Soil Dynamics and Earthquake Engineering 133 (2020) 106109

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: http://www.elsevier.com/locate/soildyn

Performance-based assessment of the seismic pseudo-static coefficient used in slope stability analysis

Jorge Macedo [a,*], Gabriel Candia [b]

[a] Georgia Institute of Technology, United States
[b] Universidad del Desarrollo, Chile. Research Center for Integrated Disaster Risk Management (CIGIDEN), ANID/FONDAP/15110017, Chile

A B S T R A C T

Pseudo-static slope stability procedures are often employed to evaluate the seismic performance of slope systems, at least in the initial evaluation stages. To yield meaningful results, these methods should rely on parameters that are representative of the existing seismic demand and the properties of the slope system being evaluated. This study proposes a performance-based probabilistic procedure to estimate the seismic pseudo-static coefficient (SPC) in a rational and transparent manner. The procedure has its cornerstone on the evaluation of seismically-induced displacement (D) hazard curves, and it provides SPC estimates that are consistent with the allowable D level that a slope system can sustain, the properties of the sliding mass, the seismic demand at the slope site, and the hazard design level or return period. The proposed procedure can be applied to evaluate the seismic performance of a wide range of slope systems potentially affected by earthquakes from different tectonic settings (i.e. subduction and shallow crustal earthquakes), and has been implemented in a computational platform that facilitates its straightforward use in engineering practice. The implementations are fully automated for South America (i.e., Peru, Chile, Ecuador), Mexico, and the United States, but the proposed framework can be applied worldwide. Finally, an illustrative example for the application of the procedure in the seismic stability assessment of a slope system is provided.

1. Introduction

The seismic performance evaluation of geotechnical slope systems is typically performed by: (a) pseudo-static slope stability analyses, (b) Newmark-based slope displacement analyses, and (c) advanced numerical procedures, such as finite elements or finite differences. Even though advanced numerical analyses are increasingly used in the industry, methods (a) and (b) are preferred in engineering practice for their simplicity, particularly in the preliminary design stages. For example, in the Peruvian mining industry, the regulators require explicitly the results from pseudo-static slope stability assessments in the design of geotechnical infrastructure (e.g., tailings dams, heap leach pads, stockpiles) for mining projects [1]. Similarly, in Chile the Supreme Decree DS248 [2] requires pseudo-static slope stability analyses in the two first phases of a 4-phase evaluation procedure [3]. Other guidelines (e.g., the APEGBC [4], in British Columbia; FHWA [5] in the United States) also recommend the use of pseudo-static based procedures, at least for preliminary evaluations of geotechnical slope systems.

Pseudo-static slope stability procedures are straightforward to be used in practice (i.e., they only require the estimation of a safety factor), and accumulate previous experience from past projects (i.e., most of the slope systems worldwide have been designed, at least in a preliminary stage, using the pseudo-static method). Therefore, their use in engineering practice is appealing [6]. The pseudo-static slope stability procedure relies on traditional limit equilibrium analyses (LEA) where the sliding mass is discretized to evaluate a safety factor (FS). In static LEA the weight of the sliding mass and the shear and normal soil resistance forces along the sliding surface are the only forces considered for equilibrium in the pseudo-static procedure; however, a constant horizontal force applied at the center of gravity of the sliding mass is added to represent the seismic loading. This horizontal force is taken as the weight of the sliding mass multiplied by seismic pseudo-static coefficient (SPC), which is typically a fraction of the peak ground acceleration (PGA). Using this additional horizontal force, the FS is evaluated using methods that rely on equilibrium equations (e.g. Ref. [7]). An adequate method should guarantee that horizontal, vertical, and rotational equilibrium are satisfied to provide reliable FS estimates [8]. Fig. 1 shows schematically the pseudo-static slope stability procedure.

Pseudo-static slope stability procedures should use parameters that are representative of the existing seismic demand and the properties of the system being evaluated [9,10]. In particular, a critical factor in pseudo-static slope stability analyses is the selection of the SPC, which typically is arbitrary, based on accumulated experience, regulatory design guidance, and engineering judgment. For example, in the

* Corresponding author.
E-mail address: jorge.macedo@ce.gatech.edu (J. Macedo).

https://doi.org/10.1016/j.soildyn.2020.106109
Received 16 August 2019; Received in revised form 5 February 2020; Accepted 24 February 2020
Available online 13 March 2020
0267-7261/

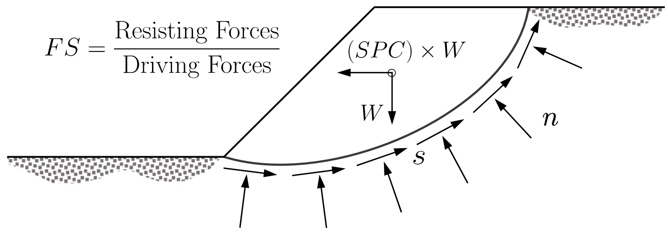

Fig. 1. Schematic representation of a stability analysis using the pseudo-static method. FS is the safety factor, the shear forces (s) and normal forces (n) are distributed along the sliding surface, the driving forces are the weight of the sliding mass (W), and the horizontal force (SPC·W) represents the seismic excitation, where SPC is the seismic pseudo-static coefficient.

Peruvian mining industry the SPC is selected as 0.5 or 0.67 times the PGA in the design for slopes in tailings dams and heap leach pads, despite the local seismicity and the properties of the system.

Recent studies (e.g. Refs. [6,11]) provide procedures to select the SPC to be used in pseudo-static slope stability assessments as a function of the allowable seismically-induced displacements (D) that the slope system can sustain, the intensity measure (IM) value (e.g., using a spectral acceleration or PGA), and in some cases the properties of the sliding mass. These efforts are a step-forward since they provide a rational basis for the SPC selection; however, they are not formulated in a performance-based robust probabilistic framework (i.e., they only consider one IM value and not the full IM hazard in the SPC selection).

This paper proposes a performance-based probabilistic procedure to estimate the SPC in a rational and transparent manner. As it will be explained later, the proposed procedure incorporates: i) the site-specific seismic demand, ii) the maximum allowable D level for the system being evaluated, iii) the properties of the sliding mass, and iv) a formal quantification of uncertainties in the IM and D. The proposed procedure is implemented in a computational platform that facilitates its use in engineering practice. The implementations are fully automated for South America (i.e., Peru, Chile, Ecuador), Mexico, and the United States, but the proposed framework can be applied worldwide. Finally, an illustrative example for the application of the procedure in the seismic stability evaluation of a slope system is shared.

2. Previous studies

The initial procedures for the selection of the SPC were based on precedence, had a regulatory basis, and they were in some extent arbitrary. For example, Seed [12] recommended SPC values of 0.10 and 0.15 for earthquakes magnitudes (M_w) 6.5 and 8.25, respectively, in the context of the seismic performance evaluation of earth dams, and the seismic performance of the dam was evaluated considering a FS $=$ 1.15 in a pseudo-static slope stability analysis. Seed [12] limited his recommendations for systems that do not suffer a significant loss of strength (i.e., no more than 15% of strength loss), and can sustain D values in the order of 1.00 m. Typically, the SPC is estimated as a fraction of the PGA expressed in units of g. For instance, Hynes-Griffin and Franklin [13] provided recommendations for the estimation of the SPC based on seismic analyses of linear elastic shear beam models, and the evaluation of Newmark-type displacements. They recommended a SPC of 0.5 times the PGA at the free field, considering an allowable D level of 1.00 m. Using this coefficient, the seismic performance is considered adequate if FS $\geq$ 1.0 in a pseudo-static slope stability evaluation that considers a 20% reduction in the strength of the materials along the sliding surface. Bray et al. [14] recommended a SPC of 0.75 times the PGA in the free field, combined with a FS $\geq$ 1, and tolerable D values between 15 cm and 30 cm for the evaluation of the seismic stability of solid-waste landfills. In the context of South America, in Peru the SPC is usually estimated as 0.5 to 0.67 times the PGA, based on the recommendations of Marcuson [15], whereas in Chile the SPC is typically estimated as a fraction of the PGA using analytical equations as the one recommended by Saragoni [16]. All these procedures do not account for the site-specific seismic hazard, and they are implicitly based on a maximum D level. Moreover, they have some assumed conservatism that differs between procedures, and they do not consider explicitly the properties of the sliding mass.

More recently, Bray and Travasarou [17], Bray et al. [18], and Bray and Macedo [11] proposed deterministic-based procedures for the evaluation of the SPC. These authors proposed to back calculate the SPC from their performance-based equations for D (using the tolerable level of D for the slope system being evaluated as an input). In this manner, the SPC is consistent with the properties of the sliding mass system, the allowable D level, and the estimated (deterministic or probabilistic) IM value. Thus, these procedures are a step-forward in the rational-based assessment of SPC. However, the hazard design level (or return period), which should be expressed in terms of the D hazard, is not considered in the evaluation. In addition, there is not a formal treatment of uncertainties in the IM assessment and the evaluation of D (only one IM value is used).

Biondi et al. [19,20] used Italian ground motion records, and following similar concepts, back calculated the SPC by introducing equivalences between the SPC and tolerable D levels in infinite slopes. Similarly, Bozbey and Gundogdu [21] proposed SPC estimations based on tolerable D levels in wedge slopes. Also, Zania et al. [22] proposed a SPC spectrum that as a function of D and the PGA. Papadimitriou et al. [6] used results from non-linear dynamic finite difference analyses and developed a procedure to estimate SPCs. In their methodology, the peak value of seismic coefficient is first estimated, and then the seismic coefficient is evaluated as a function of this peak. The seismic coefficient evaluated in this manner depends on the properties of the system, the IM level, the allowable D level, and conservative considerations based on previously published sliding-block models. This procedure is also a step forward to evaluate the seismic pseudo-static coefficient in a rational manner. However, similar to Refs. [11,17,18], this procedure provides no clear definition of the hazard level associated with the performance of the slope, and the existing uncertainties are not formally considered. Finally, Macedo et al. [10] proposed a probabilistic-based procedure that accounts for the hazard design level; however, the procedure does not account for the uncertainties in the IM and considers only partially the uncertainties associated with D (i.e., only alternative values for the slope properties are considered).

In terms of regulatory guidelines used in the past for the estimation of the SPC, the FHWA guidelines for design of geotechnical features in the United States (FHWA [23]) recommended a SPC of 0.5 times the PGA in the free field, and 15 cm of tolerable displacement for evaluating the seismic performance of slopes and retaining structures in transportation facilities. The Landslide Hazard Implementation Committee in California [24] proposed guidelines for the estimation of SPC as a function of the local seismicity and the tolerable D level [25]. Similarly, the National Cooperative highway research program (NCHRP [26]) and the updated FHWA guidelines [5] provided procedures to estimate SPC based on local PGA assessments, slope properties, and the allowable D level. The Association of Professional Engineers and Geoscientists (APEG) of British Columbia (BC) adopted the method proposed by Bray and Travasarou [17] to provide recommendations for the estimation of the SPC in the context of residential developments in BC [4]. The APEGBC selected a representative initial fundamental period of 0.33 s for an average sliding mass, and a maximum allowable D level of 15 cm to propose a SPC that was a function of the spectral acceleration at 0.5 s and the earthquake magnitude. In Peru, MEM [1] suggests a SPC in the range from 0.55 to 0.67 times the PGA, without explicit recommendations of the allowable D level. Likewise, the Chilean DS248 guidelines [2] recommend estimating the SPC using a "region-specific database" to define the design earthquake and a FS of 1.2 [27], but they do not provide allowable D levels. Instead, the engineering practice in Chile often uses the SPC as proposed by Saragoni [16].

The current procedures in regulatory guidelines have similar shortcomings as the procedures previously described in this section; indeed, most of the regulatory guidelines support their recommendations on the previously described procedures.

3. Critical design issue

Before we discuss in more detail the proposed procedure for the performance-based estimation of the SPC, it is imperative to highlight the intended applicability of the proposed procedure.

First, in any seismic slope performance assessment the critical design issue should be identified, by evaluating if a significant loss of strength in the slope system is possible. The procedures in this study are developed for cases where the materials in the slope system or its foundation do not have a significant loss of strength due to an earthquake loading. If there is a possibility for a significant loss of strength (i.e., due to soil liquefaction, or due to a flow slide) the focus of the evaluation should be the assessment of the extent of the loss and the associated consequences in the performance of the system. There are different procedures in the literature to address this scenario (e.g. Refs. [28–32], in particular [28] addresses the cyclic degradation in slopes), and it is not the focus of this study. This study addresses the case where a significant loss of strength is not likely (i.e., the FS in a slope stability assessment considering post-liquefaction conditions is higher than 1.0).

4. Probabilistic evaluation of the pseudo-static coefficient

In this section we discuss the proposed framework for the performance-based probabilistic estimation of the SPC. The proposed framework is based on the concept of a displacement hazard curve (DHC), which provides the annual rate of exceedance for a set of D values. Fig. 2 shows a general scheme for the setup of seismic sources considered in this study. We consider the seismic performance evaluation of systems that can be affected by a combination of shallow crustal, subduction interface and subduction intraslab mechanisms. Next, we discuss the DHC concept, which is the cornerstone of the proposed procedure.

4.1. Estimation of seismic-induced displacement hazard curves

The estimation of a DHC relies on the formulation of robust probabilistic procedures for the estimation of D. These procedures are typically based on seismic sliding block displacement analysis and can be broadly categorized as:

(1) Procedures to estimate D considering rigid-sliding blocks, without considering the flexibility of the sliding mass and its dynamic response (e.g. Refs. [33–41]).
(2) Procedures to estimate D considering a decoupled approximation for the dynamic response of the slope. This approximation first evaluates the dynamic response of the sliding mass assuming no relative displacement, and then uses the results to estimate the seismically-induced slope displacements in a second step (e.g. Refs. [42–44]).
(3) Procedures to estimate D based on fully coupled stick-slip sliding blocks, which capture the simultaneous occurrence of the nonlinear dynamic response of the potential sliding mass and the effects of periodic sliding episodes (e.g. Refs. [11,18,45,46]).

Alternatively, the procedures to estimate D can be also categorized by its applicability to different tectonic settings. Most procedures have been developed for tectonic settings consistent with shallow crustal seismicity (e.g., California). Whereas there are only few procedures developed explicitly for subduction zone earthquakes. For instance, Urzua and Christian ([40]) proposed a relation to estimate D in subduction zones, but this method is only applicable for rigid slopes and developed using ground motion recordings from only three Chilean earthquakes. We are aware of only one robust procedure (i.e. a procedure that considers a large number of ground motions from different subduction regions and is applicable to rigid and flexible slopes) developed by Bray et al. [18] for subduction settings.

Displacement hazard curves can be estimated using a D model and the hazard information of the relevant IM. For example, the DHC obtained from D models that use a single *IM* (e.g., as in Bray and Travasarou [45]; Bray et al. [18]; and Bray and Macedo [11] models) can be estimated from Equation (1) as [10]:

$$\lambda_D(d) = \sum_{i=1}^{nky} \sum_{j=1}^{nts} \int_0^{\infty} P\left(D > d \middle| IM, k_{yi}, T_{sj}\right) d\lambda(IM) w_{ij} \tag{1}$$

where k_{yi} and T_{sj} are realizations of the sliding mass properties defined as the yield coefficient and the fundamental period of the sliding mass, respectively; for a detailed explanation of these properties refer to Ref. [10]. The term $P\ (D > d|IM, k_{yi}, T_{sj})$ in Equation (1) is the conditional probability that D exceeds a threshold level d conditioned to IM, k_y and T_s, and $d\lambda(IM)$ is the annual occurrence rate for IM. The uncertainty in the slope system properties k_y and T_s is considered as epistemic, then it is treated considering a logic tree approach with alternative

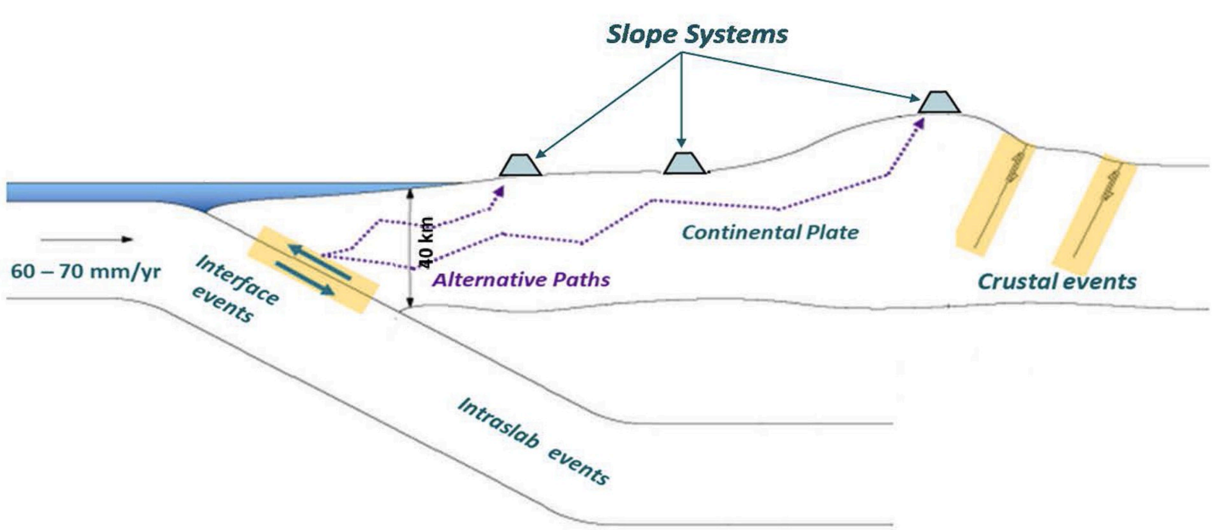

Fig. 2. General setup for the seismic sources considered in this study for the estimation of SPCs. The Figure illustrates the location of potential shallow crustal seismic sources, and subduction (interface, intraslab) seismic sources. For illustrative purpose the tectonic setting in Peru and Chile was used.

values and weights. These weights can be assigned relying on an underlying distribution for these parameters. For example, Macedo et al. [10] assigned weights using lognormal distributions. Fig. 3 shows an example of seismic hazard curves and DHCs estimated by Macedo et al. [10] for a site in the U.S.A Pacific North West with contribution from subduction interface, subduction intraslab, and shallow crustal sources. Such scenario is also typically found in engineering and mining projects in South America (e.g., Peru and Chile).

Interestingly, from the probabilistic seismic hazard assessment (PSHA) results in Fig. 3a it is apparent that shallow crustal sources contribute the most to the seismic hazard. However, the DHCs in Fig. 3b show that the interface seismic sources are actually those that contribute the most to the D hazard. Engineers design for engineering demand parameters such as D not for intensity measures such as a spectral acceleration. Thus, the selection of a SPC should consider the hazard level associated with D, in this manner a direct relation between a DHC and the selected SPC can be established. In this study the concept of a DHC is the cornerstone of the proposed procedure for the performance-based estimation of SPCs as explained in the following.

4.2. Proposed procedure

The proposed procedure for the estimation of a performance-based, hazard-consistent seismic pseudo-static coefficient is comprised of the following steps:

1. Define D_a, the allowable displacement level that the slope system can sustain to achieve the target performance level. This should be done based on regulations or defined as a design/evaluation criterion in consultation with the client.
2. Select the hazard design level Ha_d (i.e., reciprocal of the return period) to evaluate the seismic performance of the slope system. This selection is usually based on regulatory guidelines. Several hazard design levels may be selected associated with different performance levels.
3. In a d (x-axis) versus D-hazard (y-axis) space, define the performance objective point $O(D_a, Ha_d)$ as the intersection between the vertical line passing through D_a, and the horizontal line passing through Ha_d.
4. Assemble a logic tree to consider the epistemic uncertainty associated, alternative values of the sliding mass's fundamental period (T_s), and alternative displacement (D) models (If deemed necessary, logic tree branches for alternative IM models may also be considered).
5. Assign weights to the T_s-branches, and D-branches in the logic tree defined in step 4. The weights in the T_s-branches can be derived from a lognormal distribution for T_s (e.g., as in Ref. [10]), whereas the weights in the D-branches can be assigned based on sound engineering judgement.
6. For each branch in the logic tree Assume an initial k_y value and calculate an initial DHC. Then, perform iterations by changing the k_y value to evaluate new DHCs until the resulting DHC passes through O. The final calculated k_y values for each branch are realizations of the SPC that are consistent with the Ha_d in step 2 and D_a, in step1. This step requires solving the non-linear equation $\lambda_D(D_a) = Ha_d$ for k_y using iterative procedures.

A schematic representation of the 6 steps required to compute the SPC for a single branch is shown in Fig. 4a. After repeating this procedure for each branch in the logic tree, the corresponding DHC's are pinched at $O(D_a, Ha_d)$, as shown in Fig. 4b. However, each DHC corresponds to a different SPC value. These SPC realizations are consistent with the performance level defined in Step 1 and the hazard design level from Step 2.

To solve the non-linear equation for k_y described in step 6, a simple and efficient approach is to use an interval enclosing method (e.g. Ref. [47]) as shown schematically in Fig. 5. The algorithm begins by defining an initial (trial) value for k_y, and a bounding interval for k_y such that $k_L \leq k_y \leq k_U$. Then, the displacement hazard curve is computed using Equation (1) and the interval is narrowed successively until the ratio k_U/k_L approaches unity. The algorithm is very efficient and converges after few iterations.

5. Implementations

We implemented the proposed procedure in a MATLAB graphical user interface (GUI), which includes state-of-the-art PSHA capabilities, a suite of D models for different tectonic settings, and an automated generation of logic trees. In addition, the GUI provides default seismic hazard models for Peru, Chile, Ecuador, Mexico, and the United States, which fully automates the proposed procedure for the SPC estimation in these countries. These capabilities make straightforward the use of the proposed procedure in engineering practice. For other seismic regions, the user may define custom seismicity models and import them as plain text files. In these cases, all the seismic hazard calculations, are performed in the software SeismicHazard [49], embedded in the current software (Appendix A provides details on this, and the GUI inputs).

The GUI consists of a collection of interdependent modules which include: (1) a main window with a summary of the displacement hazard logic tree and the resulting DHC curves; (2) a window for the visualization and definition of the DHC logic tree (Fig. 6); and (3) a D-model explorer, which allows visualizing alternative slope displacement formulations (Fig. 7). (4) a window that shows the SPC estimation and

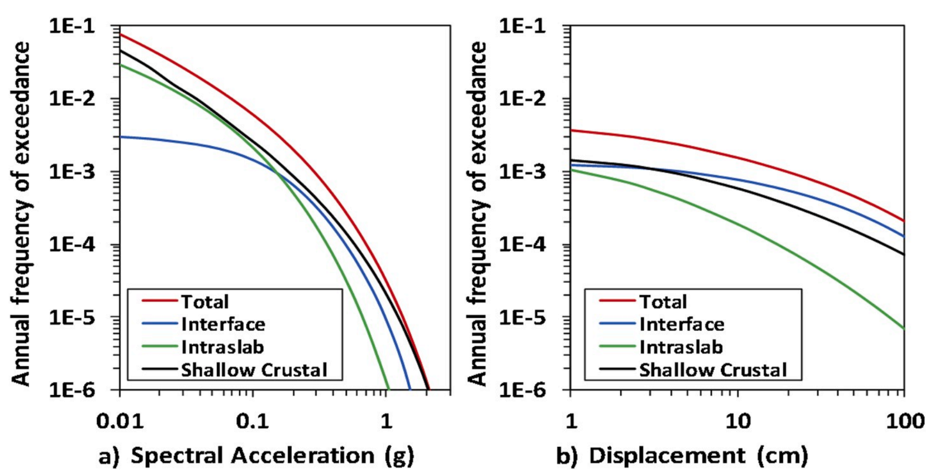

Fig. 3. Illustration of DHC estimated in an area with contribution from different tectonic settings a) IM hazard curves, b) D hazard curves. After [10].

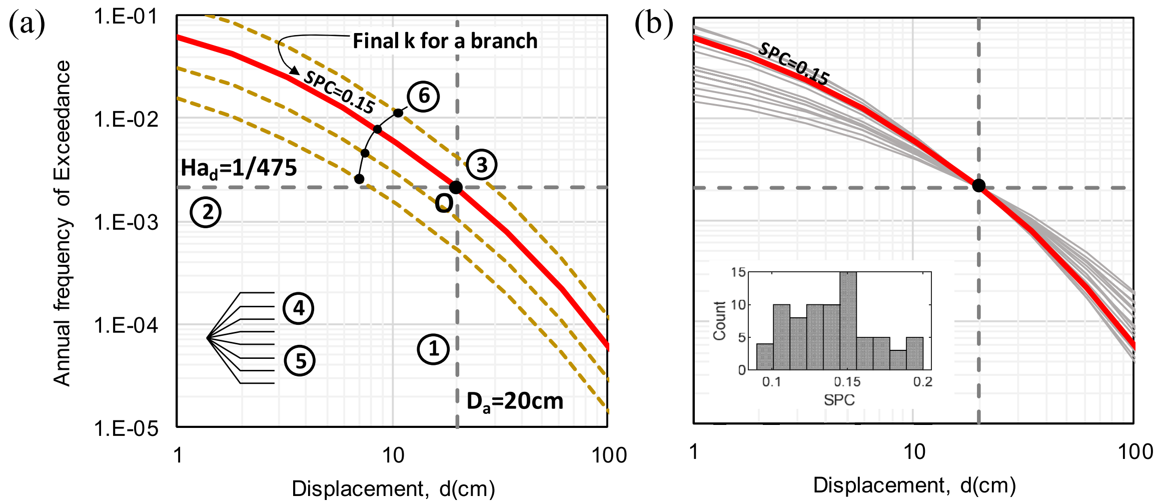

Fig. 4. (a) Schematic description of the 6-stepped procedure; this illustration shows four iterations on k_y values, the final curve is plotted in the continues red line; (b) displacement hazard curves computed for every branch of the logic tree. The multiple realizations of the SPC are shown in the histogram. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

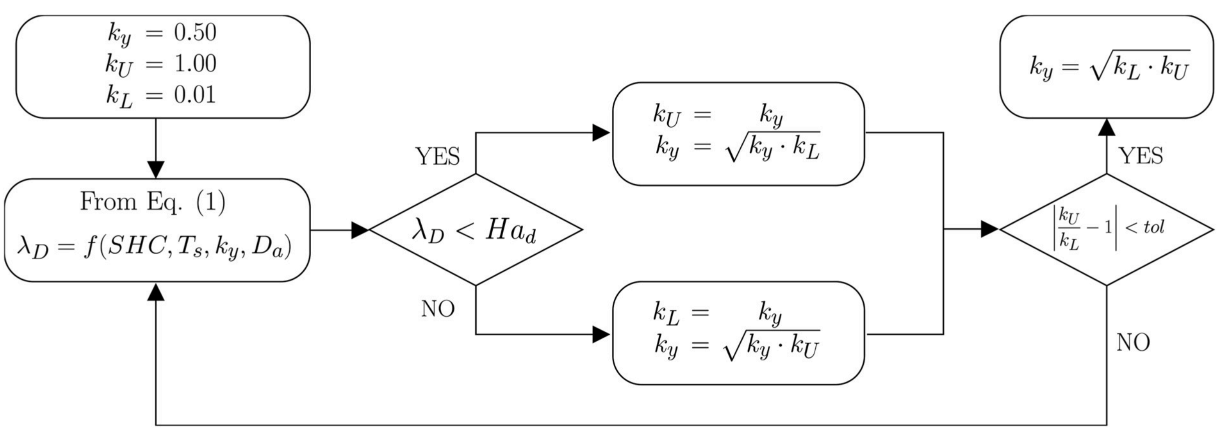

Fig. 5. Algorithm used in the step 6 for computing the SPC.

associated histograms.

In terms of computational capacity/efficiency, the illustrative example presented in the next section consists of 48 alternative SPC values; these SPC represent realizations of the epistemic uncertainty associated to the seismic hazard model, the slope parameters, and the slope D-models. The runtime of this example was approximately 9 min, measured on a 64-bit computer running MATLAB R2018a and with a relative speed of 65 in MATLAB'S benchmark tests. The GUI is publicly available through GitHub at https://github.com/gacandia/SlopeDisplacements. In the following we highlight the main GUI capabilities.

Automatic definition/treatment of uncertainties. In the context of performance-based earthquake engineering, uncertainties are typically categorized as: I) aleatory variability, which is associated with the natural randomness in a process, and can be parametrized by probability density functions (PDF); and II) epistemic uncertainty, which is the scientific uncertainty in the model of the process. This uncertainty is due to limited data and knowledge about the system; in the evaluation of slope systems it is typically characterized by alternative realizations for the properties of the sliding mass as well as alternative D models. The developed GUI allows a straightforward and automatic definition of the aleatory variability through PDF functions to evaluate D hazard curves (i.e., Eq. (1)) as well as epistemic uncertainties through the definition of logic trees for the calculation of SPC. Fig. 6 shows a screenshot of the GUI section where uncertainties are defined. In addition, the illustrative example in the next section shows the definition of uncertainties and the implementation of a logic tree using the GUI for the calculation of SPC (Fig. 10), considering alternative ground motion models, alternative properties for the system, and alternative D models.

Automatic evaluation of DHCs and the SPC. The implemented GUI has capabilities to evaluate D hazard curves (DHCs) accounting for the existing variabilities as previous discussed, and considering a variety of D models that could be formulated in terms of a single IM (e.g. Ref. [18]), but also using multiple IMs (e.g. Ref. [33], in which case a vector hazard approach is needed for the joint IM hazard assessment). For instance, Fig. 7 shows the GUI panel for exploration of alternative D models. In addition, the evaluation of DHCs can be automatically performed considering the contribution from different tectonic settings (i.e., Fig. 2). The GUI automatically uses these results to provide performance-based estimates of the SPC, according to the steps described in the previous section. See the illustrative example in the next section for some representative outputs.

Embedded Seismic Hazard Toolbox. The implemented GUI has embedded a state-of-the-art seismic hazard platform to calculate IM hazard curves (i.e., a PSHA assessment) accounting for the IM variability (i.e., hazard percentiles), if considered necessary. This variability could be considered in the evaluation of Eq. (1); however, engineering practice

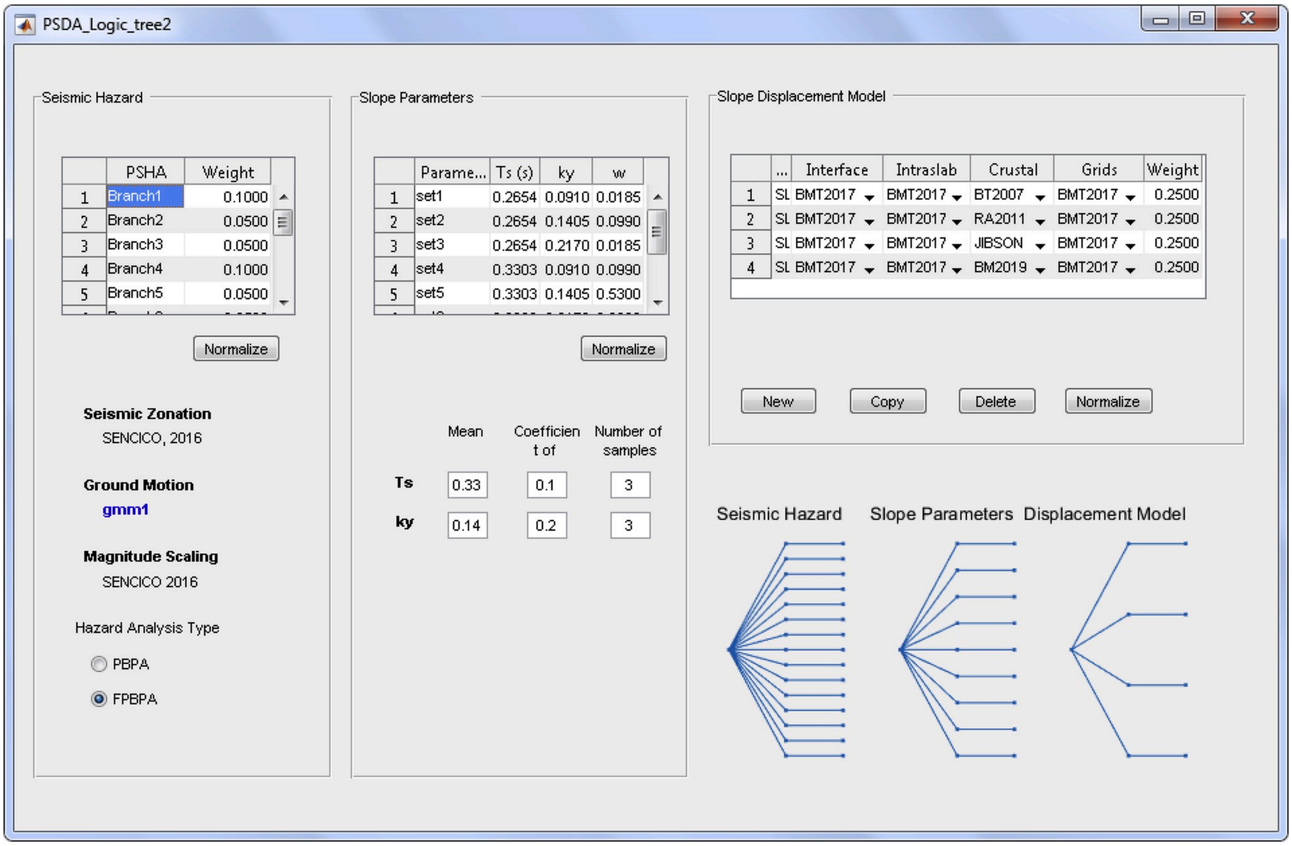

Fig. 6. Logic tree definition for displacement hazard curves within the proposed GUI. The left panel accounts for alternative seismic hazard models; the central panel defines the slope displacement parameters (T_s, k_y) sampled from their corresponding PDFs; and the right panel defines the alternative D models for each tectonic setting.

typically considers only the mean IM hazard [48]. The embedded seismic hazard code (see full documentation in Ref. [49]) includes built-in seismicity models for Peru, Chile, Ecuador, and Mexico, and provides a direct communication with the USGS website for evaluating IM hazard information in the United States. Thus, the evaluation of a SPC considering all the variabilities previously described (i.e., material properties, D alternative models) and the IM variability can be accounted for in the simple and computationally efficient manner. In addition, through interactions with the embedded platform, the GUI allows defining custom seismic sources, seismicity parameters, and ground motion models, allowing the estimation of a SPC anywhere in the globe.

6. Illustrative example

This example uses the proposed procedure in the estimation of the SPC to be used in the pseudo-static slope stability analysis of a dam. Fig. 8 shows the geometry considered for the dam, which is similar to the geometry considered by Bray and Travasarou [45] and Bray and Macedo [11] when showing examples for the application of D models.

The best estimates for T_s and k_y are reported as 0.33 s and 0.14 respectively. T_s is estimated from measured shear wave velocities, which have a best estimate of 400 m/s. As described in Ref. [45], k_y (i.e. the yield coefficient, also referred to as critical acceleration) is estimated from a pseudo-static slope stability analysis performed with the total stress strength properties of c = 14 kPa and φ = 21° based on undrained triaxial compression tests (TX-CIU). Additionally, a covariance value of 0.1 is considered for T_s, which is in the medium range of the values provided by Ref. [10].

The dam is hypothetically placed in the Peruvian Andes, at coordinates: S11.90° W76.2°, a strategic region for the local mining industry. Thus, the dam will be potentially affected by seismic sources in the subduction interface, subduction intraslab and shallow crustal tectonic settings.

Fig. 9a shows the seismic sources for Peru that are used in seismic hazard assessments, the seismic sources are provided by the government agency SENCICO [50]. Fig. 9a also shows the location of the dam site. The geometry of seismic sources and seismicity parameters for Peru are automatically available in the implemented GUI. For the evaluation of IM hazard curves, we use the Abrahamson et al. [51] ground motion from the NGA-West2 project [52] for the shallow crustal seismic sources. For the subduction seismic sources, we use the Abrahamson et al. [53], Montalva et al. [54], and the Zhao et al. [55] GMMs with weightings of 0.5, 0.25 and 0.25 respectively.

As an example, Fig. 9b shows the Sa (T = 0.5s) hazard curve estimated with the GUI (i.e., the IM at the system's degraded period as required in the Bray et al. [18] D model for subduction zones that we use later). Notice from Fig. 9b the contribution of different source mechanism to the total hazard, which in this case is dominated by intermediate depth 'intraslab' events.

For the slope system parameters, we consider eight realizations of the fundamental period T_s, sampled from its PDF. Likewise, we define two sets of alternative D models, with weighting factors of 0.5 each. The first set uses the Bray et al. [18] D model for subduction seismic sources and the Bray and Travasarou [45] D model for shallow crustal seismic sources, and the second set uses the Bray et al. [18] D model and the Rathje and Antonakos [44] for the subduction and shallow crustal seismic sources respectively. Notice that the Bray et al. [18] D model is to date, the only existing robust model developed for subduction tectonic settings. In summary, the resulting logic tree, shown in Fig. 10, has 48 branches that combine three GMM models, eight sets of slope

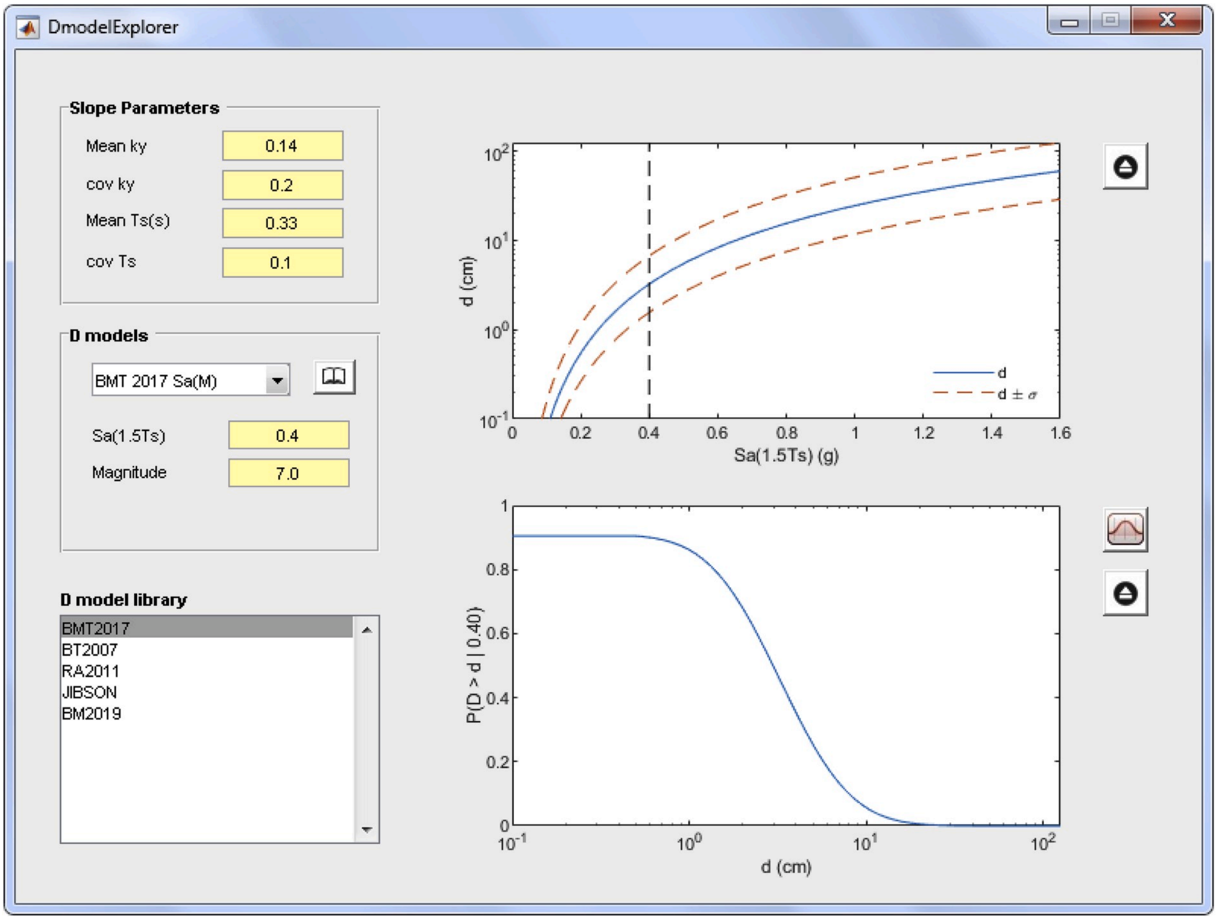

Fig. 7. Exploration module for D models, which allows the user to explore D models.

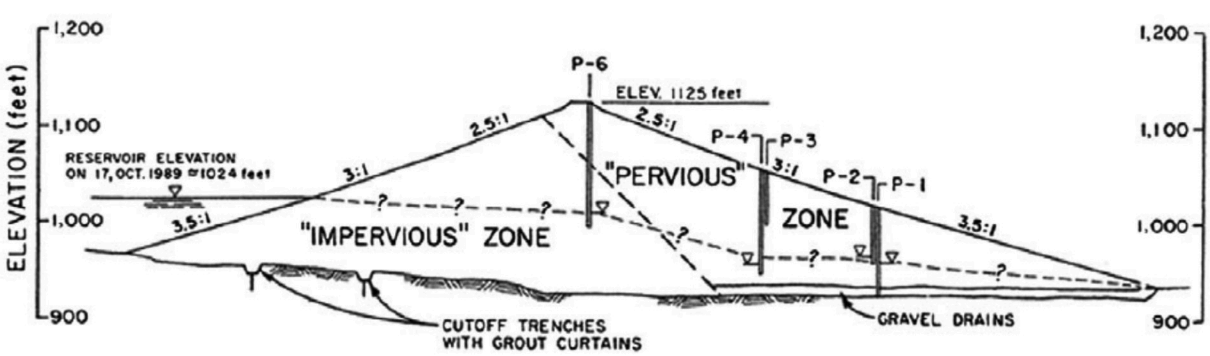

Fig. 8. Dam cross section for the illustrative example (Adapted from Ref. [11]). The dam is hypothetically located in the Peruvian Andes, at coordinates: S11.90° W76.2°.

parameters, and two D models. The weights along each branch are obtained as the product of the seismic hazard weight, the slope parameter weight and the D model weight along that branch. Finally, for the iterations of step 6 in the proposed procedure, we consider an initial value $\boldsymbol{k_y} = 0.14$ (median value), initial bounding values $\boldsymbol{k_L} = 0.001$ and $\boldsymbol{k_U} = 0.8$, and a convergence tolerance $\boldsymbol{tol} = 10^{-4}$ (refer to Fig. 5). These inputs are automatically defined by the GUI and the solution is not sensitive to them, as they only affect the convergence rate of iterations.

Using the inputs defined above, we perform the probabilistic evaluation of the SPC for a design displacement of Da = 30 cm. The analysis was conducted for two return periods: (i) 475 years, and (ii) 2745 years. Appendix B shows step by step procedures within the developed GUI for this illustrative example.

The results from this analysis are shown in Fig. 11, for the two return periods defined. Notice that the 48 DHSs pass through the performance objective point $O(D_a = 30\ cm, Hd_a)$ and have median k_y values of 0.08 ($\mathrm{Ha_d} = 1/475$ years) and 0.16 ($\mathrm{Ha_d} = 1/2475$ years).

The histograms in Fig. 12 summarize the realizations of the SPC for the two performance levels; considering all the branches in the defined logic tree. For a 475 year return period, for instance, the 48 alternative SPC coefficients have median value of 0.08, and the 16th-84th percentiles are 0.07–0.100, respectively. Likewise, for a return period of 2475 years, the median SPC is 0.16, and the 16th-84th percentiles are 0.14–0.19, respectively. In terms of the SPC selection for real projects, the decision maker may choose the median SPC or the percentile 84th as a function of the failure consequences and overall project risk

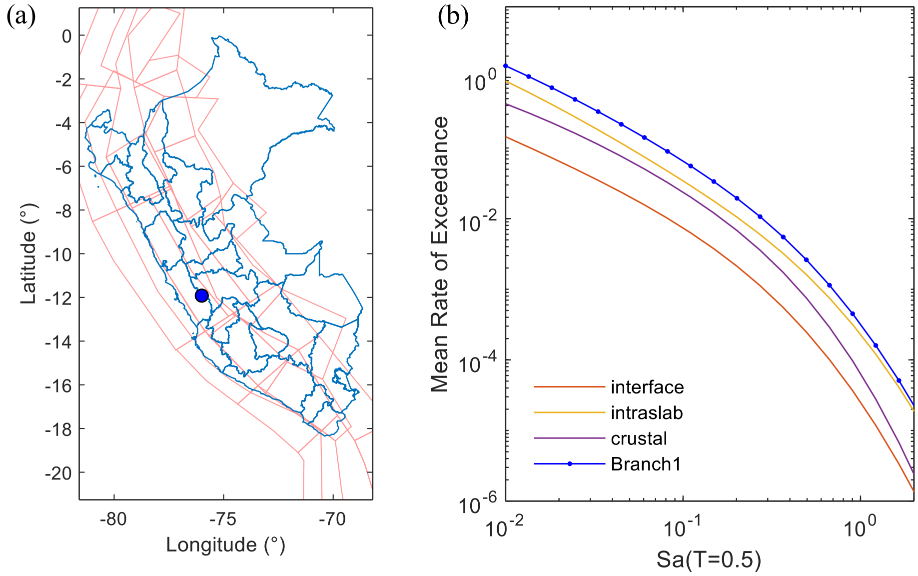

Fig. 9. (a) Sencico's [50] seismic source model and site location; and (b) illustration of the IM (spectral acceleration) hazard curves at the system's degraded period of 0.5 s to be used in the Bray et al. [18] D model. In this figure part, the total hazard and the deaggregation by mechanism correspond to the first branch in the logic tree.

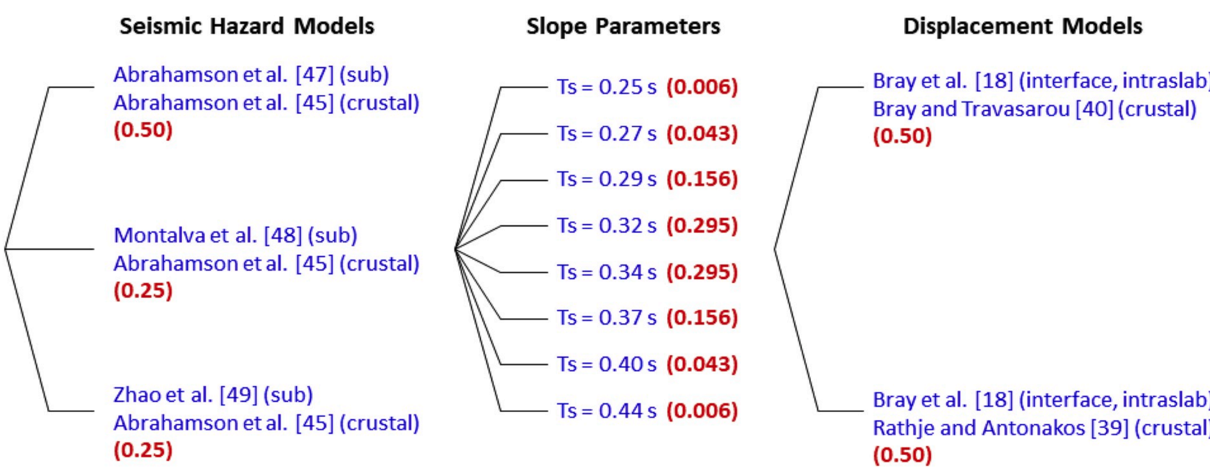

Fig. 10. Logic tree used to compute SPC realizations based on the proposed procedure. The weights for each alternative model are shown in parenthesis. In this example, the logic tree consists of 48 branches (thus 48 SPC realizations), resulting from the combination of three seismic hazard models, eight T_s values, and two alternative D models.

assessment. If the estimated FS in a pseudo-static slope stability analysis that uses the selected SPC is greater than one, the seismic performance of the dam is preliminarily considered adequate and seismically-induced slope displacements larger than 30 cm are not likely to occur. The final decision in terms of the seismic performance of the dam may depend on the design criteria adopted by the designer, which changes from project to project.

7. Conclusions

Pseudo-static slope stability procedures are popular in engineering practice, specially when official regulators request explicitly the use of these procedures, at least in the initial stages of a project. Pseudo-static slope stability analyses should use parameters that are representative of the existing seismic demand and the properties of the geotechnical slope system being evaluated or designed. The seismic pseudo-static coefficient (SPC) employed in the pseudo-static procedure should be selected in a rational manner if this procedure is to form a sound basis for the seismic performance assessment of a slope system. However, this selection has been typically rather arbitrary based on accumulated experience, regulatory design guidance, and engineering judgment.

In this study, we propose a performance-based probabilistic procedure for the rational selection of the pseudo-static coefficient to be used in slope stability analyses. The procedure has its cornerstone on the evaluation of displacement hazard curves (DHC) and incorporates 1) the site-specific seismic hazard (through the mean IM or the full set of IM hazard curves percentiles, if needed), 2) the maximum allowable D that the system can sustain for the desired seismic performance, 3) the properties of the slope system, and 4) the full set of existing uncertainties in the IM and D hazards.

The procedure can be applied on different tectonic settings such as subduction and shallow crustal tectonic settings. Importantly, through the estimation of DHCs the procedure links the SPC estimates to the D hazard, which is more directly related to the seismic performance of a

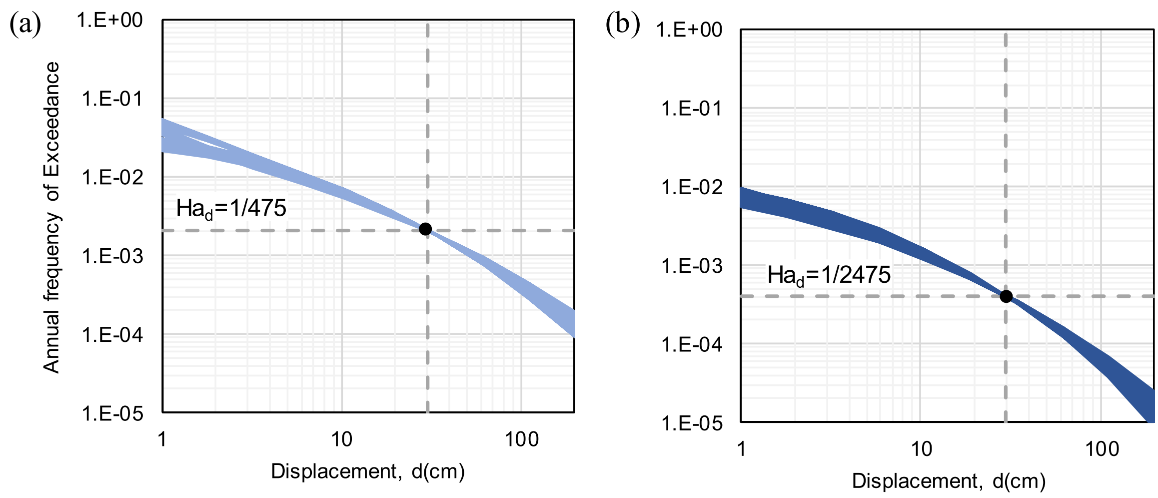

Fig. 11. Displacement hazard curves obtained using the estimated SPC after iterations in the 48 branches of the logic tree (one DHC per branch) for return periods of (a) 475 years, and (b) 2475 years; the median SPC values are $k_y = 0.08$ and $k_y = 0.16$, respectively.

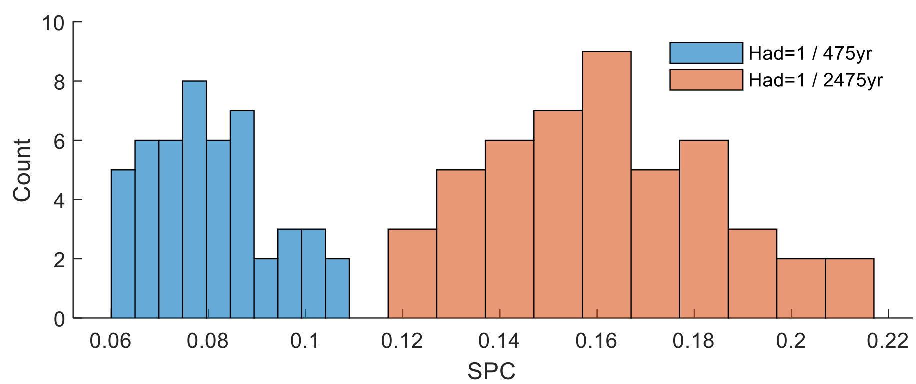

Fig. 12. Histogram showing all the realizations of the estimated SPC according to the proposed procedure.

slope system compared to the IM hazard typically used in engineering practice.

We have implemented the proposed procedure in a Graphical User Interface (GUI) tool, which allows for a straightforward computation of the SPC in engineering practice. The proposed framework can be applied worldwide, but in particular, the SPC estimation has been fully automated through built-in seismicity models in South America (i.e., Peru, Chile, and Ecuador) and Mexico. Similarly, the implementations are fully automated for the United States. In the latter case, the GUI retrieves the NSHM2008 (or NSHM2014) IM hazard information directly from USGS's online tool. The implemented GUI is publicly available at:https://github.com/gacandia/SlopeDisplacements.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Jorge Macedo: Conceptualization, Methodology, Software, Writing - original draft, Writing - review & editing. **Gabriel Candia:** Methodology, Software, Writing - original draft, Visualization.

Acknowledgement

This research was sponsored by the Georgia Institute of Technology. Dr Gabriel Candia was funded partially by the Research Center for Integrated Disaster Risk Management (CIGIDEN), ANID/FONDAP/15110017, Facultad de Ingeniería Civil at Universidad del Desarrollo, FONDECYT Grant N° 1170836 "SIBER-RISK: SImulation Based Earthquake Risk and Resilience of Interdependent Systems and NetworKs", and FONDECYT Grant N°11180937 "Seismic Risk of Mined Tunnels". The authors are grateful for these supports.

Appendix C. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.soildyn.2020.106109.

APPENDIX A

Figure A1 shows the structure of an input file for the general case where seismic sources have not yet been defined. As mentioned in the manuscript, the user may input custom models to compute SPCs for slopes located in any seismic environment. The input file in the general case consists of three sections: Seismic Hazard Options, Displacement Hazard options, and SPC options. For regions that have seismic sources already defined (i.e. USA (through the USGS), Peru, Chile, Ecuador, Colombia, Mexico), only the SPC options (options 9 to 15) are required. The inputs can be entered as plain text (i.e. Figure A1), or through mouse-click operations directly in the GUI (see Figs. 6 and 7 in the main manuscript as well as appendix B). Options 1 to 8 correspond to the embedded seismic hazard code, which is detailed in Ref. [49].

In the general case, the GUI imports the inputs through a text file that contains the seismic hazard model (i.e., geometry of seismic sources, magnitude recurrence laws, rupture scaling laws, ground motion model, site coordinates), the slope parameters (mean and covariance of T_s and k_y), and SPC parameters (*Had*, *Da*). The structure and syntax of the input text file are explained in detail in the following example.

Consider a hypothetic slope ($T_s = 0.333$ *s*, $k_y = 0.2$) located at coordinates XYZ (0,0,0) affected by subduction earthquakes from two line sources. The first source (interface), has a length of 400 km, a uniform depth of 40 km and generates earthquakes with magnitude 7 at a rate of 4.0 events per year; the second source (intraslab) also has a length of 400 km, a uniform depth of 90 km, and generates earthquakes with magnitude 7 at a rate of 6.5 events per year. The relative position of the slopes and the line sources is shown schematically in Figure A2. The ground motion at the site is modelled using the Youngs et al., 1997 GMM with hypocentral distance, and without sigma truncation. The slope displacements are evaluated with the Bray et al. (2017) model. The input text file for this academic example is shown in Figure A1; it consists of three sections: Seismic Hazard Options, Displacement Hazard Options, and SPC options.

Table A1
Description of analysis options for the input text file used in the currentsoftware

Seismic Hazard Options. <u>Note:</u> They correspond to the embedded PSHA code (Additional details in Ref. [49]). These options are required only when the seismic sources need to be defined.	Option 0	Describes general parameters and definitions required to PSHA calculations, including the coordinate projection, shear modulus of earth crust, the list of intensity measures to compute hazard and the discretization of the IMs test values. Other options, not required in this example, have been omitted.
	Option 1	Declaration of the weights assigned to each branch in the PSHA logic tree. The software supports branches for alternative source models, alternative ground motion models, and alternative magnitude scaling relations
	Option 2	Geometry and mechanism of seismic sources. Supported source geometry include point sources, lines, areas, volumes.
	Option 3	GMPE library.
	Option 4	Declaration of GMPE groups. Each row in this option is a list of pointers to the GMPE library, which allows to assign different GMPEs to different sources in a multisource models
	Option 5	Magnitude recurrence relations for each source declared in Option 2. Supported models include: delta, truncated exponential, truncated normal, characteristic, user defined model through table of rates of occurrences.
	Option 6	Magnitude - rupture area scaling law and source discretization for each source declared in Option 2
	Option 7	Site coordinates and Vs30 assignment. The coordinates must be defined using the projection defined in Option 0 (e.g., ECEF, WGS84, WGS80, spherical). The user may specify a single Vs30 value for each site, Vs30 raster files, or shapefiles with regions where Vs30 is known
	Option 8*	(placeholder) This section is used to import seismic hazard benchmark solutions for comparison purposes
Displacement Hazard Options **<u>Note:</u>** (All these options can be entered alternatively with straightforward click-mouse operations- See Figs. 6 and 7, and Appendix B)	Option 9	Global parameters used in the calculation of DHCs, including the discretization of displacement test values, and an optimization flag (on/off) to integrate magnitude-dependent displacement models.
	Option 10	Slope parameters. Mean, covariance, and number of samples used to describe the slope's fundamental period (T_s) and yield coefficient (k_y). The combinations of Ts and ky pairs define the slope parameter branches in the logic tree structure.
	Option 11	Library of slope displacement models. The current software version includes displacement models for slopes on subduction zones and models for slopes in shallow crustal regions.
	Option 12	Declaration of D-model branches. Each row of this option defines a set of displacement models consisting with the earthquake mechanism (subduction interface, subduction intraslab, crustal, grid seismicity).
	Option 13*	(placeholder) this section defines the continuous displacement models.
	Option 14*	(placeholder) This section is used to import displacement hazard benchmark solutions for comparison purposes.
SPC Options (mouse-click operations)	Option 15	Definition of hazard level or return period to estimate the SPC, and the allowable level of displacements for the slope being evaluated in units of centimeters.

* not shown in the example of Figure A2.

```
# ------------------  SEISMIC HAZARD OPTIONS ------------------------------------

Option 0 - Global Parameters
Projection   : ECEF
ShearModulus : 3e11  #dyne/cm2
IM           : 0.5
im           : logsp(0.001,3,200)

Option 1 - Logic Tree Weights
Geom Weight : 1
Gmpe Weight : 1
Mscl Weight : 1

Option 2 - Source Geometry
geometry 1 Line Source
S1 type line mechanism interface gmpe 1 vertices 0 -80 -40 400 -80 -40
S2 type line mechanism intraslab gmpe 2 vertices 0 0 -90 400 0 -90

Option 3 - GMPE Library
YOUNGS97inter handle Youngs1997 mechanism interface media rock
YOUNGS97intra handle Youngs1997 mechanism intraslab media rock

Option 4 - GMPE GROUPS
Youngs 1997 pointers 1 2

Option 5 - MAGNITUDE SCALING RELATIONS
seismicity 1
S1 handle delta NMmin 4   bvalue 0.9 M 7
S2 handle delta NMmin 6.5 bvalue 0.9 M 7

Option 6 - RUPTURE AREA SCALING
S1 type null spacing 0.01
S2 type null spacing 0.01

Option 7 - Pre defined sites (Optional)
source 760
site1  0 0 0

# ------------------  DISPLACEMENT HAZARD OPTIONS ---------------------------------
Option 9 - PSDA setup
d         : logsp(1,200,40)  # cm
optimize  : on

Option 10 Slope Parameters
ky       : mean 0.2     cov 0.3 samples 0
Ts       : mean 0.33333 cov 0.1 samples 0

Option 11 Library of Slope Displacement Models
DISP1  handle psda_BMT2017M

Option 12 Slope Displacement Models (regular)
Slope Weights 1
Slope1 interface DISP1 intraslab DISP1 crustal DISP1 grid DISP1

#------------- SEISMIC PSEUDOSTATIC COEFFICIENT OPTIONS ---------------------------
Option 15 SPC Options
Had     : 1/475
Da      : 20
```

Fig. A1. Input textfile example for SPC calculations.

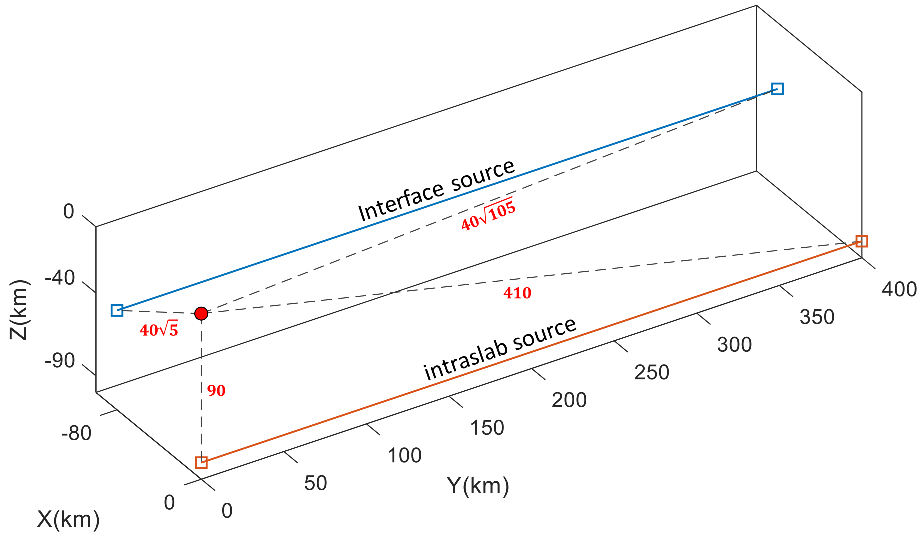

Fig. A2. Geometry and seismic sources for the illustrative example.

APPENDIX B

This appendix provide access to a Youtube video that shows a step by step procedure for the illustrative example presented in the manuscript. The Youtube video is available at:===https://www.youtube.com/watch?v=k6nF0lsXGVQ&list=PLs7YV1kZOt6nlSL8oBDLqJv2uNFx1L3DV&index=3.

References

[1] MEM. Guía ambiental para la estabilidad de taludes de depósitos de desechos sólidos de mina. Ministerio de energía y minas; 1997.

[2] de Minería Ministerio. Reglamento para la aprobación de proyectos de diseño, construcción, operación y cierre de los depósitos de relaves. Retrieved from, http://bcn.cl/1uvyi; 2007 [In spanish)].

[3] Moreno JJ, Kendall S. Considerations for preparing design criteria for dewatered tailings facilities. 6th international seminar in Tailings Management. 2019.

[4] APEGBC. Guidelines for legislated landslide assessments for proposed residential development in British Columbia. Association of Professional Engineers and Geoscientists of BC; 2010.

[5] FHWA. LRFD seismic analysis and design of transportation geotechnical features and structural foundations. *Geotechnical engineering circular* No. 3. 2011. p. 592. Report No. FHWA-NHI-11-032.

[6] Papadimitriou AG, Bouckovalas GD, Andrianopoulos KI. Methodology for estimating seismic coefficients for performance-based design of earth dams and tall embankments. Soil Dynam Earthq Eng 2014;56(57–73).

[7] Spencer E. A Method of analysis of the stability of embankments assuming parallel inter-slice forces. Geotechnique 1967;17:11–26. https://doi.org/10.1680/geot.1967.17.1.11.

[8] Duncan JM, Wright SG. Soil strength and slope stability. New Jersey: John Wiley & Sons, Inc.; 2005.

[9] Bray JD, Travasarou T. Pseudostatic slope stability procedure. Santiago, Chile: 5th International Conference on Earthquake Geotechnical Engineering; 2011. p. 10–3 [January].

[10] Macedo J, Bray J, Abrahamson N, Travasarou T. Performance-based probabilistic seismic slope displacement procedure. Earthq Spectra 2018;35(2):673–95.

[11] Bray J, Macedo J. Procedure for estimating shear-induced seismic slope displacement for Shallow Crustal Earthquakes. J Geotech Geoenviron Eng 2019; 145(12):04019106.

[12] Harry Seed. Considerations in the earthquake-resistant design of earth and rockfill dams. Geotechnique 1979;29(3):215–63.

[13] Hynes-Griffin ME, Franklin AG. Rationalizing the seismic coefficient method. Vicksburg, MS: U.S. Army Engineer WES; 1984. Misc. Paper No. GL-84-13.

[14] Bray JD, Rathje EM, Augello AJ, Merry SM. Simplified seismic design procedure for geosynthetic-lined, solid-waste landfills. Geosynth Int 1998;5:203–35.

[15] Marcuson WF, Franklin AG. Seismic design, analysis, and remedial Measures to improve the stability of existing earth dams - corps of Engineers approach. In: Howard TR, editor. Seismic design of embankments and caverns; 1983 [New York, ASCE].

[16] Saragoni R. Análisis de riesgo sísmico para la reconstrucción del Puerto de Valparaíso. 6tas Jornadas Chilenas, 9-13 de Agosto de 1993, Santiago, Chile 1993: 165–78.

[17] Bray JD, Travasarou T. Pseudostatic coefficient for use in simplified seismic slope stability evaluation. J Geotech Geoenviron Eng 2009;135(9):1336–40.

[18] Bray JD, Macedo J, Travasarou T. Simplified procedure for estimating seismic slope displacements for subduction zone earthquakes. J. Geotech. Geoenviron. Eng. ASCE 2018;144(3). https://doi.org/10.1061/(ASCE)GT.1943-5606.0001833.

[19] Biondi G, Cascone E, Rampello S. Performance-based pseudo-static analysis of slopes. In: Fourth international conference on earthquake geotechnical engineering, Thessaloniki, Greece; 2007. p. 25–8. June.

[20] Biondi G, Cascone E, Rampello S. Valutazione del comportamento dei pendii in condizioni sismiche. Riv Ital Geotec 2011;XLV(1):11–34. 2011.

[21] Bozbey I, Gundogdu O. A methodology to select seismic coefficients based on upper bound Newmark displacements using earthquake records from Turkey. Soil Dynam Earthq Eng 2011;1(31):440–51.

[22] Zania V, Tsompanakis Y, Psarropoulos PN. Seismic slope stability of embankments: a comparative study on EC8 provisions. In: Maugeri M, editor. Proceedings, ERTC-12 workshop on evaluation of geotechnical aspects of EC8, Athens, Sept.11. Patron Editore – Quarto Inferiore – Bologna; 2011.

[23] FHWA. Design guidance: geotechnical earthquake engineering for highways. Geotechnical engineering circular No. 3, 1. Federal Highway Administration; 1997. and Vol. 2, Reports No. FHWA-SA-97-076 and FHWA-SA-97-077.

[24] Blake TF, Hollingsworth RA, Stewart JP. Recommended procedures for implementation of DMG special publication 117 guidelines for analyzing and mitigating landslide hazards in California. Southern California Earthquake Center, University of Southern California; 2002. available at, http://www.scec.org/resources/catalog/hazardmitigation.html.

[25] Stewart JP, Blake TF, Hollingsworth RA. A screen analysis procedure for seismic slope stability. Earthq Spectra 2003;19(3).

[26] NCHRP 12-70. Seismic analysis and design of retaining walls, buried structures, slopes and embankments. NCHRP report 611. Washington, DC: Transportation Research Board; 2008.

[27] Moreno JJ, Kendall S. Considerations for preparing design criteria for dewatered tailings facilities. *6th international seminar in Tailings Management*. 2019.

[28] Ingegneri S, Biondi G, Cascone E, Di Filippo G. Influence of cyclic strength degradation on a Newmark-type analysis. In: Earthquake geotechnical engineering for protection and development of environment and constructions. Rome: Proc. 7 ICEGE; 2019. p. 2996–3004.

[29] Bray JD, Sancio RB. Assessment of the liquefaction susceptibility of fine-grained soils. J. Geotech. Environ. Eng. ASCE 2006;132(9).

[30] Boulanger RW, Idriss IM. CPT-based liquefaction triggering procedure. J. Geotech. Environ. Eng. ASCE 2015:04015065. https://doi.org/10.1061/(ASCE)GT.1943-5606.0001388.

[31] Bray JD, Macedo J. 6th Ishihara lecture: simplified procedure for estimating liquefaction induced building settlement. Soil Dynam Earthq Eng 2017;102: 215–31.

[32] Macedo J, Bray JD. Key trends in liquefaction-induced building settlement. J. Geotech. Environ. Eng. ASCE 2018;144(11).

[33] Saygili G, Rathje EM. Empirical predictive models for earthquake-induced sliding displacements of slopes. J Geotech Geoenviron Eng 2008;134(6):790–803.

[34] Randall Jibson. Regression models for estimating coseismic landslide displacement. Eng Geol 2007;91(2–4):209–18.
[35] Watson-Lamprey J, Abrahamson N. Selection of ground motion time series and limits on scaling. Soil Dynam Earthq Eng 2006;26(5):477–82.
[36] Lin J-S, Whitman RV. Decoupling approximation to the evaluation of earthquake induced plastic slip in earth dams. Earthq Eng Struct Dynam 1986;11:667–78.
[37] Newmark NM. Effects of earthquakes on dams and embankments. Geotechnique 1965;15(2):139–60.
[38] Richards R, Elms DG. Seismic behavior of gravity retaining walls. J. Geotech Eng Div, ASCE 1979;105(4):449–64.
[39] Urzúa A, Christian JT. Sliding displacements due to subduction-zone earthquakes. Eng Geol 2013;166:237–44.
[40] Du W, Wang G. A one-step Newmark displacement model for probabilistic seismic slope displacement hazard analysis. Eng Geol 2016;205:12–23.
[41] Biondi G, Cascone E, Maugeri M. Displacement versus pseudo-static evaluation of the seismic performance of sliding retaining walls. Bull Earthq Eng 2014;12(3): 1239–67.
[42] Makdisi FI, Seed HB. Simplified procedure for estimating dam and embankment earthquake-induced deformations. J. Geotech. Eng. 1978;104:849–67.
[43] Bray JD, Rathje ER. Earthquake-induced displacements of solid-waste landfills. J. Geotech Geoenviron. Eng. ASCE 1998;124(3):242–53.
[44] Rathje EM, Antonakos G. A unified model for predicting earthquake-induced sliding displacements of rigid and flexible slopes. Eng Geol 2011;122(1):51–60.
[45] Bray JD, Travasarou T. Simplified procedure for estimating earthquake-induced deviatoric slope displacements. J Geotech Geoenviron Eng 2007;133(4):381–92.
[46] Du W, Wang G, Huang D. Evaluation of seismic slope displacements based on fully coupled sling mass analysis and NGA-West2 database. J. Geotech. Geoenviron. Eng. ASCE 2018;144(8):6018006.
[47] Kubica BJ. Interval methods for solving underdetermined nonlinear equations systems. In: SCAN, conference. Texas, USA: El Paso; 2008.
[48] Wang Y, Rathje E. Probabilistic seismic landslide hazard maps including epistemic Uncertainty. Eng Geol 2015;196:313–24.
[49] Candia G, Macedo J, Jaimes MA, Magna-Verdugo C. A new state of the art platform for probabilistic and deterministic seismic hazard assessment. Seismol Res Lett 2019;90(6):2262–75. https://doi.org/10.1785/0220190025.
[50] Sencico. Actualización del programa de cómputo orientado a la determinación del Peligro Sísmico en el país. Lima, Peru: Servicio Nacional de Capacitación para la Industria de la Construcción; 2016 [In Spanish].
[51] Abrahamson NA, Silva WJ, Kamai R. Summary of the ASK14 ground Motion relation for active crustal regions. Earthq Spectra 2014;30(3):1025–55.
[52] Bozorgnia Y, Abrahamson NA, Atik LA, Ancheta TD, Atkinson GM, Baker JW, Baltay A, Boore DM, Campbell KW, Chiou BS-J, et al. NGA-West2 research project. Earthq Spectra 2014;30(3):973–87. https://doi.org/10.1193/072113EQS209M.
[53] Abrahamson C, Shi M, Yang B. Ground-motion prediction equations for Arias intensity consistent with the NGA-West2 ground-motion models. PEER 2016/05 Report 2016. available at, https://peer.berkeley.edu/new-peer-report-201605-%E2%80%9Cground-motion-prediction-equations-arias-intensity-consistent-nga-west2.
[54] Montalva GA, Bastías N, Rodriguez-Marek A. Ground-motion prediction equation for the Chilean subduction zone. Bull Seismol Soc Am 2017;107(2):901–11.
[55] Zhao JX, Zhang J, Asano A, Ohno Y, Oouchi T, Takahashi T, Ogawa H, Irikura K, Thio HH, Somerville P, Fukushima Y. Attenuation relations of strong ground motion in Japan using site classification based on predominant period. Bull Seismol Soc Am 2006;96(3):898–913.