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Measuring chronic multidimensional poverty

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Abstract

This paper adopts a new approach to the measurement of chronic multidimensional poverty. It relies on the counting approach of Alkire and Foster (2011) for the measurement of multidimensional poverty in each time period, and then on the duration approach of Foster (2009) for the measurement of multidimensional poverty persistence across time. The proposed indices are sensitive both to (i) the share of dimensions in which people are deprived and (ii) the duration of their multidimensional poverty experience. A related set of indices is proposed to measure transient poverty. An empirical illustration is provided for Chile between 1996 and 2006.

JEL classifications: D63; I3; I32; O1

Keywords: Chronic and transient poverty; Multidimensional poverty; Multidimensional poverty indices; Poverty dynamics; Chile

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1. Introduction

Sen (1976) argued that an index of poverty should identify persons who live in poverty and measure the extent of individual poverty. His seminal contribution inspired numerous proposals of unidimensional indices of poverty based on cross-sections of income or consumption data. It continues to inspire policy frameworks such as the Sustainable Development Goals, whose first goal seeks to "end poverty in in all its forms", with the clear pledge that "no one will be left behind" (UNGA, 2015, pp. 2, 15).

In order to leave no one behind it is necessary to redress poverty 'in all its dimensions' among the long-term poor as well as the recently poor, or the episodic poor. It could be useful to analyse whether the drivers of poverty *reduction* differ for poor groups who have experienced different durations of poverty: There is evidence that the socioeconomic covariates of poverty vary between chronic and transient poverty experiences (Jalan & Ravallion, 2000), and that common covariates may have differential effects depending on the duration of poverty (Bayudan-Dacuycuy & Lim, 2014). If transient poverty rather reflects vulnerability to occasional undesirable fluctuations in well-being (Ward, 2016), whereas chronic poverty reflects a more fundamental inability to raise long-term living standards (McCulloch & Baulch, 2000), then identifying the chronically and transiently poor populations is of paramount importance for policy responses (Carter & Barrett, 2006; Lybbert, Barrett, Desta, & Coppock, 2004).

The duration of poverty at the individual or household level is also a crucial issue for understanding how people experience poverty. Persistent conditions of insufficiency might precipitate detrimental effects on well-being. For instance, an increase in the duration of poverty increases the likelihood of impairment and illness. A person stricken by long-lasting poverty can become socially excluded and/or lose allegiance to the wider community (Walker, 1995). This, in turn, may lead to social unrest (Salvatore, 2007). Likewise, chronic insufficiency of income may be the main driver of multiple deprivations in non-monetary dimensions of wellbeing (Mahadevan & Hoang, 2016). Therefore it often becomes desirable to measure and analyse individual poverty dynamically using panel data.

An important recent development in poverty measurement research has been the definition of a robust multidimensional framework. The reason for its emergence is that well-being depends on both monetary and non-monetary dimensions of life (see Sen & Anand, 1997; Foster & Sen, 1997; Kolm, 1977; Sen, 1985, 1987; Streeten, 1981). Examples of non-income dimensions are housing, schooling, nutrition, etc. A person with a sufficiently high income may not always be well-off with respect to some non-monetary dimensions of life, and conversely, certain achievement are not related with income. It may not be possible to trade off income and some non-income dimensions. It also may be necessary to develop policies to address specific deprivations or combinations of deprivations. If so, then the construction of a multidimensional poverty index and its analysis may be worthwhile. Besides these intrinsic reasons to consider non-monetary dimensions of life for poverty alleviation policies, collecting information on non-monetary indicators is usually cheaper and more accurate than surveying income or consumption. Additionally, this kind of indicators can reduce the leakages from geographic targeting (Bigman & Srinivasan, 2002) and analysis focused on population subgroups, such as child poverty (Roelen, Gassmann, & de Neubourg, 2010). In fact, this is the very reason why some countries use a score based on non-monetary indicators as eligibility criteria for their numerous poverty alleviation programmes.

It is extremely important to combine these two approaches for the study of chronic multidimensional poverty. Hulme, Moore, and Shepherd (2001) and Hulme and McKay (2008) argued explicitly that the measurement of chronic poverty should focus on multidimensional situations. 'Chronically poor are commonly multi-dimensionally deprived' (CPRC, 2004–5, p. 6). Furthermore, interesting analysis can be carried out when chronic and transient poverty measures are broken down by dimension. For example, one can perform an analysis to see whether chronic poverty has distinctive components that may comprise 'poverty traps'. Empirically, Baulch and Masset (2003) show that low-performance in monetary indicators can be less persistent than non-monetary indicators (e.g. child malnutrition or school attendance), which could lead to underestimation of the prevalence and intensity of chronic poverty. This further justifies the assessment of multidimensional poverty from a dynamic perspective emphasizing different poverty duration experiences.

This paper extends the Alkire–Foster multidimensional counting approach to the measurement of chronic poverty using the Foster (2009) duration approach. The latter is chosen because it is parsimonious and easy to understand, and is based on the same axiomatic foundations as the Alkire–Foster family of multidimensional poverty indices. Moreover, unlike other inter-temporal poverty approaches, Foster's identification criteria *explicitly identify the chronically poor*; but can easily be adjusted to identify the transiently poor—which is useful when analysing determinants of poverty reduction. The Alkire and Foster (2011) approach has the practical advantage that it can be computed with ordinal or ratio-scale data and is widely applied. We discuss policy applications of our class of indices so that they can be treated as a tool for understanding the sources of poverty. Such an analysis enables us to figure out origins of poverty at a more disaggregated level. Using this analysis it will be possible to identify some determinants of poverty and design policy to fight it. Alternatively, we can carry out the analysis with the Borguignon and Chakravarty (2003) index which also qualifies as a satisfactory indicator of multidimensional poverty and possesses these characteristics (see Duclos & Tiberti, 2016).

In a nutshell, our class of chronic multidimensional poverty measures identifies the poor in three stages. Firstly, we apply deprivation cut-offs to each person's achievement vector to determine the indicators in which they are deprived. Secondly, we identify each person as multidimensionally poor or non-poor in each period based on their weighted deprivation score. Thirdly, we count the periods in which each person experienced multidimensional poverty. We identify as chronically multidimensionally poor those persons who have experienced multidimensional poverty in at least the number of periods specified by the analyist or policymaker. Our measurement method also generates a range of intuitive and consistent partial and sub-indices. These include the incidence and intensity of chronic multidimensional poverty and the censored headcount ratios from the Alkire–Foster method. New statistics include the average duration of poverty and the average duration of deprivation in each indicator, as well as period-specific indicators of incidence and intensity. Thus, our method proposes a way to identify and evaluate the experience of the chronically poor in a multidimensional sense.

Our proposed measurement approach is unique across the literature in the way it combines the multidimensional counting approach to poverty in any given period with the duration approach to chronic poverty. Other notable contributions either adopt a multidimensional counting approach without identifying the chronically poor (e.g. Nicholas & Ray, 2011), identify the chronically poor without adopting a multidimensional counting approach (e.g. Foster, 2009; Foster & Santos, 2014; Jalan & Ravallion, 1998; Porter & Quinn, 2014), or measure a concept of inter-temporal poverty with one continuous indicator (usually income or consumption) and without distinguishing between the chronically and transiently poor (e.g. Bossert, Chakravarty, & D'Ambrosio, 2012; Bossert, Ceriani, Chakravarty, & D'Ambrosio, 2014; D'Ambrosio, 2013; Dutta, Roope, & Zank, 2013; Gradin, del Rio, & Canto, 2012; Hojman & Kast, 2009; Hoy & Zheng, 2011; Mendola

& Busetta, 2012). Nicholas, Ray, and Sinha (2013) do propose a class of measures combining a multidimensional counting approach with an aim to identify the chronically poor. However the manner in which they identify the poor is fundamentally different from our method. Among other things, in their framework, they skip the intermediate step of identifying the multidimensionally poor in each period, as well as the measurement of distributional intensity.[1]

We illustrate the usefulness of our measurement framework with an empirical application to Chile, relying on its CASEN dataset. The case of Chile is particularly interesting. After the Pinochet regime, the country experienced high levels of GDP growth, improvement in welfare and reduction of income poverty. The income poverty rate halved, from 45.1% in 1987 to 23.2% in 2006 and average GDP growth reached 7.9%.

As of 1997, the Asian crisis slowed down the expansion of the economy and the pace of poverty alleviation. In 1999, the economy shrank by almost 1% and poverty reduction only reached 0.75 points per year in the period 1998–2000. Conversely, after 2000, strong economic growth and a set of well-targeted public policies reduced the incidence of poverty from 20.2% in 2000 to 13.7 in 2006.

In early 2002, a programme called "Chile Solidario" concluded a contract between families and the State. Families committed to meet 53 minimum conditions to overcome extreme poverty and the State was to provide "psychosocial support, protection bonds, guaranteed cash subsidies, and preferential access to skill development, work and social security programmes" (Packard, 2004; Palma & Urzua, 2005). Implicitly, poverty alleviation policies were linked to multidimensional strategies of poverty identification and intervention. However, traditional poverty measurement was still only based on monetary indicators.

Despite further improvements in levels of income, targeted policies proved to be less effective in reducing traditional poverty after 2006. The consolidation of a persistent type of extreme income poverty which is less related to economic development presented a new challenge for public policies. Additionally, during the last decade, traditional income measures have been questioned in Chile. Civil society and academia showed how limited the association of these measures is with people's perception (FNSP, 2010) and that there is a lack of up-to-date techniques and institutions for poverty measurement (CMP, 2014). The use of a consumption bundle from 1987 and external corrections to the income levels are but two examples.

The implementation of a multidimensional poverty measure might help not only the understanding but also the reduction of poverty. In fact, in 2014, the Chilean government and civil society presented a complementary measure of poverty. The new multidimensional measurement of poverty comprises information on education, health, housing and employment. However, only a longitudinal measure can capture the persistent characteristics of poverty in a broader time perspective. Multidimensional analysis of chronicity provides information on families that remain in poverty and it helps the policy maker to acknowledge and learn from those who have overcome poverty.

The next section presents some notation and definitions. Section 3 introduces our class of chronic multidimensional poverty measures. We also introduce a family of transient multidimensional poverty measures. Section 4 discusses the policy relevance of key properties fulfilled by the class of chronic multidimensional poverty indices introduced in the previous section. Section

[1] A discussion of the key differences between our proposal and the measures of Nicholas et al. (2013) can be found in Alkire et al. (2014).

5 offers two empirical illustrations that use ratio scale and, separately, ordinal variables, using the CASEN panel datasets in Chile with observations for 1996, 2001, and 2006. Section 6 concludes.

2. Preliminaries

We have observations on d dimensions or attributes of well-being for a set of N individuals at T different time points. Let x_{ij}^t sand for the quantity of attribute j possessed by person i in period t. Let $\mu(\nu)$ stand for the arithmetic mean of ν. It is assumed that $x_{ij}^t \geq 0 \forall i, j, t$. Let X^t denote the matrix whose ith row is the row vector $x_{i.}^t = (x_{i1}^t, x_{i2}^t, \ldots, x_{id}^t)$. X^t is the $N \times d$ achievement matrix in period t. The distribution of attribute j in period t is represented by the column vector $x_{.j}^t$.

2.1. *The Alkire–Foster dual-cutoff approach to the identification of the multidimensionally poor*

In this multidimensional set-up, a deprivation cut-off z_j is defined for each attribute; these are fixed across periods. These deprivation cut-offs give the minimal quantities necessary to be non-deprived in each attribute. Let $z = (z_1, \ldots, z_d)$ be the vector of deprivation cut-offs in different periods and $z_j > 0 \forall j$. Person i is regarded as deprived in dimension j in period t if $x_{ij}^t < z_j$. Person i is non-deprived in dimension j in period t if $x_{ij}^t \geq z_j$. Note that deprivation cut-offs can be applied to ordinal or cardinal data.

When some data are ordinal or binary – a common situation in multidimensional poverty measurement – we create an $N \times d$ deprivation matrix for period t; $G^t(0)$, whose typical element, $g_{ij}^t(0)$, takes the value of 1 if $x_{ij}^t < z_j$, and 0 if $x_{ij}^t \geq z_j$. If all data are cardinal, we create an $N \times d$ powered deprivation gap matrix for period; $G^t(\alpha)$, whose typical element, $g_{ij}^t(\alpha)$, is constructed as follows. For any triplet (i, j, t), let $x_{ij}^t \equiv \min\{x_{ij}^t, z_j\}$. The powered deprivation shortfall of person i in dimension j at period t is: $g_{ij}^t(\alpha) \equiv \left(1 - \frac{x_{ij}^t}{z_j}\right)^{\alpha}$, where $\alpha \geq 0$. Clearly, individuals deprived in j at t have a positive deprivation gap, whereas otherwise $g_{ij}^t(\alpha) = 0$. Since we are using the Alkire–Foster method of identification and aggregation, we use $g_{ij}^t(\alpha)$.

Different dimensions can be assigned different positive weights in order of importance, such that $\sum_{j=1}^{d} w_j = 1$, where w_j is the non-negative weight assigned to dimension j.

Identification of the multidimensionally poor in period t proceeds according to the following steps. Having defined a d-dimensional column vector of weights: $W = (w_1, w_2, \ldots, w_d)$, we generate an N-dimensional counting vector, $C^t = G^t(0)W'$. A typical element of C^t, e.g. C^t, gives the weighted sum of deprivations for person i in period t. Formally, $c_i^t = \sum_{j=1}^{d} w_j g_{ij}^t(0)$.[2]

Following Alkire and Foster (2011) we identify the multidimensionally poor using a second poverty cutoff k, which is defined as the share of total dimensions in which a person must be

[2] Recall that $g_{ij}^t(0) = 1$ when individual i is deprived in dimension j.

deprived in order to be identified as poor, thus $0 < k \leq 1$. Hence if $0 < k \leq \min\{w_1, w_2, \ldots, w_d\}$, we obtain the union method of identification. And $k = 1$ yields the intersection method.[3]

We apply this cutoff to generate an N-dimensional identification (column) vector for period t, $I^t(k)$, such that a typical element, $\rho_i^t(k)$, is defined by: $\rho_i^t(k) = \mathrm{II}(c_i^t \geq k)$.[4] The identification vector elements take two values: 0 and 1. The entry $\rho_i^t(k) = 1$ if and only if individual i is multidimensionally poor, according to deprivation cut-offs z, weights W and poverty cut-off k; and $\rho_i^t(k) = 0$ otherwise.

2.2. The duration approach

Having identified the poor in every period, the next step is to identify the chronically poor. As mentioned above, we assume that the attribute quantities have been appropriately transformed to take into account variations across time periods (e.g. due to discount factors) and hence for each dimension a common threshold can be used. Let $z = (z_1, z_2, \ldots, z_d)$ be the vector of common deprivation cut-offs.

Given the Alkire–Foster method of identification of the multidimensionally poor, Foster's (2009) duration approach says that a person is chronically poor if she remains in poverty for at least a certain proportion τ of the total number of time periods, T (that is, $0 < \tau \leq 1$). We refer to τ as the duration cut-off. Thus, this is a triple-cutoff approach.

We apply the deprivation cut-off across the number of periods in which each individual is multidimensionally poor. First, we count the periods of poverty by constructing a $N \times T$ matrix, $I(k)$, in which each of the t column vectors is the identification vector for the tth period, $I^t(k)$. Then we generate the N-dimensional chronic counting vector, L, whose typical element, $l_i = \frac{1}{T}\sum_{t=1}^{T}\rho_i^t(k)$, gives the proportion of periods in which person i is multidimensionally poor for a given k. Finally, we apply the cut-off τ to the chronic counting vector, to identify the chronically poor. We generate an N-dimensional column vector, $P^c(k;\tau)$, for the identification of the chronically poor, such that a typical element, $\rho_i(k;\tau)$, is defined by: $\rho_i(k;\tau) = \mathrm{II}(l_i \geq \tau)$. $\rho_i(k;\tau) = 1$ if and only if individual i is chronically multidimensionally poor, according to deprivation cut-offs z, weights W, poverty k and duration cut-off τ.[5]

Finally, let X denote the $(N \times d) \times T$ achievement matrix for all periods. For a given $T > 1$ and >1, we denote the set of all inter-temporal achievement matrices of the form X by M^N.

3. A class of chronic multidimensional poverty measures

Closely following the functional forms proposed by Alkire and Foster (2011) and Foster (2009), we propose the following normalized population average of powered deprivation gaps, in which only the deprivation gaps of the chronically poor are considered. In essence, this measure is the

[3] For a discussion of counting poverty identification methods see Alkire et al. (2015).

[4] $\mathrm{II}(a)$ is an indicator function whose value is 1 if and only if a is true. Otherwise, it is equal to 0.

[5] The measures presented subsequently could also use different identification strategies, such as the average deprivation level across years $\rho_i(k;\tau) = \mathbb{I}(\frac{1}{T}\sum_{t=1}^{T} c_i^t \geq k)$ or the inclusion of a functional form (or weights) to allow for different valuation across years; however, the axioms satisfied by such an approach would change; also, the resulting measures would not be associated with the set of intuitive partial indices of H^C, A^C, D^C presented below.

mean *across people and time* of the weighted sum of deprivation gaps, $\sum_{j=1}^{d} w_j g_{ij}^t(\alpha)$, which are censored for individual i if $\rho_i(k;\tau) = 0$:

$$M_C^\alpha(X; z, W, k, \tau) = \frac{1}{NT} P^{c\prime} \sum_{t=1}^{T} G^t(\alpha) W' \tag{1}$$

where, W' is the transpose of W, $G^t(\alpha)W'$ is a N-dimensional column vector whose typical element is $\sum_{j=1}^{d} w_j g_{ij}^t(\alpha)$, and $P^{c\prime}$ is the transpose of P^c, i.e. a N-dimensional row vector whose typical element is $\rho_i(k;\tau) = \mathrm{II}(l_i \geq \tau)$ as defined in Section 2.2. An alternative way of writing M_C^α is:

$$M_C^\alpha(X; z, W, k, \tau) = \frac{1}{N} \sum_{i=1}^{N} \rho_i(k;\tau) \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{d} w_j g_{ij}^t(\alpha) \tag{2}$$

M_C^α is the population sum of powered censored normalized deprivation gaps divided by the maximum possible value, NT; which arises if and only if $x_{ij}^t = 0 \forall (i, j, t) \in [1, N] \times [1, d] \times [1, T]$, for $\alpha > 0$.[6] If $\alpha = 0$ then the maximum is attained if and only if $x_{ij}^t < z_j \forall (i, j, t) \in [1, N] \times [1, d] \times [1, T]$.

M_C^α is an extension of the Alkire–Foster multidimensional poverty index to chronic poverty and is an extension of the Foster index to the multidimensional space. M_C^α can be expressed in terms of intuitive partial indices that convey meaningful information on different features of a society's experience of chronic multidimensional poverty. We focus particularly on the first measure in our class, the adjusted headcount ratio of chronic multidimensional poverty, M_C^0, because it can be constructed using ordinal data. The multiplicative decomposition is the following:

$$M_C^0(X; z) = \frac{1}{N} \sum_{i=1}^{N} \rho_i(k;\tau) \frac{1}{T} \sum_{t=1}^{T} c_i^t = H^C \times A^C \times D^C$$

where $\boldsymbol{H^C}$ is the headcount ratio of chronic multidimensional poverty, the percentage of the population that are chronically multidimensionally poor according to k and τ:

$$H^C = \frac{1}{N} \sum_{i=1}^{N} \rho_i(k;\tau)$$

$\boldsymbol{A^C}$ is the average intensity of poverty among the chronically multidimenisonally poor, or the share of weighted deprivations that chronically poor people experience in the periods in which they are multidimensionally poor:

$$A^C = \frac{P^{c\prime} \sum_{t=1}^{T} C^t}{T \times P^{c\prime}(k;\tau) L} = \frac{\sum_{i=1}^{N} \rho_i(k;\tau) \sum_{t=1}^{T} c_i^t}{\sum_{i=1}^{N} \rho_i(k;\tau) \sum_{t=1}^{T} \rho_i^t(k)}$$

[6] The intervals $[1, N]$, $[1, d]$ and $[1, T]$ are all subsets of the set of natural numbers.

$\boldsymbol{D^C}$ reflects the average duration of poverty among the chronically poor (i.e. $N \times H^C$)—the average share of T periods in which they experience multidimensional poverty:

$$D^C = \frac{P^{c\prime}(k;\tau)L}{N \times H^C} = \frac{\sum_{i=1}^{N}\rho_i(k;\tau)\sum_{t=1}^{T}\rho_i^t(k)}{N \times H^C \times T}$$

It may also prove useful to assess the duration of dimensional deprivations among the chronically poor. Construct an $N \times d$ censored deprivation duration matrix Q, whose typical entry q_{ij} reflects the share of periods in which person i was chronically poor (by k and τ) *and* was deprived in dimension j. For the chronic poor, $0 \le q_{ij} \le 1$ in each dimension, whereas $q_{ij} = 0$ for non-poor persons in all dimensions. Thus, the matrix has at least one positive entry for $H^C N$ rows, while the rest of the rows, corresponding to people who are not chronically poor, only have zeroes.

Then the dimensional duration index for dimension j is:

$$D_j = \frac{1}{N \times H^C}\sum_{i=1}^{N} q_{ij}$$

The value of D_j provides the average percentage of periods in which chronically poor people are deprived in dimension j. The relationship between the weighted mean across all D_j and the adjusted headcount ratio of chronic multidimensional poverty is elementary:

$$M_C^0 = H^C\sum_{j=1}^{d} w_j D_j \quad \text{And :} \quad \sum_{j=1}^{d} w_j D_j = A^C \times D^C$$

Another interesting relationship between the adjusted headcount ratio of chronic poverty and partial indices pertains to per-period censored headcount ratios. These represent the proportion of people who are chronically poor *and* deprived in dimension j in period t:

$$CH_j^t = \frac{1}{N}\sum_{i=1}^{N}\rho_i(k;\tau)g_{ij}^t(0)$$

Across time, the inter-temporal or longitudinal censored headcount ratio can be defined as:

$$CH_j = \frac{1}{T}\sum_{t=1}^{T} CH_j^t = H_{ch} \times D_{ch} = \frac{1}{N}\sum_{i=1}^{n} I[q_{ij} > 0] \times \frac{1}{N \times H_{ch}}\sum_{i=1}^{N} q_{ij}$$

where H_{ch} is the percentage of individuals who are chronically poor and deprived in at least one period in dimension j over the total population. D_{ch} is the average duration of that deprivation among chronically poor individuals. Weights can be applied to portray the contribution of each dimension to overall chronic poverty in period t. Of tremendous advantage for policy: our chronic multidimensional poverty adjusted headcount ratio across all periods is simply the mean of the weighted average censored headcount ratios across all periods:

$$M_C^0 = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{d} w_j CH_j^t$$

When data are ratio scale and $\alpha = 1$, we compute the adjusted poverty gap, M_C^1, which can also be expressed as follows in an analogous way:

$$M_C^1(X; z, W, k, \tau) = H^C \times A^C \times D^C \times G^C$$

where:

$$G^C = \frac{1}{N \times T \times M_C^0} P^{c\prime} \sum_{t=1}^{T} G^t(1) W'$$

That is, G^C is the average normalized gap that chronically poor people experience in those dimensions in which they are deprived. Likewise, when data are ratio scale and $\alpha = 2$, the adjusted squared gap measure of chronic poverty, M_2^C, is expressed as the product of the following partial indices:

$$M_C^2(X; z, W, k, \tau) = H^C \times A^C \times D^C \times S^C$$

where:

$$S^C = \frac{1}{N \times T \times M_C^0} P^{c\prime} \sum_{t=1}^{T} G^t(2) W'$$

That is, S^C is the average severity, or squared gap, that chronically poor people experience in those dimensions in which they are deprived.

3.1. A class of transient multidimensional poverty measures

Using the same framework we also propose a family of indices of *transient* (multidimensional) poverty,M_{tr}^{α}. The main difference between the two families is in the identification of the poor. We identify a person as transiently poor if $0 < l_i < \tau$. Hence we use a different N-dimensional vector, $P^{tr}(k; \tau)$, for the identification of the transiently poor, such that a typical element, $\omega_i(k; \tau)$, is defined by $\omega_i(k; \tau) = \mathrm{II}(0 < l_i < \tau)$. $\omega_i(k; \tau) = 1$, if and only if, individual i is transiently multidimensionally poor, according to deprivation cut-offs z, weights W, multidimensional cut-off k and duration cut-off τ. The family is:

$$M_{tr}^{\alpha}(X; z, W, k, \tau) = \frac{1}{NT} P^{tr\prime} \sum_{t=1}^{T} G^t(\alpha) W' \tag{3}$$

An alternative way of expressing M_{tr}^{α} is:

$$M_{tr}^{\alpha}(X; z, W, k, \tau) = \frac{1}{NT} \sum_{i=1}^{N} \omega_i(k; \tau) \sum_{t=1}^{T} \sum_{j=1}^{d} w_d g_{ij}^t(\alpha) \tag{4}$$

4. Policy relevance of key properties

The class of indices $M_C^{\alpha}(X; z, W, k, \tau)$ satisfies a set of desirable properties included in Alkire, Apablaza, Chakravarty, and Yalonetzky (2014). In this section we discuss the policy relevance of a handful key properties. Firstly, all indices in our class fulfil Additive Subgroup Decomposability (ASD), which implies that for any partitioning of the population into $m(\in \mathbb{N})$ subgroups, overall chronic poverty is given by the population-share weighted average of the subgroup chronic poverty levels. Thus, if chronic poverty in one subgroup decreases (increases), while remaining unchanged in other subgroups, then global poverty falls (rises). For designing poverty alleviation policy it becomes appropriate to isolate the population subgroups and/or dimensions that are more afflicted by chronic poverty. Given that M_C^α fulfils ASD, the percentage contribution of subgroup i to total poverty is given by $\frac{N_i M_C^\alpha(X_i;z,W,k,\tau)}{NM_C^\alpha(X;z,W,k,\tau)}100$ (where i represents subgroup i). Total chronic poverty will reduce by this percentage if poverty in subgroup i is eliminated.

Secondly, when $\alpha=0$, the index satisfies a dimensional decomposability condition which says that the overall index can be expressed as a weighted sum of chronic dimensional indices (Chakravarty, Mukherjee, & Ranade, 1998). The percentage contribution of dimension j to the overall index is given by $\frac{w_j \sum_{t=1}^{T} CH_j^t}{TM_C^0(X;z,W,k,\tau)}100$. These statistics become helpful in identifying the dimensions that contribute more to the overall chronic poverty and hence in formulating relevant anti-poverty policies.

Thirdly, when $\alpha \geq 1$, the indices satisfy a property of chronic strong transfer (CHTS), whereby a reduction in the degree of inequality among the poor decreases the value of the index. Essentially, this property ensures that the poorest among the poor are prioritized by policy whenever the attributes are measured with cardinal variables, in the sense that the poverty index will decrease further whenever people suffering from more acute deprivations are targeted first.

Generally, a complementary form of prioritization of the poorest among the poor, for policy purposes, can also be achieved for any $\alpha \geq 0$, by a combination of: (1) increasing the value of any deprivation cut-off z; (2) increasing the value of the multidimensional cut-off k; and/or (3) increasing the value of the duration cut-off τ. Any of these three adjustments is bound to keep or reduce the value of the chronic poverty index, thereby identifying a narrower, but more chronically deprived, group of people.

5. Empirical illustration

We illustrate the usefulness of our class of chronic multidimensional poverty indices with a case-study of Chile, relying on a panel dataset with data points in 1996, 2001 and 2006. We provide one empirical illustration with ordinal variables and another one with cardinal variables. The next subsection discusses the data and the choice of well-being indicators. Then the application with ordinal variables is presented, followed by the application with cardinal variables. We also provide estimates of dimensional and period contributions to overall chronic multidimensional poverty. This section illustrates the proposed indices, and provides tables and figure showing its consistent partial and sub-indices. It does not propose an ideal set of indicators or index specifications for use in policy.[7]

5.1. Data and indicators

The CASEN (National Survey of Economic Characterization) panel follows households in three regions (covering 60% of Chile's population) in three rounds: 1996, 2001 and 2006. The

[7] To do so it would be necessary to clarify the purpose of the exercise, to justify the selection of the unit of identification, indicators, and deprivation cutoffs, both normatively and in light of the changing composition and demographic structure of households in the panel.

panel survey began with a representative subsample of 5209 households (20,942 individuals) based on the cross-sectional survey of 1996.[8] Three GDP growth experiences can be identified in the period in question. First, 1996 marks the beginning of one of the most successful decades of GDP growth and income poverty reduction in Chile (Contreras, 2003; Contreras, Larrañaga, Litchfield, & Valdés, 2001). In 2001 the country suffered from the negative impact of the Asian crisis (Corbo & Schmidt-Hebbel, 2011), and in 2006 a public policy response to lower growth rates was implemented (Galasso, 2011; Glick & Menon, 2009).

We provide two illustrations of the chronic poverty indices. In the first illustration we use ordinal, categorical or binary variables, and hence calculate only M_c^0 and M_{tr}^0. In the second, three continuous variables are used, hence we can potentially compute M_c^α and M_{tr}^α for any value of α, thus generating information on the breadth and severity of chronic poverty.

We use three equally-weighted dimensions: education, housing and employment/income. The selection of dimensions and indicators is consistent with the national measure of multidimensional poverty of 2014. There are only two important differences. First, we excluded the health dimensions due to the lack of comparable objective data. Second, we included income levels into the multidimensional measure to capture short-term changes in the labour market. Additionally, with the help of the traditional framework the income level allows us to foresee the ability of a family to reach a basic food basket and consequently meet its caloric needs.

The selection of indicators is also related with the public policies implemented by the programme "Chile Solidario" and our indicator choices were partly guided by the reliable information available in the dataset, across years. Each dimension comprises information on short- and long-term indicators. In education, for instance, school attendance captures short-term changes in public policies and schooling can be improved only in the longer run. For the ordinal illustration three equally-weighted indicators are selected in each dimension; for the cardinal illustration one indicator is used per dimension. Table 1 presents the indicators for both illustrations together with their uncensored headcount ratios.

[8] The survey is deemed one of the longest panel datasets for a developing country with longitudinal and cross-sectional representativeness (Dercon & Shapiro, 2007). By design, it tends to overestimate income poverty levels vis-à-vis national ones by approximately 5%. Inflation factors were produced in order to adjust for attrition among young (20–29 years) and elderly people (over 60) in large households, and in rented dwellings (Bendezu, Denis, Sanchez, Ugalde, & Zubizarreta, 2007). To correct for attrition, sample weights for longitudinal consistency were implemented; consequently, results are not comparable with cross-sectional data from 2006.

[9] In parentheses: lower and upper 95% confidence intervals.

[10] In 1920, the Law 3.654 defines primary education as compulsory. In 1929, the Decree 5.291 extends this regulation to 6 years. Then, in 1965, Government Decree 27.953 increases the levels of compulsory education to 8 years. Finally, in 2003, the Constitutional Law 19.876 sets the minimum compulsory schooling to 12 years.

[11] The Chilean Government defined a set of policies to promote literacy regardless the age of the individuals (Contigo Aprendo). This indicator differs from schooling because it tries to capture the skill of literacy of each individual in the household. Consequently, if one individual is deprived the entire household is deprived. Conversely, in the schooling indicator if one individual has enough school the household is immediately non-deprived.

[12] Available at http://www.ministeriodesarrollosocial.gob.cl/casen/definiciones/vivienda.html.

[13] Deprived walls: adobe, wall without interior protection, mud, thatch, artisanal construction, rubbish, cardboard, tin or rubber. Deprived roof: clinkstone, straw, bulrush, rubbish or cane. Deprived floor: no protected cement foundation.

[14] There is no additional qualitative information regarding the type of toilet.

[15] In Narayan (2000), individuals remark about the relevance of employment not only for the pecuniary benefits but also due to social and other outcomes.

Table 1
Dimensions, indicators, weights and uncensored headcount ratios.

Dimension	Indicator	Deprivation cut-off: *An individual is deprived if he/she lives in a household with...*	Weights		Uncensored headcount ratios[9]		
			Cardinal illustrat.	Ordinal illustrat.	1996	2001	2006
Education	Educational achievement	No household member fulfilling the legal number of compulsory years of education relevant to their birth cohort[10]	1/3	1/9	8% (7%–10%)	6% (5%–7%)	5% (4%–6%)
	School attendance	At least one individual of school age (6–17 years) not attending school, or evidencing a gap greater than 3 years between his/her highest achieved school year and the appropriate school year by the individual's age		1/9	9% (7%–10%)	7% (5%–9%)	5% (4%–7%)
	Illiteracy	At least one member older than 17 not able to read or write[11]		1/9	8% (7%–10%)	7% (5%–8%)	5% (4%–6%)
Housing	Overcrowding	More than 2.5 persons per bedroom as defined by the Chilean Ministry of Social Development[12]	1/3	1/9	17% (14%–20%)	12% (10%–14%)	8% (7%–10%)
	Shelter	Insufficient housing materials as defined by the Chilean Ministry of Social Development[13] (one or more deprived indicators for walls, floor or roof)		1/9	44% (39%–48%)	37% (33%–42%)	38% (34%–42%)
	Toilet	At least 1 toilet in the household[14]		1/9	19% (15%–23%)	12% (10%–15%)	6% (4%–7%)
Income–employment	Income	A per capita income lower than the relevant national poverty line defined by the Social Planning Ministry	1/3	1/9	24% (20%–27%)	21% (17%–24%)	11% (9%–12%)
	Unemployment	No member older than 17 is employed[15]		1/9	6% (5%–7%)	10% (8%–12%)	8% (7%–10%)
	Quality of employment	No member older than 17 has access to the pension system or has signed contract—excluding rentiers, pensioners and entrepreneurs as defined by the Chilean Law		1/9	22% (19%–26%)	23% (19%–26%)	22% (19%–25%)

Table 2
Cross-sectional and longitudinal poverty measures with $k = 3/9$.

Main statistics	Cross-sectional results			Longitudinal results		
	1996	2001	2006	$t = 1/3$	$t = 2/3$	$t = 1$
Headcount ratio (H/Hc)	13.9%	10.0%	5.0%	18.3%	8.5%	2.8%
Duration (Dc)	–	–	–	53.8%	77.5%	100.0%
Intensity (A/Ac)	51.1%	49.9%	50.0%	50.4%	51.8%	52.5%
Adjusted headcount ratio ($M_0/M_0{}^c$)	0.071	0.050	0.028	0.050	0.034	0.015
Censored headcount	**Cross-sectional censored headcount**			**Longitudinal censored headcount**		
Overcrowding	6.5%	4.9%	2.3%	4.5%	2.9%	1.1%
Housing	13.1%	9.1%	5.1%	9.1%	6.0%	2.4%
Toilet	10.3%	6.6%	2.6%	6.5%	4.4%	1.6%
Attendance	4.3%	2.5%	1.9%	2.9%	2.0%	1.0%
Schooling	5.4%	3.3%	2.2%	3.6%	2.8%	1.5%
Illiteracy	4.2%	2.7%	1.5%	2.8%	2.2%	1.2%
Employment	1.9%	2.3%	1.9%	2.0%	1.4%	0.8%
Employment quality	8.0%	6.2%	4.0%	6.1%	4.3%	1.8%
Income	10.3%	7.3%	4.0%	7.2%	4.6%	1.8%
Percentage contribution	**Percentage contribution to M_0**			**Percentage contribution M_0^c**		
Overcrowding	10.1%	10.4%	8.2%	10.2%	9.5%	8.2%
Housing	20.3%	19.5%	18.8%	20.3%	19.6%	18.6%
Toilet	16.1%	14.1%	9.5%	14.6%	14.4%	12.4%
Attendance	6.7%	5.4%	7.1%	6.5%	6.4%	7.4%
Schooling	8.4%	7.0%	7.9%	8.1%	9.2%	11.1%
Illiteracy	6.6%	5.7%	5.3%	6.2%	7.1%	9.3%
Employment	2.9%	5.0%	7.0%	4.6%	4.7%	5.8%
Employment quality	12.5%	13.2%	14.7%	13.6%	13.9%	13.4%
Income	16.0%	15.5%	14.5%	16.0%	15.1%	13.9%

5.2. *Ordinal illustration*

Table 2 shows the longitudinal results of chronic multidimensional poverty using the ordinal specifications and with $k = \frac{3}{9}$. Cross-sectional multidimensional poverty falls from 0.071 to 0.028 between 1996 and 2006. Most of the improvement is due to a lower headcount ratio.[16] The largest contributors to multidimensional poverty are housing, toilet, overcrowding, quality of employment and income.

The longitudinal results show that under the time union approach ($\tau = \frac{1}{3}$), 18.3% of the population is poor, experiencing poverty spells during 53.8% of the periods in 50.4% of the possible dimensions.[17] The chronic adjusted headcount ratio in this case is 0.05. When $\tau = 1$, only 2.8% of the population is chronically poor and in 52.5% of their dimensions.

Fig. 1 displays the transitions into and out of poverty spells in a way that highlights the connection between the year-specific poverty headcounts and their chronic counterparts for different choices of τ, similar to the Venn diagram in Fig. 2.

[16] This section uses the term 'headcount' as an abbreviation for headcount ratio.
[17] We note that the concept of chronic poverty would only be meaningful when $\tau > \frac{1}{T}$.

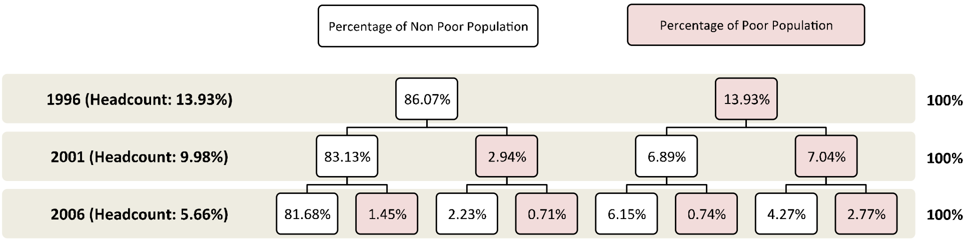

Fig. 1. Transitions entry and exit from multidimensional poverty ($k = \frac{3}{9}$).

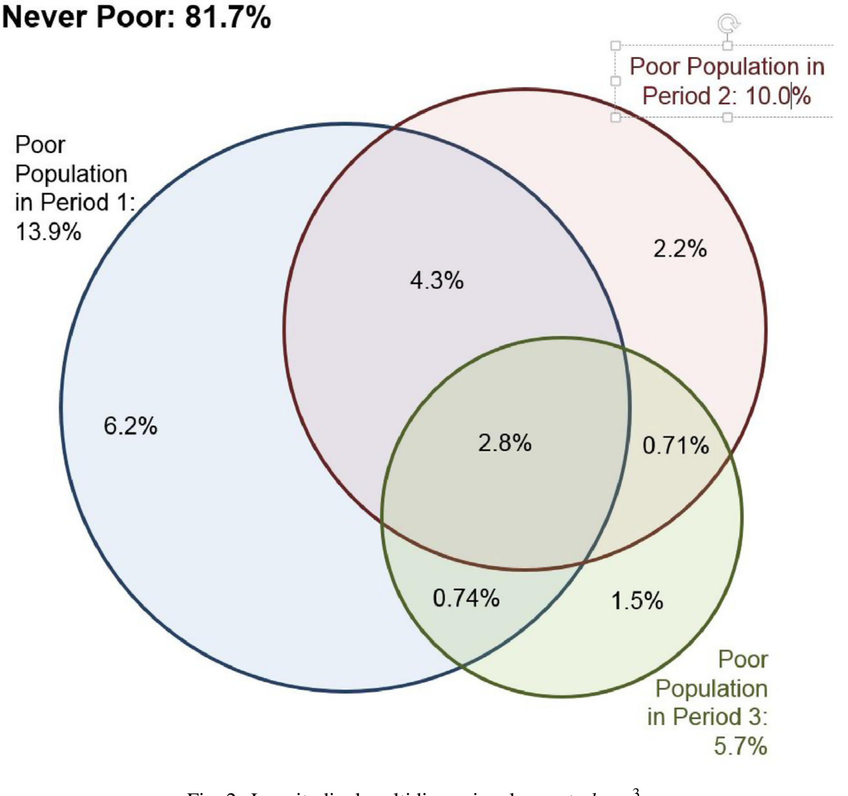

Fig. 2. Longitudinal multidimensional poverty $k = \frac{3}{9}$.

For instance, with$\tau = \frac{1}{3}$, the chronic poverty headcount of 18.3% is equal to the headcount of 1996 (13.93%) plus the new poor in 2001 (2.94%) and the new poor in 2006 (1.45%). With $\tau = 1$, the chronic poverty headcount is compounded by those who were always poor (2.8%). The longitudinal intersection approach suggests that with $\tau = \frac{2}{3}$, the chronic poverty headcount of 8.5% is equal to the percentage of individuals who are always poor (2.8%) plus those who were poor in the first and last period (0.74%) and those who became poor in the second period and remained in that condition until the last period (0.71%).

Table 3
Chronic and transient poverty for selected groups with $k = \frac{3}{9}$.

	Only once poor	Only twice	Once or twice	Chronic ($t=2/3$)	Always poor	Fall[a] 011	Rise[b] 110	Churn[c] 101
Headcount ratio (Hc)	9.8%	5.7%	15.5%	8.5%	2.8%	0.7%	4.3%	0.7%
Duration (Dc)	33.3%	66.7%	45.6%	77.5%	100.0%	66.7%	66.7%	66.7%
Av dep share (Ac)	47.8%	51.3%	49.7%	51.8%	52.4%	49.4%	52.1%	48.3%
Adj headcount ratio (M_0^c)	0.016	0.020	0.035	0.034	0.015	0.002	0.015	0.002

[a] Fall 011: Non-poor in 1996, then poor in the subsequent periods.
[b] Rise 110: Poor in 1996 and 2001, then non-poor in 2006.
[c] Churn 101: Poor in 1996, non-poor in 2001, poor in 2006. These three subgroups sum to the "only twice" poor.

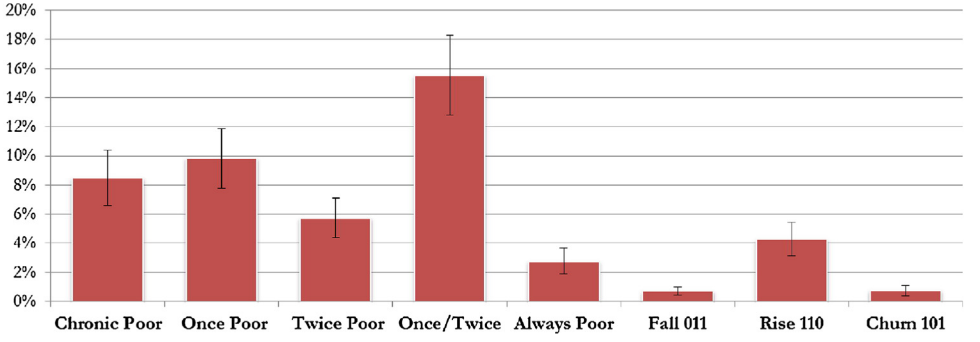

Fig. 3. Headcount ratios of chronic and transient poverty for selected groups with $k = \frac{3}{9}$.

Following Figs. 1 and 2, we can compute different headcount ratios of chronic and transient poverty using different time cut-offs. For each group of poor people we can also compute measures of incidence, duration and intensity using the methods described in Section 4.

Table 3 shows the adjusted headcount ratio and its components for different groups of poor people identified by different criteria of chronicity and transiency, and Fig. 3 depicts the headcount ratio with confidence intervals at 95%. Clearly, transient poverty is more prevalent than chronic poverty, although the average intensity of poverty (second-to-last row in Table 3) is lower among the chronically poor in this Chilean case.

Additionally, we can assess the contribution of each deprivation to the adjusted headcount ratio of each one of the above poverty groups. The contributions are based on the censored headcounts, i.e. the proportions of people who are poor (e.g. chronically or transiently) *and* deprived in a specific variable.

Fig. 4 shows the composition of poverty among those groups. Interestingly, we can see that those who fall into poverty have much larger deprivations in employment, suggesting that many of them lost their jobs. Among the always poor we see the highest contribution of the education deprivations, suggesting either that measured education is a stock variable, or that there were few educational opportunities for those who dropped out of school. More censored headcounts and relative contributions, for different choices of k and τ are available in the Appendix of Alkire et al. (2015).

The contribution results are based on the longitudinal censored headcount of each indicator, and they can be calculated as the average of censored headcounts across time for those individuals

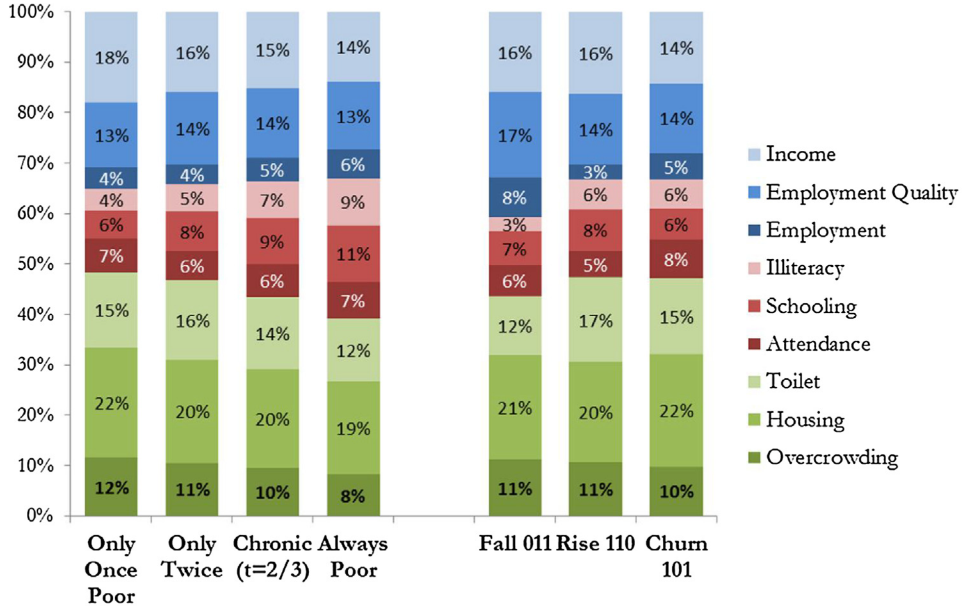

Fig. 4. Dimensional breakdown of longitudinal poverty in selected poverty groups ($k = \frac{3}{9}$).

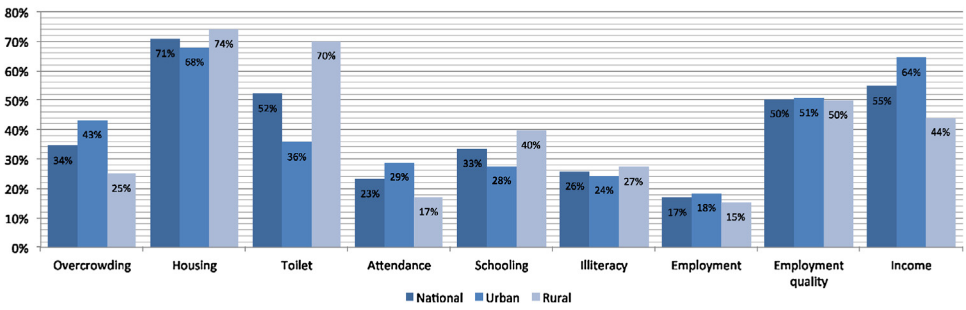

Fig. 5. Duration of deprivation (D_j) in chronic multidimensional poverty by zone ($k = \frac{3}{9}$, $\tau = \frac{2}{3}$).

living in each condition of chronic or transient poverty. We now present the new statistics that capture explicitly the duration of the deprivation.

Fig. 5 shows the duration of the deprivation in each dimension (D_j) at the national, urban and rural levels.[18] The figure shows the persistence of each deprivation among those individuals who are identified as chronic multidimensional poor. On an average, an individual in chronic multidimensional poverty is deprived in overcrowding 34% of the periods. For urban and rural areas, the respective values are 43% and 25%. Housing shows the highest duration at the national level. The duration of deprivation is higher in urban areas for overcrowding, school attendance, employment and income.

Fig. 6 plots the level and duration of dimensional deprivations among the chronically poor. The vertical axis shows the percentage of people who are chronically poor and deprived in each indica-

[18] Note that this example is illustrative; the sample may not be representative at higher levels of geographic disaggregation.

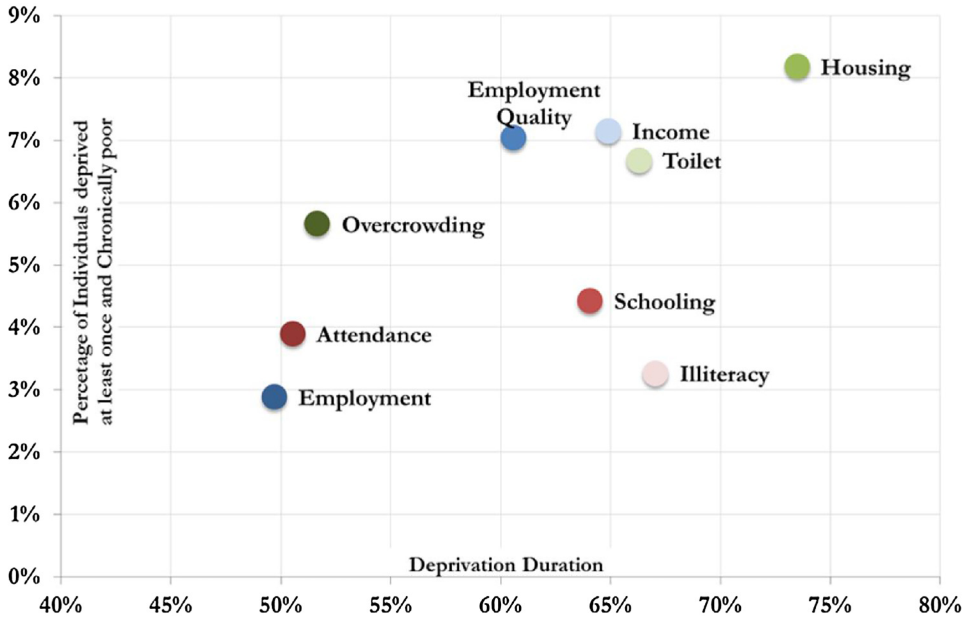

Fig. 6. Indicator censored headcounts (H_{ch}) and duration (D_{ch}) of chronic poverty ($k = \frac{3}{9}$, $\tau = \frac{2}{3}$).

tor for at least one period (H_{ch}). The horizontal axis shows the average duration of the deprivation in this indicator (D_{ch}). For instance, more than the 8% of the population have experienced housing deprivation and chronic poverty; on an average, they have been deprived in housing for 74% of the periods. It is important to note that the percentage of individuals deprived in employment and illiteracy are similar (around 3%). However, illiteracy is a more persistent deprivation.

Finally, Fig. 7 illustrates a three-dimensional graphic of four regions according to the average deprivation share (A^c)—or intensity, the duration of poverty, and the chronic poverty headcount ($k = \frac{3}{9}$, $\tau = \frac{2}{3}$). The metropolitan region has the lowest headcount but the highest duration and an intermediary intensity. Compared to the metropolitan region, the III region has twice the percentage of poverty but with a lower duration and intensity. Region VII has the highest proportion of chronically poor people (nearly 21%), but its duration is below that of the Metropolitan region. In each case, the volume represented by the headcount times the duration times the intensity represents the level of multidimensional poverty.

Fig. 7 have illustrated the possibility of examining the duration of chronic poverty across subnational regions or dimensions (or both). Such information from a well-specified measure may indeed provide powerful and useful in policy, particularly for those seeking to ‘leave no one behind’.

Results suggest that 2.7% of individuals remain in poverty in all periods and they are mainly deprived in housing and employment. Chronic poverty is not only related to the economic cycle through unemployment and income, but also to the capacity of a family to find a dwelling with a set of minimum characteristics. The analysis by indicator highlights the persistence of deficient housing and toilet facilities, especially in rural areas (VII and VIII regions). Interestingly, it also informs on difficulties to meet a minimum income and basic employment conditions.

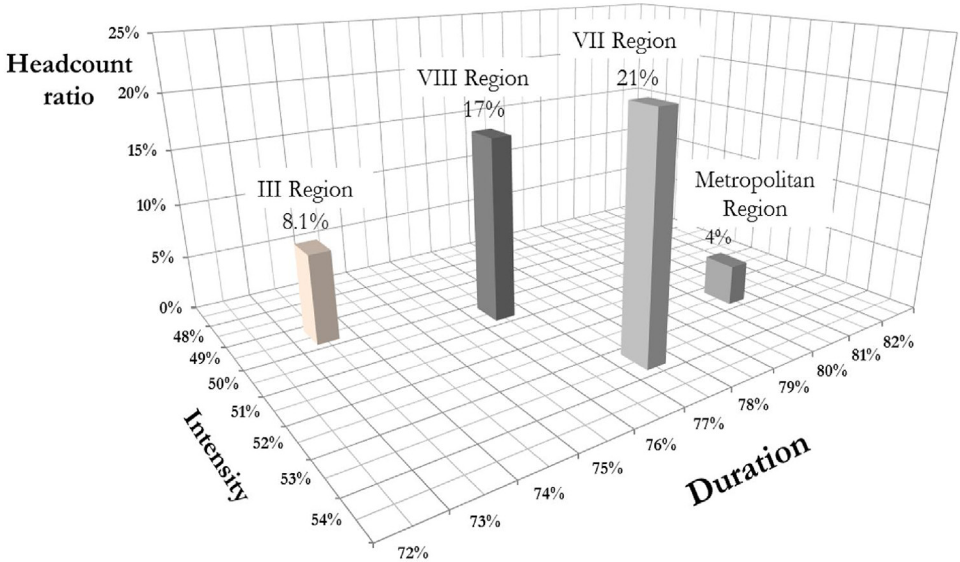

Fig. 7. Chronic multidimensional poverty by region ($k = \frac{3}{9}$, $\tau = \frac{2}{3}$).

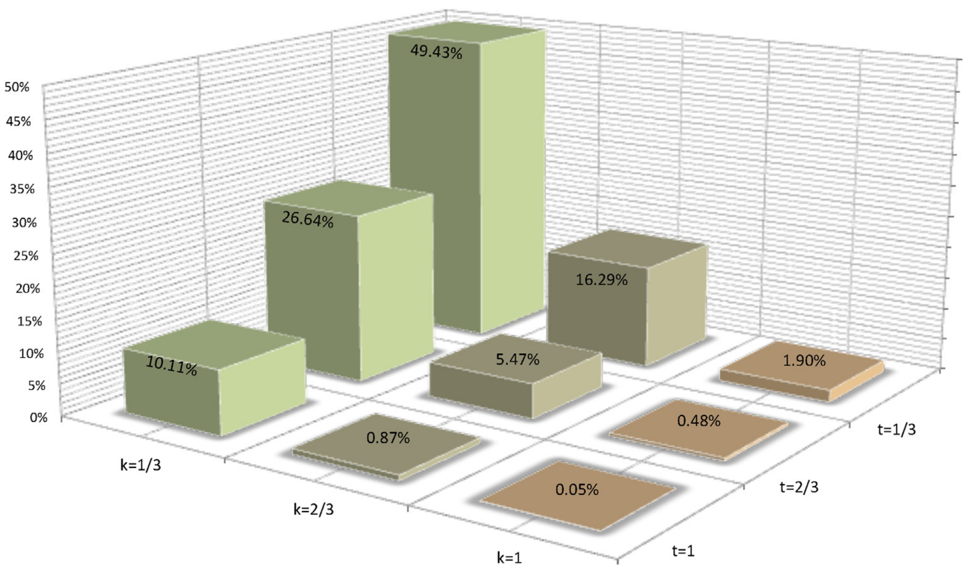

Fig. 8. Headcount ratio with all possible poverty (k) and time (τ) cut-offs.

5.3. Cardinal illustration

We now turn to illustrate the additional analyses undertake when variables are cardinal and $\boldsymbol{\alpha} \geq 0$ using the three variables described in Table 1: income, schooling, and overcrowding.

Beginning with the previous measure in which $\alpha = 0$, Fig. 8 and Table 4 show the headcount ratio for all possible combinations of poverty (k) and time (τ) cut-offs. A double union approach

Table 4
Cardinal illustration with relevant values of k and τ.

Poverty cutoff (k)	$k = 1/3$			$k = 2/3$			$k = 1$		
Duration cutoff (τ)	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$	$\tau = \frac{1}{3}$	$\tau = \frac{2}{3}$	$\tau = 1$
Headcount ratio (Hc)	49%	27%	10%	16%	5%	1%	0%	0%	0%
Duration (Dc)	58%	79%	100%	46%	72%	100%	43%	70%	100%
Intensity (Ac)	43%	45%	48%	70%	72%	75%	100%	100%	100%
Normalized gap (Gc), $\alpha = 1$	0.138	0.153	0.183	0.233	0.265	0.288	0.363	0.367	0.368
Squared gap (Sc), $\alpha = 2$	0.067	0.077	0.095	0.110	0.132	0.144	0.181	0.187	0.159
Adj headcount ratio ($M_0{}^c$)	0.124	0.095	0.049	0.053	0.028	0.007	0.008	0.003	0
Adj gap ratio ($M_1{}^c$)	0.040	0.032	0.018	0.018	0.010	0.002	0.003	0.001	0
Adj squared gap ratio ($M_2{}^c$)	0.019	0.016	0.010	0.008	0.005	0.001	0.001	0.001	0

($k = \frac{1}{3}$ and $\tau = \frac{1}{3}$) identifies 49.4% of the population as chronically poor with an average duration (D^c) of 58.1% periods and an intensity (A^c) of 43.0%. On the other extreme, a double intersection approach ($k = 1$ and $\tau = 1$) identifies only 0.05% of the population as chronically poor, with an average duration and intensity equal to 1. With an intermediate approach of $k = \frac{2}{3}$ and $\tau = \frac{2}{3}$, 5.5% of the population would be identified as chronically poor with an intensity of 72.1% and a duration of 72.0%.

Table 4 presents the findings for all values of the poverty and duration cutoff combinations, and for $\alpha = 0$, 1, 2. We saw in the ordinal illustration how to interpret the headcount ratio, average duration of chronic poverty, and average intensity of deprivation among the chronically poor. We move now to interpret the average normalized gap. For example, for $k = \frac{2}{3}$ and $\tau = \frac{2}{3}$, it is 26.5%. This means that on average, poor people's achievements fall 26.5% below the deprivation cutoff in their indicators, in the periods in which they were poor and deprived in each indicator. We see, sadly, as the poverty cutoff and duration cutoffs rise, that the average gap increases. That means that the people who are deprived in a greater share of deprivations, and experience those deprivations for a longer duration, on average have achievements that fall shorter and shorter of the deprivation cutoffs of each indicator. *This is not necessarily intuitive*. We know that intensity rises as k rises. Yet we might reasonably expect that the people who are deprived in several dimensions (or over long periods) fall less short of the deprivation cutoff than those with fewer periods and deprivations, not more. This analysis brings into view a finding which, if replicated in different datasets, is indeed troubling but policy-relevant to leaving no one behind: *the chronically poor are more deeply deprived in breadth as well as intensity*.

Moving now to the squared gap, we see as expected that it follows the gap and increases across cutoffs. However if we were to compare the ratio of the measures, we would see that the squared gap increases only mildly with increases in the poverty and duration cutoffs, relative to the normalized gap. Thus, in this dataset, the inequality among the poor exists, but is relatively low. Put differently, there are relatively few poor people whose achievement levels are dramatically beneath the average normalized gap. This is quite a meagre achievement, however, given the extent and duration of deprivations and their average gap shown by the other indicators.

Fig. 9 depicts the same transitions as were presented for the ordinal case, for the case of identification of who is poor using a different measure, with only three indicators. We see that 10.1% of people are multidimensionally poor in all three periods whereas 49.4% of people are multidimensionally poor by a union approach, in at least one of the three.

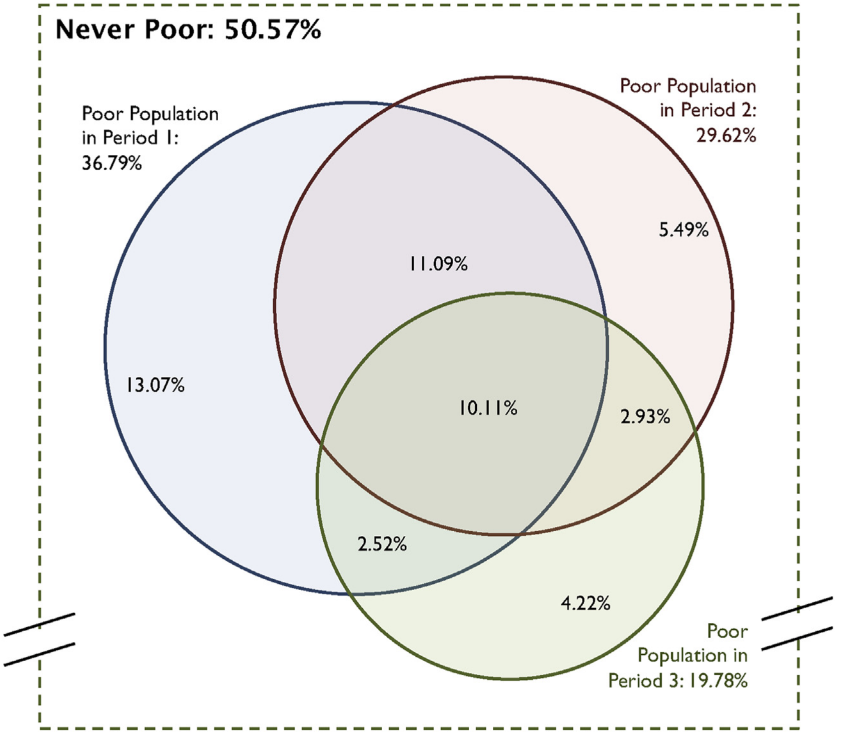

Fig. 9. Multidimensional transitions 1996–2001–2006 ($k = \frac{1}{3}$).

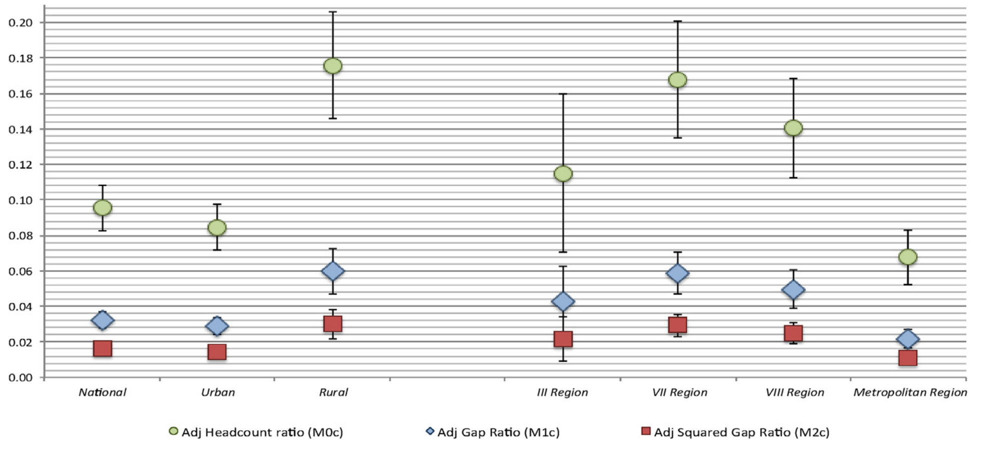

Fig. 10. Chronic multidimensional poverty by region with $k = \frac{1}{3}$ and $\tau = \frac{2}{3}$, for different α.

Fig. 10 illustrates some regional breakdowns of chronic poverty for $\alpha = 0, 1, 2$, using $k = \frac{1}{3}$ and $\tau = \frac{2}{3}$. We see that in this case the ranking of the regions is similar across all three measures—the adjusted chronic multidimensional headcount ratio, poverty gap, and squared gap measures.

In addition to providing information on the percentage of chronically poor individuals, the experimental cardinal measure compares the depth of poverty in each geographic area. In the case of Chile, despite similar results in terms of the percentage of chronically poor individuals, it is clear that the severity is stronger in rural and non-metropolitan areas. Incidence results practically coincide with previous literature (Neilson, Contreras, Cooper, & Hermann, 2008). However, we extend the period of analysis and provide extra information regarding the composition and evolution of the different traits of poverty. The additional information might help to connect current public policies with the outcomes and to design new dimensionally or subgroup targeted interventions.

6. Conclusions

It has been argued explicitly in the literature that poverty should be measured multidimensionally in terms of shortfalls of well-being attributes from minimally acceptable levels defined for different individuals in a society. Since, for many people worldwide, poverty is a situation from which it is difficult to escape over time, often it becomes important to track it over multiple periods. This, of course, requires panel data on different dimensions of well-being.

Following Foster's (2009) income-based analysis, we have considered the spell, or duration, approach to chronic multidimensional poverty. We have defined multidimensional poverty following Alkire and Foster (2011). Two notions of identification are present: the identification of the multidimensionally poor in each period and the minimum number of periods a person has to spend in poverty in order to be identified as chronically poor. The properties of the proposed class of chronic poverty measures are in some cases jointly restricted by this triple-cutoff identification approach as well as the aggregation method.

The indices of chronic and transient poverty proposed in this paper represent the most straightforward merger between the snapshot multidimensional poverty and the duration approaches to chronic poverty. Being both counting approaches to poverty measurement, they blend naturally. Besides, our indices of chronic poverty satisfy a set of relevant axioms, among which chronic strong transfers, dimensional breakdown and additive subgroup decomposability seem to be crucial requests for a policy-pertinent measure of poverty (Alkire & Foster, 2013; Chakravarty et al., 1998).

The implementation of the multidimensional poverty measure in Chile helps to understand the phenomena of chronic multidimensional poverty, complementing the use of income as the sole indicator of wellbeing manifold. Firstly, it shows which areas are less affected by fast economic development and, at the same time, facilitate the implementation of targeted policies in terms of population as well as dimensions of welfare. Secondly, it combines information of the income measure with households' and individuals' long-term characteristics unaffected by the economic cycle, such as education and quality of the dwelling. Finally, it provides additional short-term information about the evolution of welfare and the accountability of public programmes hidden behind traditional income measures, such as school attendance, for instance.

Besides the aforementioned benefits, a longitudinal measure also presents the possibility to identify and describe the harshest forms of poverty, in terms of breadth and persistence, simultaneously. It provides information on possible strategies of intervention and design of safety nets for families leaving poverty or becoming poor, respectively.

Alternative counting-based measures could also be explored. For example, by altering the order of aggregation in this paper (first across dimensions and then across time), one could aggregate across deprivations inter-temporarily first, then construct a chronic multidimensional poverty measure that identified as deprived anyone who had experienced deprivations for τ or more periods per dimension in at least k dimensions. This class of measures can easily be implemented, and could be explored (see Apablaza & Yalonetzky, 2012). We chose the former order of aggregation because policy actors must monitor and analyse change in the most-recent period in comparison with others, which the class of measures proposed in this paper permit. Future research should study the theoretical, empirical and policy implications of combining different approaches to the identification and measurement of multidimensional poverty with different ways of understanding, indentifying and measuring chronic and transient poverty.

Acknowledgements

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