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Multiperiod Optimisation of Irrigated Crops under Different Conditions of Water Availability

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Abstract: We propose a nonlinear optimisation model which maximises profits by resource allocation on a monthly time scale, considering a monthly crop yield model. The proposed model was applied to six management scenarios (two seasonal and four monthly), nine conditions of water availability, and two situations of resource availability under Chilean conditions. These situations provided the same seasonal amount of resources, but different distributions over time. The model included improvements in water resource management such as water storage and water transactions, being the latter a monthly decision variable that can increase farmers' profits. According to our results, monthly scenarios gave high profits, even better with appropriate resource distribution. When water costs are high, water transactions allow loss reduction of up to 50%. Regarding labour, the lack of availability is more critical than the wages.

Keywords: crop yield; water allocation; multiperiod optimisation; water transactions; water storage

1. Introduction

Current world population is around 7.5 billion, but by 2025 there will be an expected nine billion people [1]. To feed this population, the worldwide irrigated area should increase in 30 M ha, with 40% higher water and energy demand for the next 20 years [2]. According to McMichael et al. [3], agriculture has depleted natural resources, contributing to water scarcity. To face this problem, strategies have been proposed such as increasing water use efficiency [4], desalination [5,6], water transactions [7,8], use of infrastructure for water storage [9,10], and optimisation of resources [11–14].

Optimisation assigns resources among competitive activities, subject to some restrictions and solved through mathematical algorithms [15]. In agriculture, optimisation has been employed for resource management, especially regarding water allocation and cropping patterns. Models have been developed in the literature to maximise profits by optimal cropping patterns. For example, Sethi et al. [16] considered groundwater management through linear programming to maximise profits. Mishra et al. [17] developed a multiobjective optimisation model to determine optimal cropping pattern and optimal size of an auxiliary storage reservoir. Fasakhodi et al. [18] used a multiobjective fractional goal programming method to find the optimal cropping pattern and sustain water availability. Ponce et al. [19] developed a nonlinear water supply model for analysing the economic impacts of changes in crop yields due to climate change. Su et al. [20] improved agricultural water use efficiency and the proportion of green water utilization by multiobjective optimisation. Das et al. [21] developed a menu-driven user friendly software based on a linear programming model for optimal land and water allocation. Tan et al. [22] developed a multiobjective fuzzy robust programming model for allocation of water and land resources and Varade and Patel [23] determined an optimal cropping pattern to maximise the net annual returns in order to conserve natural resources. On the other hand, there are researches that have included crop production functions that relate yield reduction as a result of the relative loss in evapotranspiration [24]. In these studies, the equation proposed by Doorenbos and Kassam [25–28] and polynomial regressions were considered [11,29,30]. Moreover, there are investigations which include multiperiod crop yield functions related to crop yield reduction as a result of the water stress at an intraseasonal timescale [31]. These studies considered the multiplicative approach of the equation proposed by Doorenbos and Kassam [25,32–34] and Jensen [13,35–38].

Considering the importance of sub-seasonal management in food production, we formed the following research question: What is the method to cope with seasonal changes, and how does variability affect profits for a given number of resources? None of the mentioned studies included a monthly optimisation model subject to monthly constraints, such as labour and capital (separately) and the possibility of including improvements in water resource management such as water transactions and water storage. Therefore, the principal objective was to develop and test nonlinear optimisation models at seasonal and monthly time scales and then compare them under Chilean conditions. This started with developing a monthly crop yield equation based on the multiperiod model proposed by Raes et al. [31] which depends mainly on crop features and sowing date. A monthly optimisation model allows adequate resource allocation due to monthly demand for labour and capital, often reported by technical-economic reports. Improvements in water resource management such as water storage and water transactions, are also monthly decision variables that can increase profits.

2. Methodology

Our proposed nonlinear optimisation model consists of an objective function which includes a monthly crop yield function and constraints on a monthly basis for resources. The objective is to maximise profits by the allocation of water and land to be cultivated on a monthly time scale.

2.1. Multiperiod Crop Yield Function

For planning with limited data availability, a simple equation was proposed by Doorenbos and Kassam [25], which describes crop yield reduction due to water scarcity:

$$\left(1 - \frac{Y_i}{Ym_i}\right) = Ky_i \left(1 - \frac{ETa_i}{ETc_i}\right) \tag{1}$$

where i represents crop type. Y_i and Ym_i (in yield unit ha^{-1}) are the actual and maximum crop yields, respectively. Ky_i is the yield response factor, which has been documented by Doorenbos and Kassam [25] for many crops at different stages and corresponds to the slope of the yield reduction due to a decrease of applied water (Figure 1a). ETa_i and ETc_i (both in mm) are the actual and crop evapotranspiration for the whole growing period, respectively. Later, Raes et al. [31] proposed a multiperiod crop yield equation at constant time scales smaller than growth periods.

$$\frac{Y_i}{Ym_i} = \prod_{k=1}^{t} \left[1 - Ky_{i,s} \left(1 - \frac{ETa_{i,k}}{ETc_{i,k}}\right)\right]^{\Delta t_{i,k}/L_{i,s}} \tag{2}$$

where t is the number of periods, $Ky_{i,s}$ is the yield response factor at growth stage s, $ETa_{i,k}$ and $ETc_{i,k}$ are the actual and crop evapotranspiration at time k; $\Delta t_{i,k}$ is the length of time (in days) of each step during the growth stage s (1 if daily) and $L_{i,s}$ is the growth stage length (in days). Figure 1b represents the daily yield response to water [31] for maize, where relative yield stays constant when water demand is satisfied, and decreases when it is not. We propose modification to this equation for this research for monthly time steps, considering a parameter $\Delta t^*_{i,k}$ as an array which maintains the number of the days in each month while the crop is growing (instead of considering it as scalar); and $L^*_{i,k}$ as a fit parameter when it compared to the relative yield at daily time step:

$$L^*_{i,k} = \frac{\Delta t^*_{i,k}}{ln\left(\frac{Y_{i,d}}{Y_{i,k-1}}\right)} ln\left[1 - Ky_{i,k}\left(1 - \frac{ETa_{i,k}}{ETc_{i,k}}\right)\right] \quad (3)$$

where $Y_{i,d}$ is the actual crop yield at day d which is equivalent to the difference of the last day of the month k, and the sowing day for the first month and the last day of each month for the following months while the crop is growing. For example, if the sowing and harvest day of crop i are 15 November and 3 March, respectively, d corresponds to 15 for November, 46 for December (15 + 31), 77 for January, 105 for February, and 108 for March (105 + 3). Finally, the monthly crop yield equation has the following form:

$$\frac{Y_i}{Ym_i} = \prod_{k=1}^{t}\left[1 - Ky_{i,k}\left(1 - \frac{ETa_{i,k}}{ETc_{i,k}}\right)\right]^{\Delta t^*_{i,k}/L^*_{i,k}} \quad (4)$$

A comparison between the daily and monthly approach for maize is illustrated in Figure 1c. Here, yield reduction is due to seasonal water shortage as a function of time, showing that there are not significant differences between the approaches.

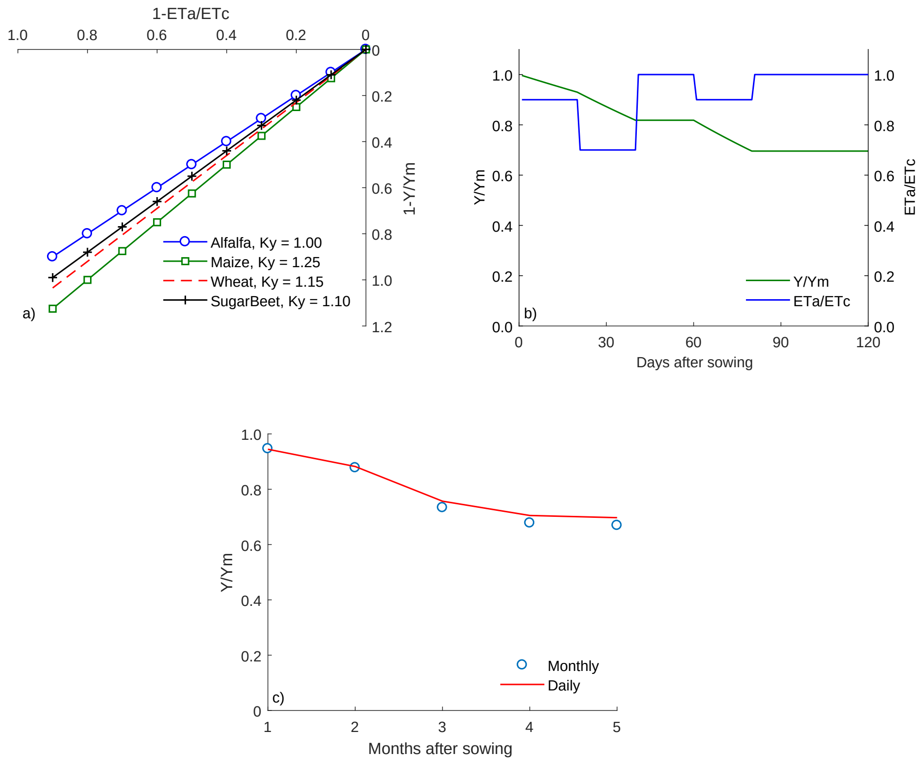

Figure 1. Representation of crop yield reduction, where (**a**) is the seasonal crop yield reduction function proposed by Doorenbos and Kassam [25] where *Ky* is the slope; (**b**) is the daily crop yield reduction for maize proposed by Raes et al. [31] where relative yield stays constant when the water demand is satisfied and decreases when it is not; and (**c**) represents daily and monthly approaches for maize.

2.2. Optimisation of Irrigated Crops

The objective function to maximise has the following form:

$$Max\ U = \sum_{i=1}^{n} P_i A_i Y_i - \sum_{i=1}^{n} \sum_{k=1}^{t} A_i C_{i,k} \tag{5}$$

where P_i is the price per crop i (in US\$ yield unit^{-1}), A_i is the area to be cultivated with crop i (in ha), and $C_{i,k}$ represents the production costs per unit area (in US\$ ha^{-1}). Some components of $C_{i,k}$ are [29]: labour and other costs such as seed, fertiliser and pesticides.

Thus, the complete form of the objective function is:

$$\begin{aligned} Max\ U = \sum_{i=1}^{n} P_i A_i Ym_i \prod_{k=1}^{t} \left[1 - Ky_{i,k}\left(1 - \frac{ETa_{i,k}}{ETc_{i,k}}\right)\right]^{\Delta t^*_{i,k}/L^*_{i,k}} - LC \sum_{i=1}^{n} \sum_{k=1}^{t} A_i NL_{i,k} - \\ \sum_{i=1}^{n} \sum_{k=1}^{t} A_i OC_{i,k} - Wcr \sum_{k=1}^{t} Wr_k - Wcb \sum_{k=1}^{t} Vwb_k + Wcs \sum_{k=1}^{t} Vws_k \end{aligned} \tag{6}$$

where LC is the labour cost (in US\$ person-day^{-1}), $NL_{i,k}$ is the labour needed per unit area (in person-day ha^{-1} month^{-1}), $OC_{i,k}$ corresponds to other costs, and Wr_k is the amount of water rights in month k (in m^3 month^{-1}) with its respective cost (Wcr) (in US\$ m^{-3}). If farmers have the possibility to buy water (to increase the area to be irrigated) and to sell water (when not using it), they can obtain higher profits. Therefore, the monthly amounts of water to buy (Vwb_k) and to sell (Vws_k) are also included, with the corresponding costs to buy (Wcb) and to sell (Wcs) water (in US\$ m^{-3}).

Constraints of resources on a monthly basis are as follows:

1. Water availability: Assuming that the farmer has the infrastructure to store water at monthly scale (Rc m^3 of capacity), available water is defined as:

$$0 \le \sum_{k'=1}^{k} \left[(Wr_{k'} + Vwb_{k'}) - \left(10 \sum_{i=1}^{n} A_i \frac{ETa_{i,k'}}{AE_i} + Vws_{k'} \right) \right] \le Rc, \quad \forall k \tag{7}$$

   where AE_i is the application efficiency of the irrigation system for crop i. It is important to mention that $ETa_{i,k}$ refers to the water contained in the soil after applied an initial volume of water $ETa_{i,k}/AE_i$, where there are losses due to irrigation system efficiency. This variable is multiplied by 10 for conversion to m^3 per hectare.
2. Land availability: This constraint defines the area to be cultivated.

$$\sum_{i=1}^{n} A_i \le At \tag{8}$$

   where At is the land availability (in ha).
3. Labour availability: Assuming that the labour availability can change for each month, this constraint is defined as:

$$\sum_{i=1}^{n} A_i NL_{i,k} \le La_k, \quad \forall k \tag{9}$$

   where La_k is the labour availability at month k (in person-d month^{-1}).
4. Capital availability: Assuming that farmers can save money if it is not spent, the monthly capital availability is considered as:

$$\sum_{k'=1}^{k}\left[(Wcr \cdot Wr_{k'} + Wcb \cdot Vwb_{k'}) + LC\sum_{i=1}^{n} A_i NL_{i,k'} + \sum_{i=1}^{n} A_i OC_{i,k'}\right] \leq \sum_{k'=1}^{k} Ca_{k'}, \quad \forall k \tag{10}$$

where $Ca_{k'}$ is the economic capital availability at month k' (in US\$ month^{-1}).

5. Crop area considerations: It is necessary to consider agricultural, market and productive diversity management criteria to restrict the maximum or minimum crop areas. This is due to marketing situations, rotations, or other agricultural limitations. These constraints are expressed as:

$$min\ S_i \leq A_i \leq max\ S_i, \quad \forall i \tag{11}$$

where $min\ S_i$ and $max\ S_i$ are the minimum and maximum areas assigned to farm with crop i, respectively.

6. Complementary considerations: To force the crop water requirement to be zero when the cultivated area is also zero, the constraint is expressed as:

$$Kz \cdot A_i - \sum_{k=1}^{t} ETa_{i,k} \geq 0, \quad \forall i \tag{12}$$

where Kz is a positive constant (Kz = 10,000 mm ha^{-1}). In order to not apply more water than required by the crop, the following constraint is also considered:

$$ETa_{i,k} \leq ETc_{i,k}, \quad \forall i,k \tag{13}$$

Finally, there are non-negativity constraints expressed as:

$$A_i, ETa_{i,k}, Vwb_k, Vws_k \geq 0 \tag{14}$$

2.3. Case Study

Our proposed model was applied to conditions characteristic of the Central Valley of Chile (Figure 2). Annual mean precipitation for this area is about 1025 mm, and the average high and low temperatures are 20.6 and 7.6 °C, respectively [39]. This sector contains about 28% of the national crop production surface. Some of the most produced crops are wheat (34.3%), maize (11.6%), and sugar beets (6%), which contribute to the national planted surface with 27.9, 22.5, and 60%, respectively. On the other hand, fodder crops represent 6.1% of the regional cultivated area (11% national) [40], alfalfa being the most common, mainly destined to intensive dairy farming.

2.3.1. Model Inputs

In this research, alfalfa, maize, wheat, and sugar beets were the crops considered for the case study. Crop yield parameters (Ky and L) were extracted from the database of CROPWAT 8.0 [41,42]. Then, values were fitted to the study area according to crop sowing date, as recommended by Faiguenbaum [43]. This gave the monthly parameters of crop yield equations presented in Table 1. Price, costs, and maximum yield for each crop considered in Equation (6) (Table 2) were extracted from technical-economic reports from the Office of Agricultural Studies and Policies (ODEPA) and the National Institute of Agricultural Research (INIA). Prices were adjusted to May 2017 using the Consumer Price Index (CPI) calculator which is available on the National Statistics Institute (INE) website. A value of 575 Chilean Pesos (CLP) per US dollar $ was considered for this study as the long-term mean. Monthly water demand for each crop was determined using the ASCE Standardised Reference Evapotranspiration Equation [44]. Monthly water availability was estimated using streamflow data from distribution channels. Then, a decile analysis was carried out to assess different conditions of water availability. Deciles 1 and 9 represented the driest and wettest conditions, respectively.

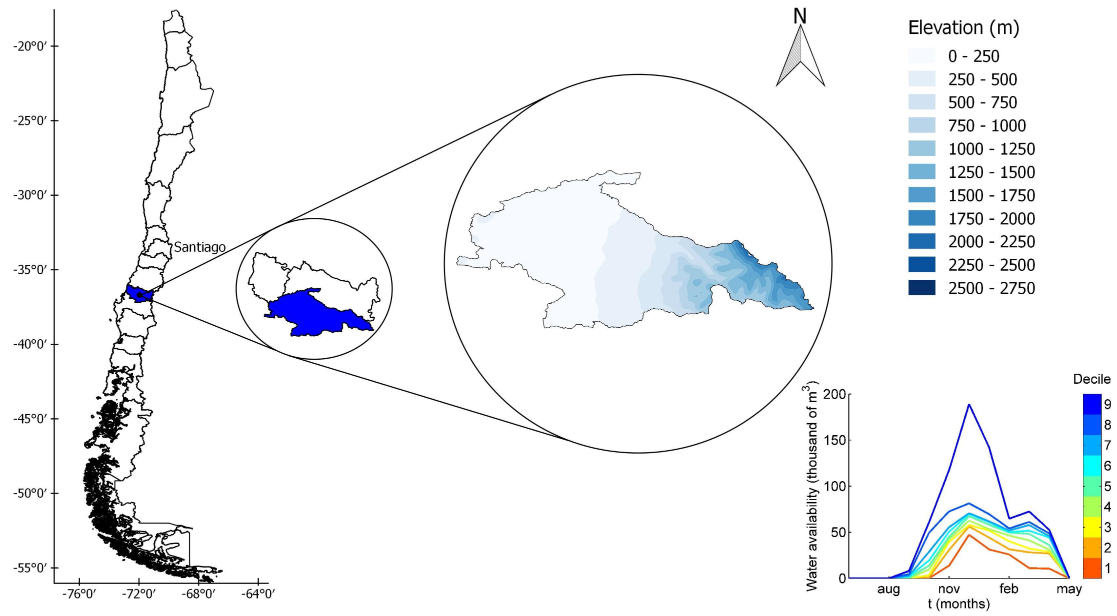

Figure 2. Study area location. The right-bottom panel shows available water based on streamflow records of a distribution channel.

Table 1. Parameters used for monthly crop yield functions based on the CROPWAT 8.0 database and the sowing date recommended by Faiguenbaum [43].

Crop	Parameter	Month							Sowing
		Sep	Oct	Nov	Dec	Jan	Feb	Mar	
Alfalfa	*Ky*	1.0	1.0	1.0	1.0	1.0	1.0	1.0	01-Sep
	Δt^*	30.0	31.0	30.0	31.0	31.0	28.0	15.0	
	L^*	49.0	49.0	49.0	49.0	49.0	49.0	49.0	
Maize	*Ky*	-	-	0.5	0.7	1.2	0.5	0.1	01-Nov
	Δt^*	-	-	30.0	31.0	31.0	28.0	5.0	
	L^*	-	-	28.4	35.1	35.6	34.3	33.4	
Wheat	*Ky*	0.4	0.6	0.7	0.5	0.1	-	-	01-Sep
	Δt^*	30.0	31.0	30.0	31.0	8.0	-	-	
	L^*	30.0	30.9	32.5	32.6	8.5	-	-	
Sugar beet	*Ky*	0.6	0.8	1.0	1.0	0.9	0.6	-	01-Sep
	Δt^*	30.0	31.0	30.0	31.0	31.0	7.0	-	
	L^*	29.5	31.5	35.2	37.3	38.8	38.4	-	

Table 2. Price and maximum yield for each crop.

Crop	Price		Maximum Yield		Source
	Value	Units	Value	Units	
Alfalfa	5.1	US$ bale^{-1}	400	bales ha^{-1}	INIA [45]
Maize	22.3	US$ qqm^{-1}	150	qqm ha^{-1}	ODEPA [46]
Wheat	22.5	US$ qqm^{-1}	70	qqm ha^{-1}	ODEPA [46]
Sugar beet	62.7	US$ ton^{-1}	100	ton ha^{-1}	ODEPA [47]

2.3.2. Model Application

The model was applied to two situations that provided the same seasonal amount of resources (for labour and capital) but different distributions in time (Figure 3). The seasonal availability of resources (labour and capital) was the same for both situations, but its distribution differed. Moreover, the model was applied to six scenarios, and their features can be summarised as follows:

- Scenario 1: Optimisation subject to seasonal constraints. This scenario assumes that resources are available for the season, but does not consider intraseasonal variability. In this scenario, for the whole growing period, only one value of Ky and ETa for each crop i was considered. Water storage and water transactions were not considered.
- Scenario 2: Optimisation subject to seasonal constraints. For the whole growing period, monthly values of Ky and ETa for each crop i were considered. In this scenario, water storage and water transactions were not considered.
- Scenario 3: Optimisation subject to monthly constraints, i.e., water and other resources availability at a monthly scale are considered. In this scenario, water storage and water transactions were not considered.
- Scenario 4: Optimisation subject to monthly constraints with water transactions.
- Scenario 5: Optimisation subject to monthly constraints with water storage.
- Scenario 6: Optimisation subject to monthly constraints with water storage and transactions. This scenario is the most complete, considering all possible factors involved in the process.

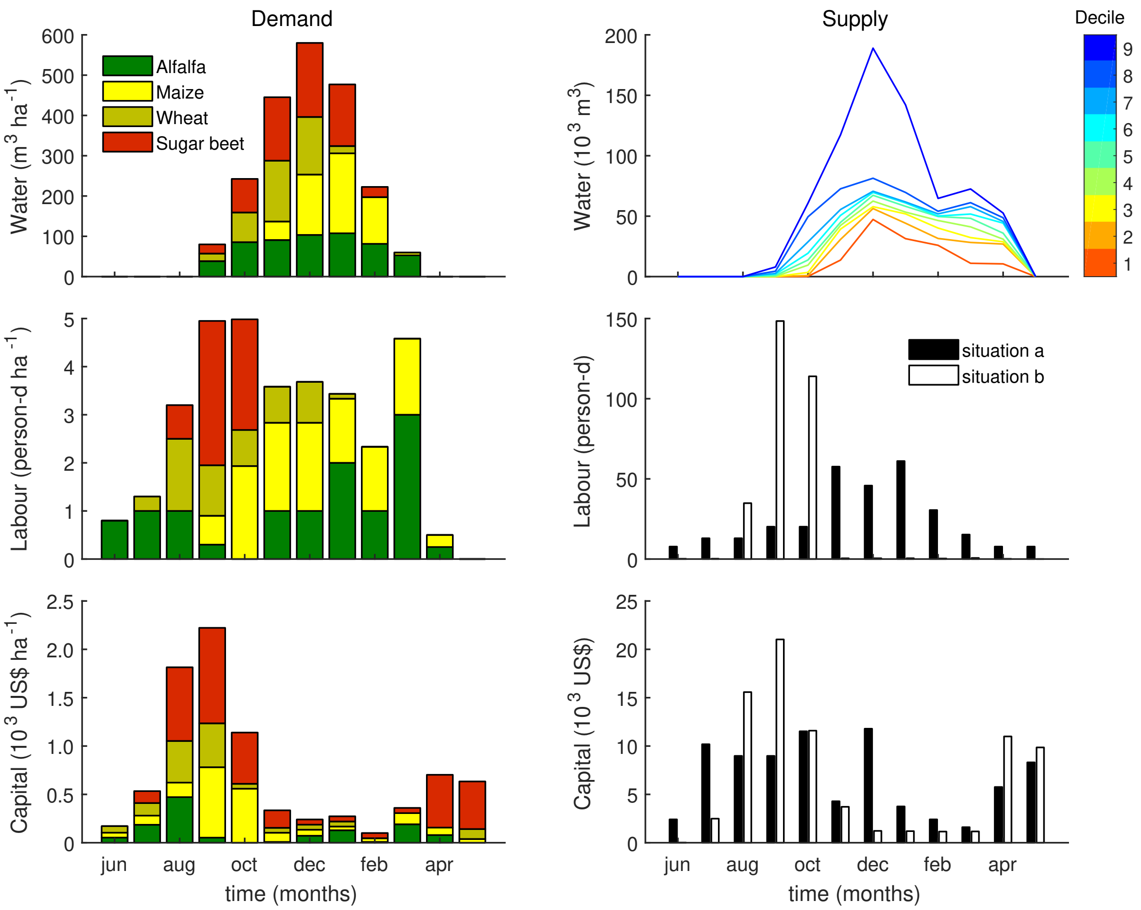

Figure 3. Monthly demand and supply of resources using collated data. Deciles 1 and 9 are the driest and the wettest conditions, respectively. Distribution of labour and capital are provided by the situation a (black bar) and b (white bar). Seasonal amount of labour and capital availability is 300 person-day and 80,000 US$, respectively. See ODEPA [46], ODEPA [47], and INIA [45] for more detailed sources of information.

A summary of the equations considered in the optimisation algorithm is presented in Table 3 for each scenario. Figure 4 summarises the aforementioned methodology, where parameters such as price received for crop and its maximum yield, monthly demand and supply of water, labour, and economic capital are available in the database. The six scenarios described above were assessed by the model proposed by Doorenbos and Kassam [25] for scenario 1, and by the monthly proposed crop yield equation (scenarios 2 through 6), subject to seasonal constraints (scenarios 1 and 2), and monthly constraints (scenarios 3 through 6).

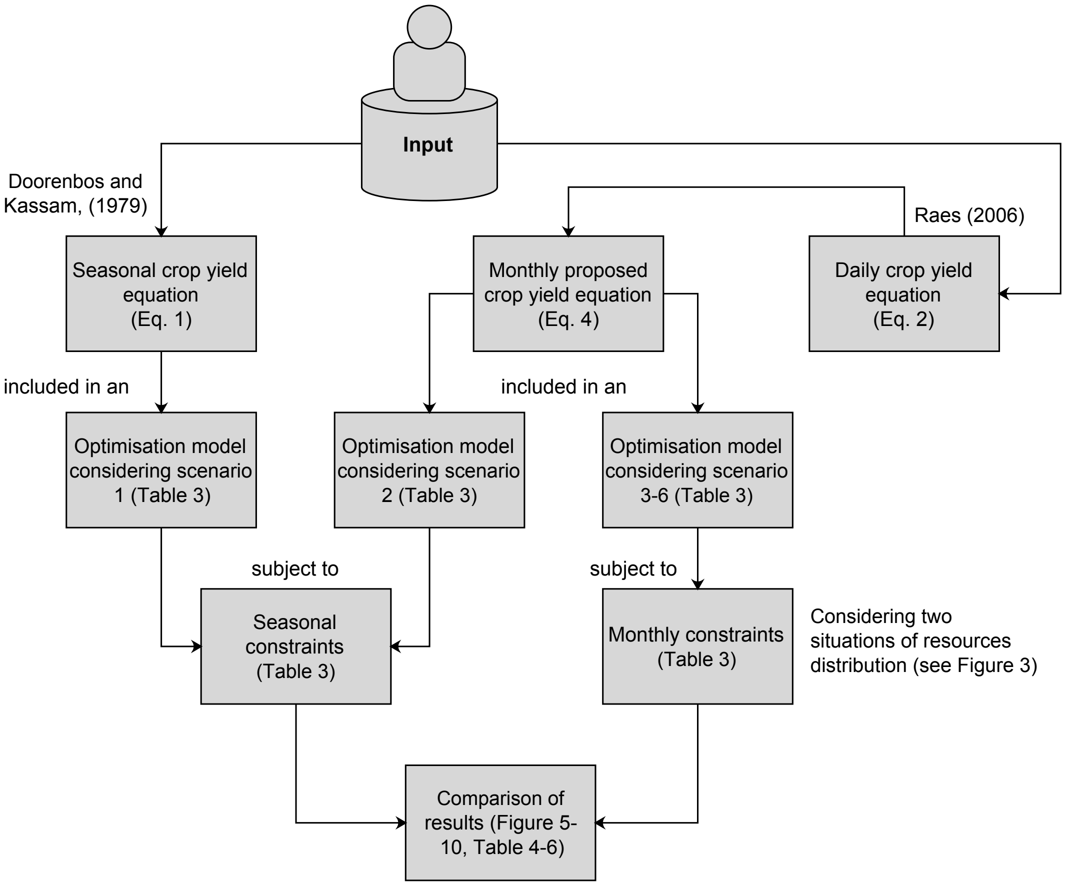

Figure 4. Methodology used for this research. The six scenarios were assessed by two situations of time and resource distribution.

The model was run using the General Algebraic Modelling System (GAMS) [48] software, Version 24.7, and solved with the CONOPT 3 solver.

For implementation of the six scenarios, the following values were considered:

For the whole scenarios, At was 25 ha, Wcr was 0.0016 US$ m^{-3}, and LC was 20 US$ person-day^{-1} [47]. In this study, it was considered that alfalfa, maize, and sugar beet were watered by sprinklers (AE_i = 0.75) and wheat by furrow (AE_i = 0.60). For scenario 1, Ky_i values of 1.00, 1.25, 1.15, and 1.10 were considered for alfalfa, maize, wheat, and sugar beet, respectively, according to the recommended values by Doorenbos and Kassam [25] (Figure 1a). For scenarios 2 through 6, the values considered are shown in Table 1. For scenarios where water transactions were considered (4 and 6), Wcb and Wcs corresponded to 0.0018 and 0.0014 US$ m^{-3}, respectively. On the other hand, for scenarios where water storage was considered (5 and 6), Rc assumed a value of 30,000 m^3.

On the other hand, the same amount of resource availability (labour and capital) was considered for both situations, but at different distributions in time, where labour and capital availability was 300 person-days and 80,000 US$, respectively.

Table 3. Objective functions and constraints used in each scenario.

Function or Constraint	Equation	Scenarios
Objective	$Max\ U = \sum_{i=1}^{n} P_i A_i Ym_i \left[1 - Ky_i \left(1 - \frac{ETa_i}{ETc_i}\right)\right] - LC \sum_{i=1}^{n}\sum_{k=1}^{t} A_i NL_{i,k} - \sum_{i=1}^{n}\sum_{k=1}^{t} A_i OC_{i,k} - Wcr \sum_{k=1}^{t} Wr_k$	1
	$Max\ U = \sum_{i=1}^{n} P_i A_i Ym_i \prod_{k=1}^{t} \left[1 - Ky_{i,k} \left(1 - \frac{ETa_{i,k}}{ETc_{i,k}}\right)\right]^{\Delta t^*_{i,k}/L^*_{i,k}} - LC \sum_{i=1}^{n}\sum_{k=1}^{t} A_i NL_{i,k} - \sum_{i=1}^{n}\sum_{k=1}^{t} A_i OC_{i,k} - Wcr \sum_{k=1}^{t} Wr_k$	2, 3, 5
	$Max\ U = \sum_{i=1}^{n} P_i A_i Ym_i \prod_{k=1}^{t} \left[1 - Ky_{i,k} \left(1 - \frac{ETa_{i,k}}{ETc_{i,k}}\right)\right]^{\Delta t^*_{i,k}/L^*_{i,k}} - LC \sum_{i=1}^{n}\sum_{k=1}^{t} A_i NL_{i,k} - \sum_{i=1}^{n}\sum_{k=1}^{t} A_i OC_{i,k} - Wcr \sum_{k=1}^{t} Wr_k - Wcb \sum_{k=1}^{t} Vwb_k + Wcs \sum_{k=1}^{t} Vws_k$	4, 6
Capital	$Wcr \sum_{k=1}^{t} Wr_k + LC \sum_{i=1}^{n}\sum_{k=1}^{t} A_i NL_{i,k} + \sum_{i=1}^{n}\sum_{k=1}^{t} A_i OC_{i,k} \le \sum_{k=1}^{t} Ca_k$	1, 2
	$\sum_{k'=1}^{k} \left[(Wcr \cdot Wr_{k'} + Wcb \cdot Vwb_{k'}) + LC \sum_{i=1}^{n} A_i NL_{i,k'} + \sum_{i=1}^{n} A_i OC_{i,k'}\right] \le \sum_{k'=1}^{k} Ca_{k'}, \quad \forall k$	3–6
Water	$10 \sum_{i=1}^{n} A_i \frac{ETa_i}{AE_i} \le \sum_{k=1}^{t} Wr_k$	1
	$10 \sum_{i=1}^{n}\sum_{k=1}^{t} A_i \frac{ETa_{i,k}}{AE_i} \le \sum_{k=1}^{t} Wr_k$	2
	$10 \sum_{i=1}^{n} A_i \frac{ETa_{i,k}}{AE_i} \le Wr_k, \quad \forall k$	3
	$10 \sum_{i=1}^{n} A_i \frac{ETa_{i,k}}{AE_i} + Vws_k \le Wr_k + Vwb_k, \quad \forall k$	4
	$0 \le \sum_{k'=1}^{k} \left[Wr_{k'} - 10 \sum_{i=1}^{n} A_i \frac{ETa_{i,k'}}{AE_i}\right] \le Rc, \quad \forall k$	5
	$0 \le \sum_{k'=1}^{k} \left[(Wr_{k'} + Vwb_{k'}) - \left(10 \sum_{i=1}^{n} A_i \frac{ETa_{i,k'}}{AE_i} + Vws_{k'}\right)\right] \le Rc, \quad \forall k$	6

To test the effectiveness of the optimisation model once it reached an optimum irrigated cropping pattern and water allocation, a sensitivity analysis was conducted. This analysis included scenario 1 and 6. The sixth scenario was evaluated considering both situations of resource distribution (a and b) (Figure 3). The analysis was conducted considering land, labour, capital, and mean water availability (25 ha, 300 person-d, 80,000 US$, and 357,334 m^3 for the whole season, respectively). The analysis tested the variation in profits by changing export prices, crop area, irrigation systems, water and labour costs, labour availability and other costs, as well as capital availability.

3. Results and Discussion

3.1. Seasonal Use of Resources and Profits

Seasonal use of resources and profits are presented in the form of radar charts (Figures 6 and 7) in which each axis corresponds to a decile of a probability (DI) of water availability for each scenario. Each ring represents a resource index, organised as follows (from inside to outside): Land, water, labour, capital used, and profit (Figure 5). These resources were relativized to consider an index number from 0 to 1, and this was carried out in the following manner: (1) For land used, an index of 1 was considered to represent 25 ha of land, which is the available resource for this case study. (2) For water used, an index of 1 depended on the seasonal amount of water of every decile. (3) For labour used, an index of 1 was considered to represent the whole labour availability (300 person-day) distributed at monthly time scales. (4) For capital used, an index of 1 was considered to represent the whole capital availability (80,000 US$) also distributed at monthly time scales. (5) For profit, an index of 1 was considered to represent the maximum profit obtained from the runs (46,269 US$).

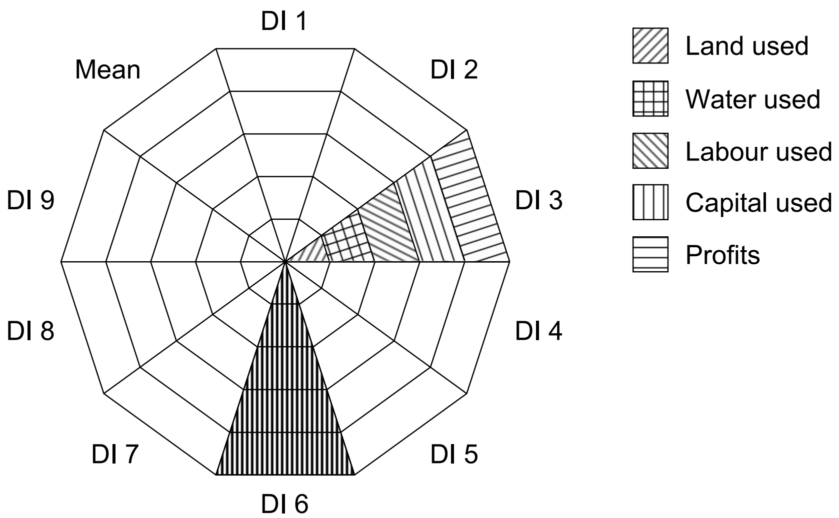

Figure 5. Interpretation of radar plots (Figures 6 and 7) showing profits and seasonal use of resources for each scenario at a situation of resource distribution. Each ring represents a resource index organised as follows (from inside to outside): Land, water, labour and capital used, and profit; each slice corresponds to a decile of water availability (DI)

3.1.1. Situation a

Figure 6 shows profits and seasonal use of resources at situation a. Scenarios 1 and 2 present the highest profits because these scenarios are subject to seasonal constraints, i.e., it is not relevant how resources are divided up in time. In these scenarios, restrictions are scalars instead of arrays, in contrast to monthly scenarios (3, 4, 5, and 6). Water is the limiting resource for these scenarios in decile 1, while capital from decile 2. The third scenario is subject to monthly constraints, and the higher the water supply, the higher the profits. In the fourth scenario, which considers water transactions, the area to be sowed does not increase as water supply increase, because of being limited mainly the labour availability in September, October, and March (Figure 10). Under the fifth scenario, which considers water storage, a similar behaviour is presented as in scenario 3, showing slightly greater profits, but lower than the fourth scenario, which includes water transactions. According to Arnell [49], De Vries and Weatherhead [50], Luo et al. [51], Xu et al. [52], water markets are regarded as an effective way for improving water-use benefits. The sixth scenario, which factors in both improvements in water resources management (water transactions and water storage) shows greater profits than the fifth scenario because it includes water markets.

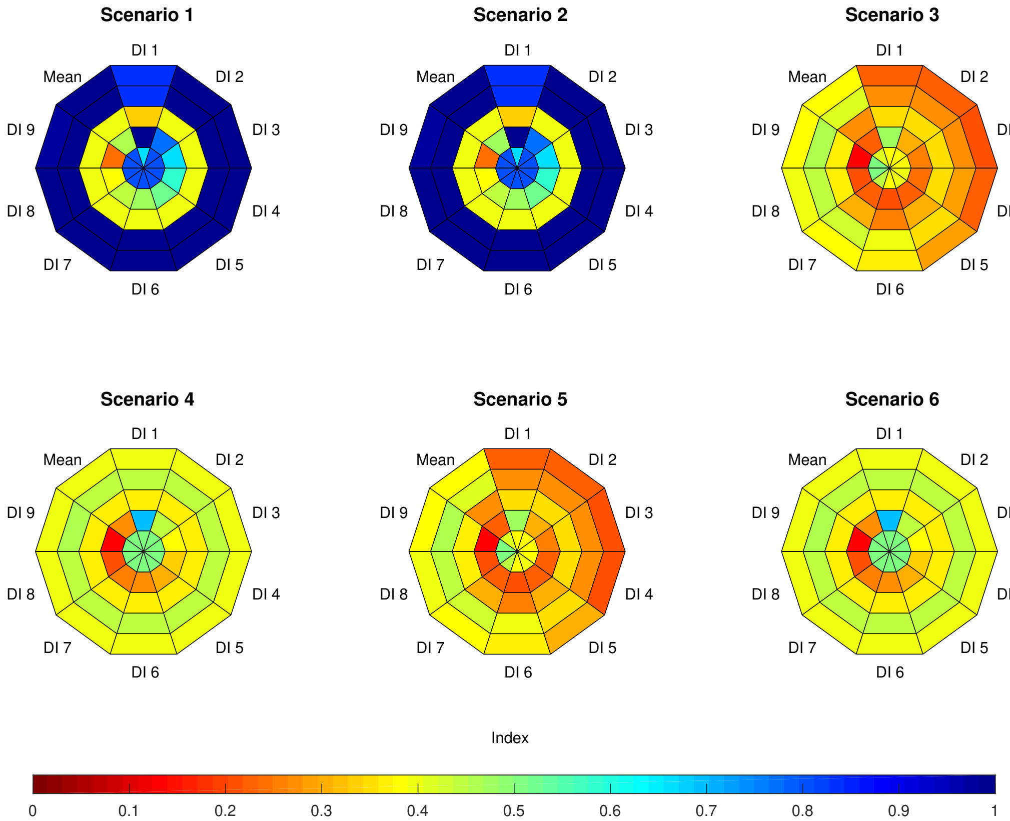

Figure 6. Profits and seasonal use of resources obtained from the optimisation process for each scenario at situation a. Each ring represents a resource index organised as follows (from inside to outside): Land, water, labour and capital used, and profit; each slice corresponds to a decile of water availability (DI).

3.1.2. Situation b

Figure 7 shows profits and seasonal use of resources under situation b. Scenarios 1 and 2 present the same values as situation a. As mentioned before, because these scenarios are subject to seasonal constraints, it is not relevant how resources are divided over time. Regarding the third and fifth scenarios, more water availability means the higher the area to sow, water, labour, and capital used, as well as profits. As far as the fourth and sixth scenarios, profits are as high as seasonal scenarios (1 and 2) and in some cases, even better (in decile 1). Due to capital limitations, profits do not increase, in spite of having the chance to buy, sell, and store water. Carvallo et al. [29] found that by reducing labour availability, profits were affected by almost 5%, compared to their optimum cropping pattern.

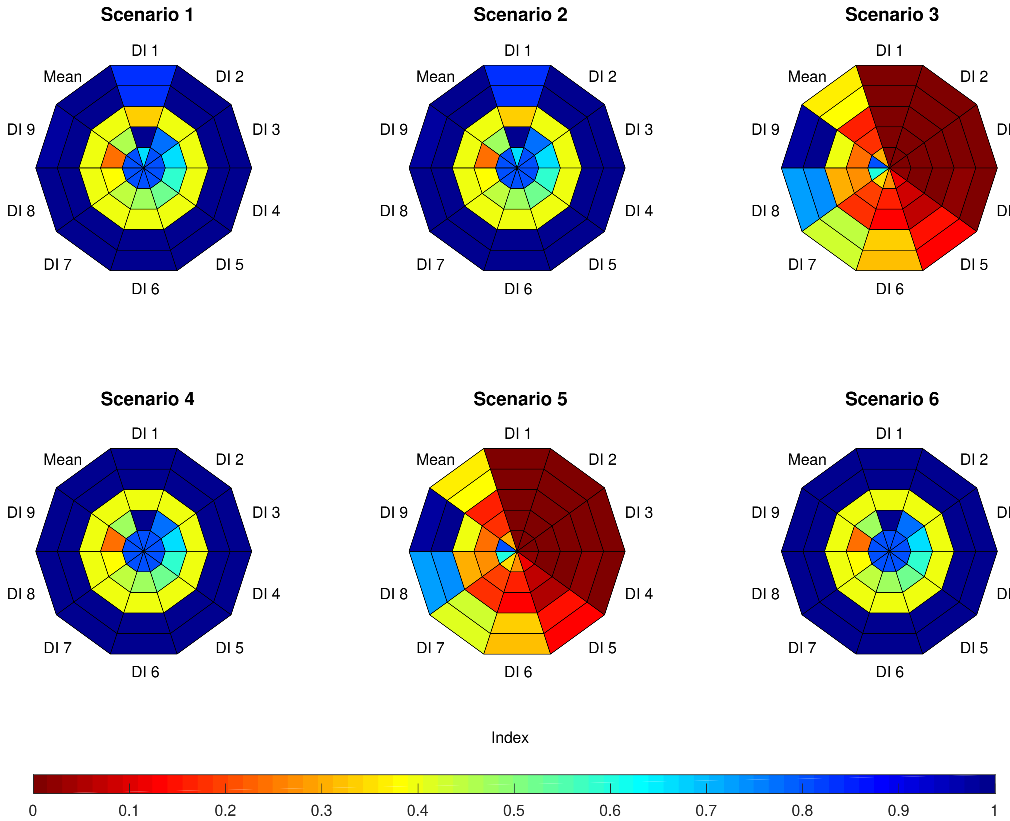

Figure 7. Profits and seasonal use of resources obtained from the optimisation process for each scenario at situation b. Each ring represents a resource index organised as follows (from inside to outside): Land, water, labour and capital used, and profit; each slice corresponds to a decile of water availability (DI).

3.2. Crop Allocation

Figures 8 and 9 show the crop allocation suggested by the runs to obtain the highest profit.

3.2.1. Situation a

Figure 8 shows crop allocation for situation a. Scenarios 1 and 2 present the same results, sowing only sugar beet is the best choice, which requires low labour but high capital (Figure 3). Regarding the third and fifth scenario, sowing maize is the best choice for the first four deciles due to the low water availability and the sowing date of this crop (1 November). From decile 5, there is enough water in September, the month when sugar beet starts to grow. From the sixth decile, the area to plant with sugar beet continues to increase and the area to sow with alfalfa appears as an option, being a better choice than maize because this crop demands less capital (Figure 3). On the other hand, in the fourth and sixth scenarios, sugar beet is the main crop suggested because there is enough water for irrigation.

3.2.2. Situation b

Figure 9 shows crop allocation for situation b. Scenarios 1 and 2 present the same results as situation a. In the third and fifth scenarios, there is not enough water before decile 5. Compared to the situation a, there is lower labour availability in November, the month when maize could be sowed (Figure 3). However, after decile 5, there is enough water to irrigate sugar beet. As far as the fourth and sixth scenarios, sugar beet is the only suggested crop because of the possibility of having more water (water transactions and water storage).

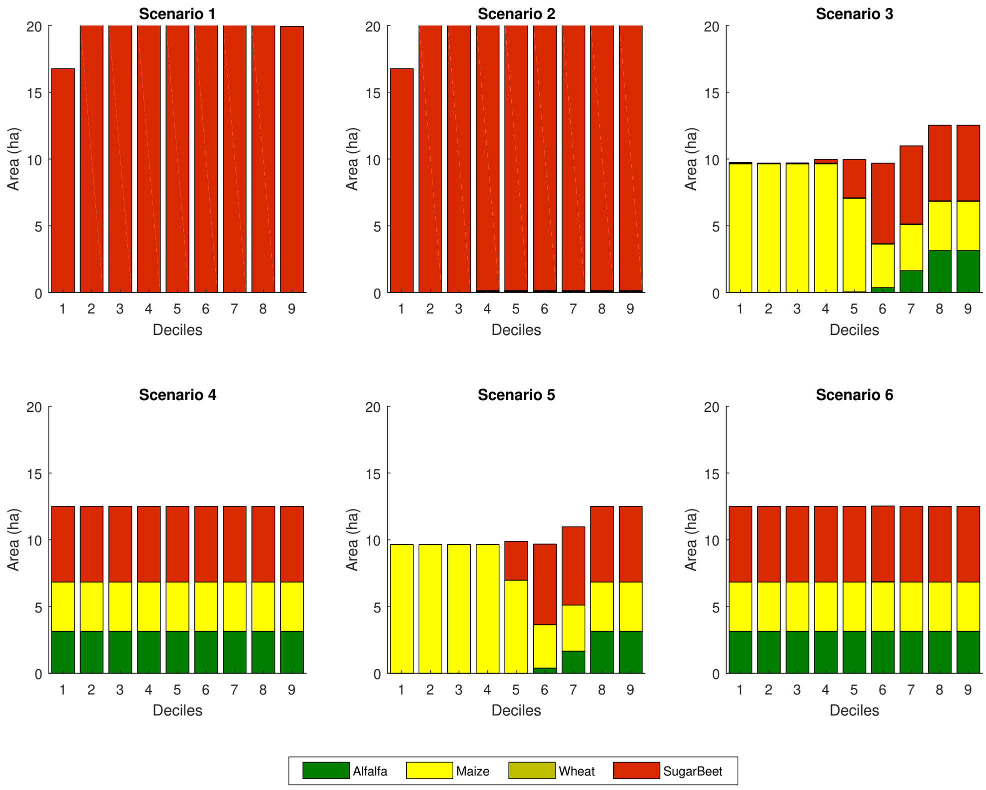

Figure 8. Crop allocation as a result of the optimisation processes for each decile of water supply and for each scenario at the situation a.

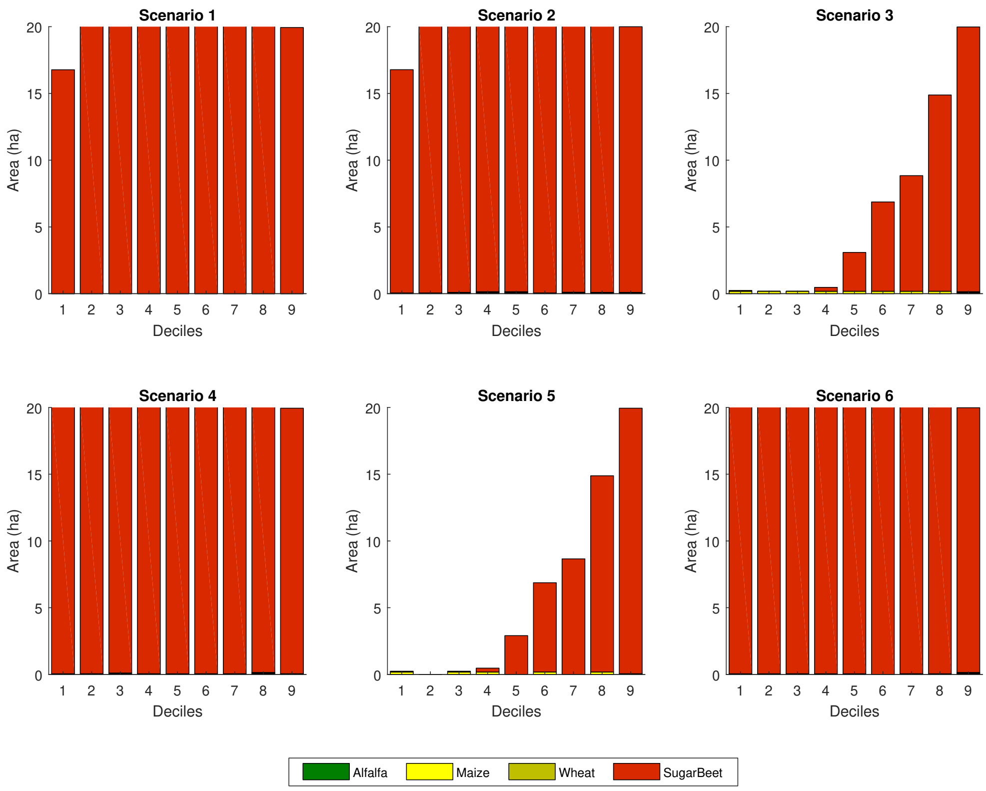

Figure 9. Crop allocation as a result of the optimisation processes for each decile of water supply and for each scenario at the situation b.

3.3. Monthly Limiting Resource

Figures 10 and 11 present resource availability for each month in order to find the monthly limiting resource. Labour and capital are relativized to consider an index number from 0 to 1. An index of 1 is used to represent the labour (300 person-day) and capital availability (80,000 US$). However, water availability indices depend on different scenarios. Regarding the third scenario which does not consider improvements in water resource management, an index of 1 represents the seasonal amount of water for every decile. For the fourth scenario, an index of 1 is considered to represent the maximum difference between income (water rights and water to buy) and outcome fluxes (water applied and water to sell). Regarding the last two scenarios, an index of 1 is considered to represent the maximum capacity of the reservoir (30,000 m^3).

3.3.1. Situation a

Figure 10 shows the monthly limiting resource presented as water, labour, and capital availability for situation a. Regarding the third scenario, which does not consider improvements in water resource management, water is the limiting resource in September for the first four deciles of water supply. For this reason, the model does not recommend establishing sugar beet. Also, during September, October, and March labour is the limiting resource for the most deciles of water supply.

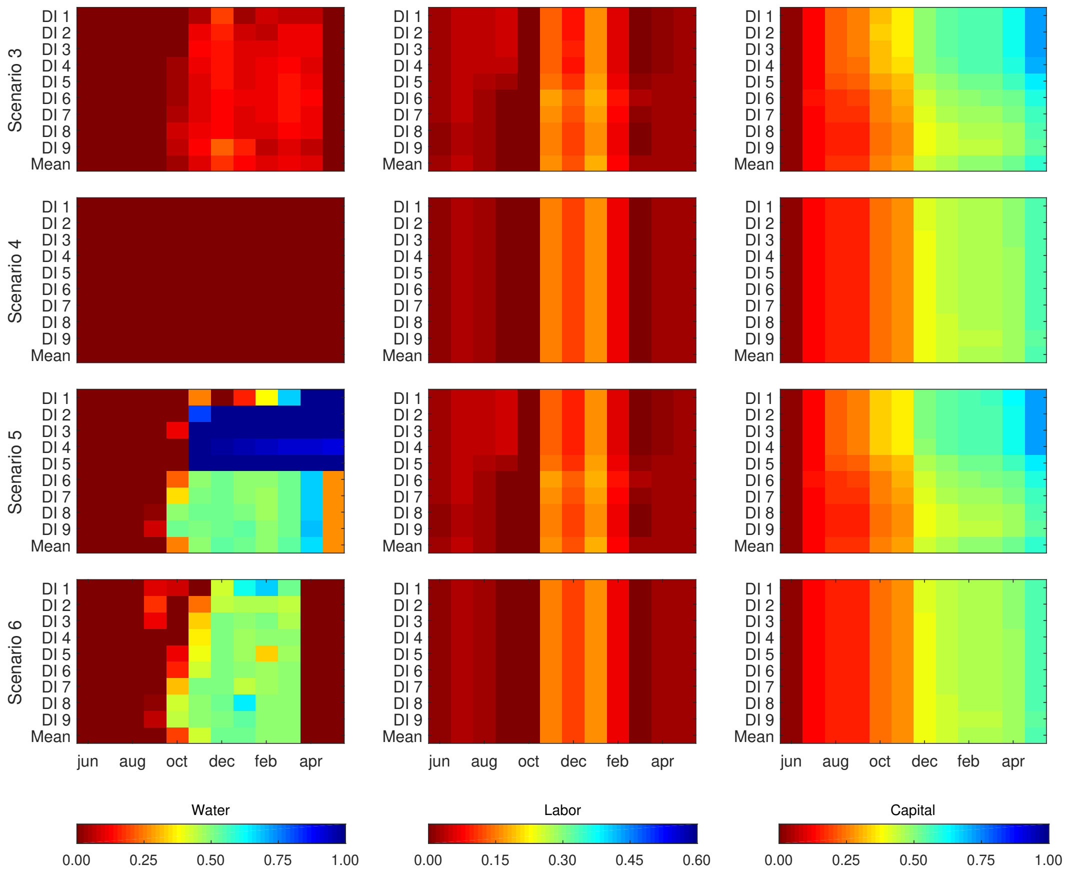

Figure 10. Monthly limiting resources presented as water, labour, and capital availability for each decile of water supply (DI) and for each monthly scenario (from 3 to 6) for situation a (see Section 3.3).

3.3.2. Situation b

Figure 11 shows the monthly limiting resources presented as water, labour, and capital availability under situation b. In the third scenario, water is a limiting resource for the first four deciles of water supply. On the other hand, labour is also a limiting resource after October (as well as in scenarios 4, 5, and 6), which is a reason not to sow maize in this scenario (Figure 3). Analysing the capital, this resource is limiting for decile 9. For the fourth scenario, which considers water transactions, water and capital are limiting resources. The water constraint equation (Table 3), does not consider storage. Regarding the fifth scenario, which factors in water storage, water is the limiting resource in September. Moreover, starting from decile 4, water availability increases due to the capacity to store water; but because of the labour constraints, it is not feasible to increase profits. Regarding the sixth scenario, capital is the limiting resource. Therefore, under this scenario, profits do not increment in spite of higher water availability.

3.4. Sensitivity Analysis

Sensitivity analysis was carried out to test the proposed optimisation model. This analysis included the first (seasonal) and the sixth scenario (the monthly scenario which considers water transactions and storage). This latter was assessed by both situations of resource distribution (a and b).

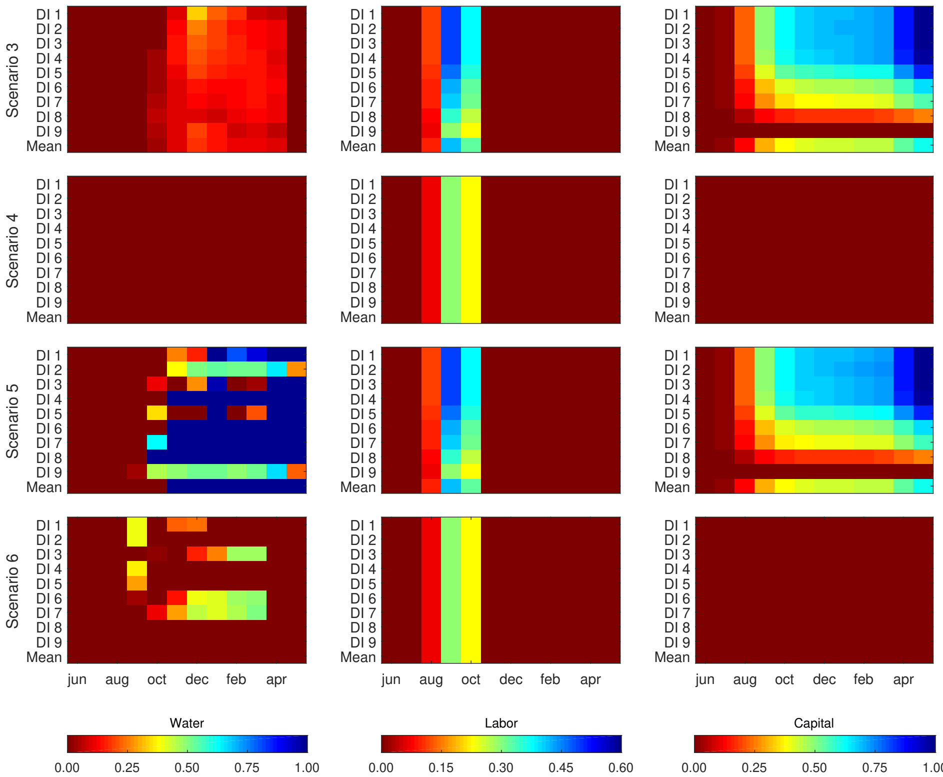

Figure 11. Monthly limiting resources presented as water, labour, and capital availability for each decile of water supply (DI) and for each monthly scenario (from 3 to 6) for situation b (see Section 3.3).

3.4.1. Scenario 1

Table 4 shows the sensitivity analysis for the scenario 1. The best profit, according to the local conditions, corresponds to 45,917 US$, allocating only sugar beet. Under this regime, capital is the limiting resource. Changes in export prices do not affect profits when maize and wheat decrease and increase their values, respectively. A reduction of 50% in the export price of sugar beet means that the farmer must only establish maize. Consequently, profits reduce by about 42%. An increase of 50% in the export price of alfalfa means that this crop and sugar beet share the available land, raising profits by 13%. When the crop area consideration is taken into account, all crops share the available area. Although profits are reduced by 9%, it is recommended to sow more than one crop due to marketing, rotations, or other agricultural limitations [21,23,26,27,29,30,36,53]. By changing the irrigation system, i.e., sugar beet irrigated by furrow (AE = 0.60) instead of sprinkler (AE = 0.75), and wheat by sprinkler instead of furrow, profits are not affected due to that the model still suggests establish only sugar beet. Moreover, water is not the limiting resource, but capital. Consequently, water use increases by 25%. An increase in water costs means that profits are reduced by almost 60%. Here, the model recommends sowing only sugar beet. In this case, improvements in water resource management such as water transactions [7,8] and water storage [9,10] can be useful to mitigate this problem. An increase in labour costs does not considerably affect profits (4%) in comparison to the lack of labour availability. The latter reduces profits by 17%. Therefore, according to Carvallo et al. [29], it is recommended to increase wages to avoid the lack of labour availability. Changes in other costs do not considerably affect profits when maize and wheat increase and decrease their values, respectively. An increment of 50% in the values for sugar beet means that the farmer sows only maize. Consequently, profits are reduced by about 43%. The same response happens when export prices for this crop decrease in 50%. A reduction of 50% in the export price for alfalfa means that this crop and sugar beet share the available land, raising profits by 10%. A decrease in capital availability reduces profits by more than 50%. For this reason, the model suggests sowing only sugar beet when capital is the limiting resource.

3.4.2. Scenario 6, Situation a

Sensitivity analysis for scenario 6, considering the distribution of resources for situation a is presented in Table 5. According to local conditions, the best choice is to allocate 3, 4, and 6 ha of alfalfa, maize, and sugar beet, respectively, resulting in profits of 18,754 US$. Changes in export prices considerably affect profits; sugar beet decreases its value by 50%, and consequently, profits are reduced by 85%. A decrease of 50% in the export price for maize means that alfalfa and sugar beet should be sowed, decreasing profits by almost 9%. When alfalfa price increases by 50%, this situation involves planting the same cropping pattern as the previous situation (reduction of 50% in the export price for maize) but profits increase by almost 20%. The reason is that the price of one of the recommended crops (alfalfa) increases, despite having the same resources consumption. When crop area is taken into account, all crops share the available area, but as a result, profits are reduced by 12%. By changing the irrigation system, profits are not considerably affected. Water use, however, increases by 12%. When water costs are high, profits are reduced by 33%. As mentioned before, an increase in labour costs does not considerably affect profits (6%) in comparison to the lack of labour availability (67%). Changes in other costs do not considerably affect profits when maize and wheat increase and decrease their values, respectively. A 50% increment of this value for sugar beet suggests that the farmer only sows 10 ha of maize, reducing profits by 44%. A 50% reduction in the export price of alfalfa leads to a rise in profits of 10%. When capital availability decreases by 50%, profits are reduced by nearly 12%. In this case, the model suggests sowing 3 and 6 ha of alfalfa and sugar beets, respectively.

3.4.3. Scenario 6, Situation b

Table 6 shows the sensitivity analysis for scenario 6, considering the resources distribution for situation b. The best profits, according to the local conditions are 46,177 US$, 0.57% higher than the seasonal scenario and 146% higher than the situation a. The latter is due to the monthly distribution of resources. According to the resource distribution of labour and capital, sugar beet is the only feasible crop to be sowed. Therefore, a reduction of 50% in the export price of this crop results in no positive returns due to costs of water rights, which farmers must pay if they use them or not. When crop area is taken into account, the model suggests establishing a minimum of 3 ha of each crop, reducing profits by 84%. However, this is an infeasible solution; sugar beet is the only feasible crop to be established for this situation of resource distribution. When water costs are high, profits decrease by 30%. That is only 50% compared to scenario 1, which does not consider water transactions. An increment of labour costs does not considerably affect profits (5%) in comparison to the lack of labour availability (18%). On the other hand, if other costs of sugar beet increase by 50%, profits decrease by almost 90%. As in the first scenario, a decrease in capital availability reduces profits by more than 50%.

Table 4. Sensitivity analysis for scenario 1.

	Crop Allocation				Use of Resources				
	Alfalfa	Maize	Wheat (ha)	Sugar Beet	Land (ha)	Water (m^3)	Labor (Person-d)	Capital (US$)	Profits (US$)
1. Optimum cropping pattern	0	0	0	20	20	167,477	120	80,000	45,917
2. Export prices									
2.1 Decrease in 50% for sugar beet	0	25	0	0	25	172,417	267	57,114	26,671
2.2 Decrease in 50% for maize	0	0	0	20	20	167,477	120	80,000	45,917
2.3 Increase in 50% for alfalfa	8	0	0	17	25	201,623	192	80,000	51,708
2.4 Increase in 50% for wheat	0	0	0	20	20	167,477	120	80,000	45,917
3. Agronomic management									
3.1 Minimum area to be sowed corresponds to 3 ha	3	3	3	16	25	196,754	178	79,578	41,695
4. Application efficiency of the irrigation system									
4.1 Sugar beet is irrigated by furrow ($AE = 0.60$) and wheat by sprinkler ($AE = 0.75$)	0	0	0	20	20	209,346	120	80,000	45,917
5. Water costs									
5.1 Costs of water rights increase to 0.05 US$/$m^3$	0	0	0	16	16	131,010	94	80,000	18,500
6. Labour									
6.1 Costs increase to 30 US$/person-d	0	0	0	20	20	164,974	119	80,000	44,036
6.2 Availability decreases to 100 person-d	0	0	0	17	17	139,015	100	66,502	38,017
7. Other costs									
7.1 Increase in 50% for sugar beet	0	25	0	0	25	172,417	267	57,114	26,671
7.2 Increase in 50% for maize	0	0	0	20	20	167,477	120	80,000	45,917
7.3 Decrease in 50% for alfalfa	6	0	0	19	25	203,014	184	80,000	50,321
7.4 Decrease in 50% for wheat	0	0	6	19	25	198,643	146	80,000	47,781
8. Capital									
8.1 Availability decreases to 50%	0	0	0	10	10	83,136	60	40,000	22,506

Table 5. Sensitivity analysis for scenario 6, situation a.

	Crop Allocation				Use of Resources				
	Alfalfa	**Maize**	**Wheat (ha)**	**Sugar Beet**	**Land (ha)**	**Water (m^3)**	**Labor (Person-d)**	**Capital (US$)**	**Profits (US$)**
1. Optimum cropping pattern	3	4	0	6	13	96,212	109	36,000	18,754
2. Export prices									
2.1 Decrease in 50% for sugar beet	5	0	0	0	5	37,995	58	8098	2805
2.2 Decrease in 50% for maize	5	0	0	6	11	89,835	95	32,687	17,122
2.3 Increase in 50% for alfalfa	5	0	0	6	11	89,835	95	32,687	22,349
2.4 Increase in 50% for wheat	3	4	5	4	16	114,625	125	36,687	19,046
3. Agronomic management									
3.1 Minimum area to be sowed corresponds to 3 ha	3	4	3	5	14	107,854	119	36,434	16,578
4. Application efficiency of the irrigation system									
4.1 Sugar beet is irrigated by furrow ($AE = 0.60$) and wheat by sprinkler ($AE = 0.75$)	3	4	0	6	13	108,040	109	36,000	18,737
5. Water costs									
5.1 Costs of water rights increase to 0.05 US$/$m^3$	3	4	0	6	13	96,212	109	53,350	12,505
6. Labour									
6.1 Costs increase to 30 US$/person-d	3	4	0	6	13	96,212	109	37,091	17,662
6.2 Availability decreases to 100 person-d	1	1	0	2	4	32,071	36	12,380	6204
7. Other costs									
7.1 Increase in 50% for sugar beet	0	10	2	0	12	81,744	115	25,683	10,416
7.2 Increase in 50% for maize	5	0	0	6	11	89,835	95	32,687	17,122
7.3 Decrease in 50% for alfalfa	3	4	0	6	13	96,212	109	34,032	20,722
7.4 Decrease in 50% for wheat	3	4	0	6	13	96,212	109	36,000	18,754
8. Capital									
8.1 Availability decreases to 50%	3	0	0	6	10	79,035	78	30,922	16,577

Table 6. Sensitivity analysis for scenario 6, situation b.

	Crop Allocation				Use of Resources				
	Alfalfa	Maize	Wheat (ha)	Sugar Beet	Land (ha)	Water (m^3)	Labor (Person-d)	Capital (US$)	Profits (US$)
1. Optimum cropping pattern	0	0	0	20	20	167,460	120	80,000	46,177
2. Export prices									
2.1 Decrease in 50% for sugar beet	0	0	0	0	0	0	0	572	−71
2.2 Decrease in 50% for maize	0	0	0	20	20	167,460	120	80,000	46,177
2.3 Increase in 50% for alfalfa	0	0	0	20	20	167,739	121	80,000	46,224
2.4 Increase in 50% for wheat	0	0	0	20	20	167,760	121	79,958	46,135
3. Agronomic management									
3.1 Minimum area to be sowed corresponds to 3 ha	3	3	3	5	14	101,803	113	36,395	7464
4. Application efficiency of the irrigation system									
4.1 Sugar beet is irrigated by furrow ($AE = 0.60$) and wheat by sprinkler ($AE = 0.75$)	0	0	0	20	20	209,292	120	80,000	46,108
5. Water costs									
5.1 Costs of water rights increase to 0.05 US$/$m^3$	0	0	0	17	17	143,133	103	77,493	32,014
6. Labour									
6.1 Costs increase to 30 US$/person-d	0	0	0	20	20	163,870	118	79,363	43,920
6.2 Availability decreases to 100 person-d	0	0	0	16	16	137,558	99	65,815	37,919
7. Other costs									
7.1 Increase in 50% for sugar beet	0	0	0	13	14	112,451	82	79,277	5221
7.2 Increase in 50% for maize	0	0	0	20	20	167,433	121	79,988	46,074
7.3 Decrease in 50% for alfalfa	0	0	0	20	20	168,256	122	79,980	46,268
7.4 Decrease in 50% for wheat	0	0	0	20	20	168,256	122	79,980	46,268
8. Capital									
8.1 Availability decreases to 50%	0	0	0	10	10	83,074	60	39,972	22,872

4. Conclusions

We developed a monthly optimisation model to obtain an optimum cropping pattern and monthly water allocation for irrigated agriculture under Chilean conditions. The objective function included a monthly crop yield model, which was developed from the Raes et al. [31] equation, being a valid alternative for handling resources on a monthly timescale. This model also included monthly water transactions as a decision variable (besides cropping pattern and monthly water allocation for crops), giving the possibility to farmers to buy water (to increase the irrigated area) and to sell water (when not using it) to improve their profits. Regarding our results, optimizing resources on a monthly basis attained higher profits as it allowed farmers to tailor their management practices and manage costs to cope with less available resources.

Scenarios based on single run optimisation at the beginning of the season assume that how resources are distributed in time is not significant, or that resources will be available. However, this approach does not account for intraseasonal changes. Thus, in seasonal-based scenarios, water is the limiting resource when available water is less than the requirements for the whole season. In the studied monthly scenarios, which include improvements in water resource management (such as water transactions and water storage), the model not only attains higher profits (even better than the seasonal approaches), but also decreases uncertainty and improves risk management. The main advantage of considering a multiperiod model is that is the best option for coping with seasonal changes because income is received from crop production and water transactions. When water transactions are taken into account, labour is the main limiting resource. According to sensitivity analysis, it is not always feasible to consider crop area criteria due to resource distribution (Table 6). On the other hand, when water costs are high, water transactions could reduce losses by up to 50%. As far as labour, the lack of availability is more critical than wages. Future studies should focus on the estimation of the yield response factor (Ky) under local conditions in order to include them in the proposed monthly optimisation model.

Seasonal scenarios for crop management are highly beneficial because they consider that certain resources are only available at the start of the season (the timescale "growing season" is longer than monthly). On the other hand, monthly scenarios increase profits when they consider improvements in water resource management, such as water transactions and water storage.

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