planning approach. The sawmill production in 2009, the year used for this evaluation, was approximately 315,000 $m^3$ of lumber that was exported to Asia, Europe, and North America and sold to the domestic market.

The three main indicators (planning objectives) identified by the manager were the total cost of operation, the amount of waste (indirectly the lumber recovery factor), and the amount of overproduction. The overproduction represents the production of lumber products that are not demanded at the time of production and that are obtained in addition to demanded products when a cutting pattern is applied. These nondemanded products are obtained due to the discrepancy among the demanded products and the set of products produced by the cutting patterns and are usually inventoried for future orders.

To evaluate the models, we considered 322 cutting patterns available in the sawmill database. For these cutting patterns, the yield of different lumber products if applied to different log classes was available.

## Deterministic Multiobjective Model

The model considers multiple periods and the possibility of inventory of logs and final products. To overcome the effect of the different units and magnitudes of the objectives, we translated the original objectives into their relative improvement based on their ideal and anti-ideal values (Martinson 1993). To do so, the solution for each objective was obtained independently, and from this set of solutions, the ideal and anti-ideal values of each objective were obtained. These values were then used in the multiobjective formulation to determine the relative improvement of these indicators over their minimum possible levels (see Equation 12 below).

The following notation is used to present both the deterministic and robust models. Lowercase represents decision variables, and uppercase represents parameters and coefficients of the models.

*Decision Variables*

$x_{it}$ = volume of logs of type $i$ acquired in period $t$ ($m^3$)
$y_{ijt}$ = volume of logs of type $i$ sawn with cutting pattern $j$ in period $t$ ($m^3$)
$Z_{kt}$ = volume of product $k$ produced in period $t$ ($m^3$)
$n_{kt}$ = volume of product $k$ sold in period $t$ ($m^3$)
$u_{it}$ = volume of logs of type $i$ kept as inventory in period $t$ ($m^3$)
$v_{kt}$ = volume of product $k$ kept as inventory in period $t$ ($m^3$)
$o_m$ = amount produced of objective $m$ (m = cost, waste, overproduction)
$ro_m$ = relative improvement of objective $m$ over its worst possible value

*Parameters*

$L_t$ = processing capacity (h)
$T_{ij}$ = time required for a log of type $i$ to be sawn with pattern $j$ ($h/m^3$)
$S_{it}$ = availability of logs of type $i$ in period $t$ ($m^3$)
$R_{ijk}$ = volume of product $k$ obtained if log $i$ is sawn with pattern $j$ ($m^3$ product/$m^3$ log)
$M_{ij}$ = volume of waste produced if a log $i$ is sawn with pattern $j$ ($m^3$ waste/$m^3$ log)
$D_{kt}$ = minimum demand of product $k$ in period $t$ ($m^3$)
$P_k$ = price of product $k$ ($\$/m^3$)
$CA_t$ = acquisition cost of a log of type $i$ ($\$/m^3$)
$CS_{ij}$ = cutting cost of a log of type $i$ if sawn with cutting pattern $j$ ($\$/m^3$)
$CL_i$ = inventory cost of a log of type $i$ ($\$/m^3$)
$CP_k$ = inventory cost of product $k$ ($\$/m^3$)
$UO_t$ = initial inventory of logs of type $i$ ($m^3$)
$VO_k$ = initial inventory product $k$ ($m^3$)
$bo_m$ = best possible value of objective $m$
$wo_m$ = worst possible value of objective $m$
$W_m$ = weight of objective $m$

*Objective Function*

We maximize the weighted relative improvement over the worst result

$$\text{Max} \sum_{m=1}^{M} W_m ro_m \tag{1}$$

*Constraints*

Log availability

$$x_{it} \leq S_{it} \quad \forall i,\ t \tag{2}$$

Processing capacity

$$\sum_{i=1,j=1}^{I,J} T_{ij} y_{ijt} \leq L_t \quad \forall t \tag{3}$$

Log balance

$$x_{it} + UO_i - u_{it} = \sum_{j=1}^{J} y_{ijt} \quad \forall i,\ t = 1 \tag{4}$$

$$x_{it} + u_{it-1} - u_{it} = \sum_{j=1}^{J} y_{ijt} \quad \forall i,\ t > 1 \tag{5}$$

Product balance

$$\sum_{i=1,j=1}^{I,J} R_{ijk} y_{ijt} + VO_k - v_{kt} = n_{kt} \quad \forall k,\ t = 1 \tag{6}$$

$$\sum_{i=1,j=1}^{I,J} R_{ijk} y_{ijt} + v_{kt-1} - v_{kt} = n_{kt} \quad \forall k,\ t > 1 \tag{7}$$

Demand

$$n_{kt} \geq D_{kt} \quad \forall k,\ t \tag{8}$$

*Objective Calculations*

$$o_1 = \sum_{i=1,t=1}^{I,T} CA_i x_{it} + \sum_{i=1,j=1,t=1}^{I,J,T} CS_{ij} y_{ijt} + \sum_{i=1,t=1}^{I,T} CL_i u_{it} + \sum_{k=1,t=1}^{K,T} CP_k v_{kt} \tag{9}$$

$$o_2 = \sum_{i=1,j=1,t=1}^{I,J,T} M_{ij} y_{ijt} \tag{10}$$

$$o_3 = \sum_{k=1,t=1}^{K,T} (n_{kt} - D_{kt}) + \sum_{k=1}^{K} v_{kT} \tag{11}$$

$$ro_m = \frac{(wo_m - o_m)}{(wo_m - bo_m)} \quad \forall m \tag{12}$$

Equation 1 represents the weighted sum of the relative improvement over the worst possible values of the three indicators, and Equations 2 and 3 limit the use of resources (log availability and processing capacity) to their maximum availability. Equations 4–7 are balance constraints that relate acquired, processed, and inventoried logs with the production, inventory, and sale of products. Equation 8 ensures that the minimum demand is met, and Equations 9–11 compute the levels of the three indicators. Finally, Equation 12 calculates the relative improvement of the indicators based on their best and worst possible values. These are the three indicators used as objectives of our model.

We assumed, without loss of generality, that the weights for the three objectives were equal; that is, $W_m = 1$ for all objectives $m$. We note that our weight definition is equivalent to the most commonly used definition in which the sum of weights equals 1. We opted to set each weight to 1 to make it easier for the decisionmaker to compare the relative importance of the objectives when uncertainty in their weights was introduced.

### Robust Multiobjective Model

Several approaches of robust optimization to find decisions that are less sensitive to uncertain data have been proposed. Although the first ones resulted in models that solved the worst-case scenario (Soyster 1973), other less conservative approaches have been suggested since then (El Ghaoui and Lebret 1997, Ben-Tal and Nemirovski 1998). These approaches transform a deterministic model into its robust counterpart in a way that the latter seeks the best objective value that simultaneously allows changes in the model parameters within their range of possible values. The main disadvantage of these transformations is that the complexity of the original models increases; that is, linear models become nonlinear, and the problems become more difficult to solve. However, Bertsimas and Sim (2004) proposed an approach that does not increase this complexity. The approach has been extensively used in different areas, and in the forest context some applications can be found at the strategic (Palma and Nelson 2009, 2010), tactical (Palma and Nelson 2014), and operational (Alvarez and Vera 2014) levels.

In this approach, uncertainty is assumed to distribute uniformly within a range of values, and robustness is modeled by ensuring constraint feasibility through the inclusion of a buffer term, $\beta(\mathbf{ro}^*, \Gamma)$, that depends on a given vector of decision $\mathbf{ro}^*$ (relative improvement of the objectives) and the level of protection required, $\Gamma$. This level of protection, $\Gamma$, can take on values from 0 (deterministic case) to the number of uncertain coefficients and allows decisionmakers to handle the level of robustness of the solutions. If applied to the objective function, this concept of robustness translates into searching for solutions that guarantee that a good value of the objective function will be achieved even if the objective coefficients vary.

The objective function of our model is therefore reformulated as follows

$$\text{Min } c \tag{13}$$

subject to

$$\sum_{m=1}^{M} W_m ro_m + \beta(\mathbf{ro}^*, \Gamma) \leq c \tag{14}$$

**Table 1. Payoff table of the deterministic single objective and multiobjective solutions.**

| | Performance indicator | | |
|---|---|---|---|
| Objective | Cost ($) | Waste ($m^3$) | Overproduction ($m^3$) |
| Cost | 412,003 | 9,053 | 229 |
| Waste | 500,623 | 8,805 | 242 |
| Overproduction | 504,848 | 9,127 | 206 |
| Multiobjective | 478,128 | 9,044 | 230 |

The multiobjective approach produced a compromise solution that balanced the three objectives.

The larger the buffer term is, the more the impact it has on the quality of the objective function, so an optimal value of $\beta$ has to be sought. Bertsimas and Sim (2004) estimate the buffer size through a linear programming (LP) model and then replace the buffer term with the dual of this LP model. The robust version of the model given by Equations 1–12 becomes

$$\text{Min } c \tag{15}$$

subject to

$$\sum_{m=1}^{3} \bar{W}_m ro_m + \Gamma g + \sum_{m=1}^{3} h_m \leq c \tag{16}$$

$$g + h_m \geq \hat{W}_m ro_m \quad \forall m \tag{17}$$

$$g \geq 0 \tag{18}$$

$$h_m \geq 0 \quad \forall m \tag{19}$$

plus Equations 2–12 where $\bar{W}_m$ and $\hat{W}_m$ are the weight estimate and its error size (i.e., the real unknown weight $W_m \in [\bar{W}_m \pm \hat{W}_m]$, uniformly distributed, for all objectives $m$) and Equations 16–19 and the new variables $g$ and $h_m$ come from the dual transformation of the LP model used to determine the buffer size (see Appendix for details). These new variables have no practical meaning other than representing dual values that allow us to quantify the buffer size based on the level of the objectives and their weight estimate error (Equation 17).

We defined the weight errors ($\hat{W}_m$) as 0.1, 0.2, and 0.3 for the three objectives (cost, waste, and overproduction), respectively, to evaluate the effect of different levels of errors on the decisions. This definition represents, for example, that due to the unclear objective preferences the second objective can be up to either 33% more important or 27% less important than the first objective. We note that the specific values we used for the weights and their errors do not affect the proposed methodology and the main conclusions of this work. In addition to the error size, two levels of robustness were tested, $\Gamma = 1$ (labeled as robust 1), and $\Gamma = 2$ (labeled as robust 2).

Both deterministic and robust models were implemented in IBM ILOG CPLEX Optimization Studio version 12.5.

## Results

### Deterministic Single Objective and Multiobjective Approaches

The use of a single indicator as objective function produced unbalanced solutions in terms of the dissimilar quality of the other indicators (Table 1). For example, minimizing the overproduction led to the highest cost (US$504,848) and minimizing the waste led

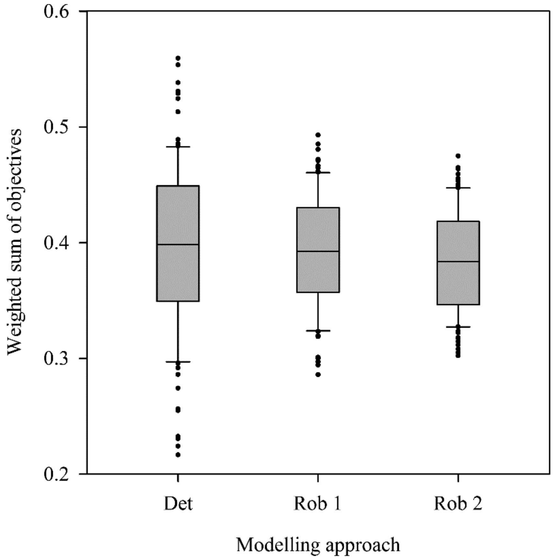


**Figure 1. A decrease in the variability of the objective function was observed with robust models (Rob 1 and Rob 2) compared with the deterministic model (Det). The lowest boundary of the box indicates the 25th percentile, the line within the box marks the median, and the highest boundary of the box indicates the 75th percentile. Error bars above and below the box indicate the 90th and 10th percentiles.**

**Table 2. Comparison of deterministic and robust multiobjective solutions to the sawmill problem with three objectives and two levels of robustness.**

| | Model | | |
|---|---|---|---|
| | Deterministic | Robust 1 | Robust 2 |
| Objective | | | |
| Cost ($) | 478,128 | 470,537 | 467,874 |
| Waste ($m^3$) | 9,044 | 9,050 | 9,052 |
| Overproduction ($m^3$) | 230 | 234 | 236 |
| Log size[a] | | | |
| Small ($m^3$) | 0 | 418 | 551 |
| Medium ($m^3$) | 1,688 | 1,715 | 1,780 |
| Large ($m^3$) | 2,978 | 2,598 | 2,452 |
| Weighted sum | 0.3954 | 0.3878 | 0.3843 |

[a] The 15 log types were grouped in three categories: small (logs 1–5), medium (logs 6–10), and large (logs 11–15).

to the highest overproduction (242 $m^3$). As expected, the multiobjective approach produced a compromise solution for the three objectives. Cost, waste, and overproduction improved their values by 29, 26, and 33%, respectively, in relation to their worst possible levels when the multiobjective approach was used.

### Deterministic and Robust Multiobjective Approaches

A set of 100 objective weights were simulated within their range of possible values ($[\bar{W}_m \pm \hat{W}_m]$ with $\bar{W}_m = 1$, m = 1, 2, 3) to represent different scenarios of real preferences. These weights were used to evaluate both the deterministic and the robust decisions. If the real objective preferences are not as estimated and can fluctuate within a range of values, the compounded objective value of the deterministic multiobjective model showed an important variability (SD = 0.073). This variability was reduced to 0.049 and 0.044 with the two levels of robustness, respectively (Figure 1).

Deterministic and robust decisions were different (Table 2). The quality of the compounded objective function (weighted sum) slightly decreased as more robustness was required, and both the use of logs and the levels obtained for the different objectives differed. More robust decisions improved the cost indicator (the cost was reduced) but increased the amount of waste and the excess of production. Unlike the deterministic solution that used the full availability of large logs, robust solutions preferred small logs as more robustness was required.

## Discussion

As expected, the multiobjective formulation of the sawmill problem produced a more similar level for the three performance indicators than the single objective formulation. Our results suggest that the value of the objective function of the decisions obtained using a point estimate of the objective preferences may be more variable, depending on the observed preferences. This variability can be reduced with a robust formulation of the problem.

Considering robustness when the objective preferences were uncertain affected both the cutting decisions and the level of achievement of the performance indicators or objectives. When the size of the preference uncertainty was different among the objectives, the values of those objectives for which the preference was more accurately defined increased in relation to what is observed with deterministic models. The more uncertain the objective preferences were, the lower the amount of the objective produced. More specifically, it can be observed from Equation 17 that $r_{om} \leq (g + h_m)/\hat{W}_m$ for all objectives $m$, i.e., the more uncertain the objective weight ($\hat{W}_m$), the smaller the amount of the objective produced. Thus, the robust approach explicitly considers both the contribution of the decisions to a higher objective function value and their impact on the variability of the solution quality. This situation is explained by the higher potential impact on the objective function variability that a large objective amount could produce if the possible weights change widely. The practical consequence of this fact is that although an objective is clearly more important than other objectives, if its weight error is large, then a low level of that objective will be obtained. To avoid this situation, when using this approach, decision-makers should make a special effort to accurately define the weights of those objectives that are more important.

As occurred with the objectives, weight error had a similar effect on the decisions that the robust formulation chose. The deterministic model preferred decisions with the best average levels of the three objectives, regardless of their variability, this is, decisions that are very good for some objectives and very bad for others. The robust models combined decisions with the best average levels of objectives with decisions that, although reported worse levels of objectives, produced a more uniform level of them. For instance, large diameter logs are more profitable for a sawmill than small diameter logs and are usually preferred in deterministic settings. However, in our case they affected the variability of the objective function in the presence of unclear objective preferences. Because large diameter logs allow more cutting possibilities, a wider variability in the total cost, waste, and overproduction is observed than in small diameter logs. Because the robust model tries to reduce this variability, small diameter logs are also part of the robust optimal decision. We note that the use of small diameter logs may sound counterintuitive as large logs allow more flexibility in terms of the products that can be obtained. However, we are not considering uncertainty in the demand, in which case this flexibility would be very useful, but uncertainty in the objective preferences in a multiobjective framework. Higher values of highly variable decisions, such as the use of large diameter logs, will translate into highly dissimilar levels of objectives.

The robust approach used is based on increasing the chance of feasibility of constraints with uncertain coefficients. A buffer for each constraint is explicitly modeled to maintain the left-hand side of a constraint far enough from the right-hand side so that the inequality holds in practice even if the coefficients are not as estimated. Whether the uncertain coefficients are in the objective function, as in the weighted sum of objectives we used, the approach can still be used by modeling the objective function as a constraint. However, in this case the philosophy of the robust optimization approach is less clear. With uncertainty in the objective coefficients, the robust model is intended to seek solutions that guarantee that a good value of the objective function will be obtained for a range of values of the uncertain coefficients. The practical implication of this fact is that, regardless of the objective weights actually observed, robust solutions will preclude the occurrence of decisions with a very low objective function. A good value of the objective function is therefore guaranteed.

This guarantee can be very important in a multiobjective decision context in which different actors are involved, such as in environmental and social problems. Because the objective preferences in this type of problem can be difficult or even impossible to determine, the approach can help decisionmakers find less risky decisions and therefore decrease the negative outcomes if the real preferences differ from their estimates. Most of the current approaches that consider uncertainty in multiobjective problems compare a reduced number of solution alternatives by assigning to each of them an index that reflects the stability of a solution (Lahdelma et al. 1998, Kangas 2006). The robust formulation presented in our study does not compare solutions already generated but generates solutions that were demonstrated to be less sensitive to uncertain weights than those generated by the traditional deterministic multiobjective models.

An approach that minimizes the variability of the solutions along with other objectives could eventually be useful to our purpose. Such an approach, as suggested by Mulvey et al. (1995), would not need a description of the uncertainties but would require a description of the decisions' covariance. These models are common in finance, where the information of (co)variance is broadly available. Another approach is one that combines optimization and scenario analysis (Pukkala and Miina 1997), in which the variability of a solution is evaluated for a set of scenarios of weights and then the model is successively reoptimized to find better solutions. The robust approach proposed here has the main advantage of producing computationally tractable models, and the use of a simple description of the uncertainties (uniform distribution), which is particularly useful when uncertainties are not statistically described as happens in many cases. However, if uncertainties are well described through probability distributions, then the simplicity of the uncertainty description becomes a drawback of the approach, as useful information will be left out of the decision process. In addition, the level of protection, $\Gamma$, used in the robust model has no clear meaning other than to control the tradeoff between the quality of the objective function and the robustness of the solutions. The approach therefore requires the evaluation of different levels of protection to obtain a range of solutions from which the decisionmaker can make a choice. Further research on a more accurate way to relate the level of protection and the solution quality and on the convenience of using this approach to other management decision problems is of interest.

In conclusion, robust optimization seems to be a good tool to handle uncertainty in the objective weights of multiobjective models. Although a larger number of variables and constraints is required, the model complexity remains the same. If weights are not accurately defined, decisions that produce more homogeneous objective outcomes are to be preferred, and the production of objectives with a wider range of possible weights tends to decrease compared with deterministic solutions.

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## Appendix

For the buffer $\beta(\mathbf{ro}^*, \Gamma)$ used in Equation 14, Bertsimas and Sim (2004) propose the following LP model for a fixed value of the decisions $\mathbf{ro}^*$ and nonnegative values of $e_m$, BLP: {max $\Sigma_{m=1}^{3} \hat{W}_m ro_m^* e_m$; subject to $\Sigma_{m=1}^{3} e_m \leq \Gamma$and $e_m \leq 1\ \forall m$}. BLP is a subproblem that finds the maximum buffer required if a given solution is made and up to $\Gamma$ of the uncertain coefficients ($W_m$) are allowed to change simultaneously. The decision variable $e_m$ identifies those objectives that contribute the most to the size of the buffer. Note that BLP assumes a fixed value of $ro_m^*$; otherwise the problem would be nonlinear.

Since objective levels are fixed and considering that we are only interested in the objective function of model BLP, its dual can be used to estimate the buffer size of Equation 14 as a linear model. With $g$ and $h_m$ as the dual variables associated with the equations of BLP, its dual is DBLP: {min $\Gamma g + \Sigma_{m=1}^{3} h_m$; subject to $g + h_m \geq \hat{W}_m ro_m\ \forall m$; $g \geq 0$; $h_m \geq 0\ \forall m$}. This dual is then embedded in Equation 14 to replace the buffer term and becomes Equations 16–19.