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1. Abstract

This paper focuses on the relationship between the venture capitalist and the entrepreneur. In particular, it analyses how both players' unobservable effort levels affect the equity share that the entrepreneur is willing to cede to the venture capitalist. We solve the entrepreneur's maximization problem in the presence of double-sided moral hazard. In this scenario, we show that the venture capitalist's share is binding and, therefore, there is no efficiency wage. We simulate the model and show that the entrepreneur's effort does not monotonically decrease in the share allocated to the venture capital, while the venture capitalist's effort does not monotonically increase in his share. We show that as efforts tend to be more complementary, the project cash flows are distributed nearly equally, at approximately 50% for each partner. This theoretical finding is actually observed in real contracts between entrepreneurs and venture capitalists.

Key words— Double-sided moral hazard; Venture Capital; Equity Share.

2. Introduction

Entrepreneurship is the driving force of economic growth. The entrepreneur's role in the process of development has been for long emphasized in the literature. Schumpeter (1934) argues that the existence of entrepreneurs, who innovate, generates the process of "creative destruction" by which new innovations cause constant change in the marketplace, which result in the exit of existing firms and the entry of new ones. Baumol (2002) argues that, through innovation, entrepreneurs are the engine of growth. Acs (2006) illustrates the way entrepreneurship is good for economic growth.

Over the past 30 years, the Venture Capital industry has played a key role on providing financing for entrepreneurs. Companies such as Google, Intel, FedEx, Apple, and Microsoft, to name a few, have all been backed by Venture Capitalists (hereinafter "VCs"). The VC industry has grown dramatically in the last decades. In particular, VC investments grew from $20 billion in 1985 to $0.6 trillion in 2014 (NVCA, 2015). Also, the number of VC-backed companies as percentage of U.S. public companies that were founded after 1979 is 42% and account for the 63% of total market capitalization. These VC-backed companies provide the 38% of the total employment and spend the 85% of total research and development (Gornall and Strebulaev, 2015). All of this highlights the importance of VC in the entrepreneurship and economic growth process.

Although the importance of the entrepreneur-VC relationship, the topic of how they share the equity of the new venture has received little attention from a theoretical point of view. The result of this allocation affects the incentives that both partners confront and thus, has major effects on the effort levels that the partners will exert in the new endeavor. In this paper, we tackle this subject emphasizing the importance of complementarity between the entrepreneur and the VC, and how it impacts the share allocation.

The literature recognizes the extra-financial value of venture capital. VCs dedicate a significant amount of time to managing their portfolios (Gorman and Sahlman, 1989). The advisory services which VCs provide become a key factor for the success of a business. As stated by Casamatta (2003), entrepreneurs are endowed with creativity and technical skills in developing innovative ideas, but they often lack business experience and require the assistance that VCs can offer. VCs provide marketing, networking, a market for the product and consulting experience, while entrepreneurs possess skills in technology and production and experience in innovation (Fairchild, 2011). The synergy that is generated by the complementarity between entrepreneurs'abilities and VCs'experience has a positive effect on the market value of the enterprise. VCs that are part of networks enjoy higher quality relationships, a set of investment opportunities, and access to information while improving the firm's cash flows (Hochberg, Ljungqvist and Lu, 2007).

When a VC funds an entrepreneur, the latter must transfer shares in the project ownership as compensation for the advisory services and financing provided by the VC. This generates a double-sided moral hazard problem. This phenomenon occurs because the entrepreneur's effort is not observable by the VC nor is the VC's effort observable by the entrepreneur. Casamatta (2003) advances a theory to describe the dual role of the VC, namely providing funding and advisory services. Casamatta (2003) argues that if the entrepreneur is more efficient than the VC, the entrepreneur will not contract the VC, meaning that he will not transfer a share of the project cash flows unless the VC contributes capital to fund the project.

Gavious and Elitzur (2003) analyze the contractual relationship between a VC and a entrepreneur. Moral harzard shows up in the model because the VC does not observe the effort of the entrepreneur. However, the model does not incorporate the VC effort. Thus, moral hazard runs in one direction.

De Bettignies and Brander (2007) develop a model in which the entrepreneur must choose between VC funding or bank financing. Unlike a bank, a VC provides advisory services to the entrepreneur. However, the VC's effort is not observable, which creates another potential moral hazard. The entrepreneur's effort is also not observable, and hence also creates a potential moral hazard. De Bettignies and Brander (2007) emphasize the double-sided moral hazard problem and a strategy to induce efficient effort levels in this scenario. The eventual ownership structure of the firm will be determined by the way in which incentives are aligned. When the VC owns a greater share of the business, his effort level is improved, but this reduces the entrepreneur's level of effort. Bank financing will give the entrepreneur complete control over the business, but this leaves the project without the advisory services provided by the VC.

It is important to highlight that De Bettignies and Brander (2007) fail to solve the double-sided moral hazard problem faced by the entrepreneur. They only work at the level of the participation constraints, which is why their model gives solutions, concerning the share given to the VC that includes real negative numbers or complex numbers. As in Casamatta (2003), De Bettignies and Brander assume that the players' efforts are perfect substitutes, meaning that in

this scenario it makes no sense to speak of the entrepreneur's skills being complemented by the experience and networking of the VC. Hence, the synergy of efforts is irrelevant. In both models, the entrepreneur's effort decreases by the VC's share, while the VC's effort increases by his share. However, this phenomenon does not occur in a scenario in which efforts are complements.

Elitzur et al. (2011) tackles the issue from the VC's point of view. They develop a model where entrepreneurs compete for VC funding, and find that having a large number of entrepreneurs who race for funding can cause under-investment in technology by entrepreneurs. And, recently, Lukas et al. (2016) study, in a multi-stage setup, how economic and technological uncertainty affect financing. They show that higher uncertainty leads the VC to increase the optimal stake in the venture.

The novelty of this paper is to design optimal contracts in the context of double-sided moral hazard but in an economy in which efforts are complements. This paper approaches the problem from a similar angle to De Bettignies and Brander (2007); however, we depart from their paper in three ways. First, we do not impose any particular functional form for the project revenue function or the disutility of the players' efforts. In this context, we do not impose the assumption that the players' efforts are perfect substitutes and we introduce the notion of complementarity. Second, we make the players' decision to invest in the project endogenous. Third, we solve the entrepreneur's maximization problem in the presence of double-sided moral hazard, and in this scenario, we show that the venture capitalist's share is always binding and, therefore, contrary to the argument by De Bettignies and Brander, there is no efficiency wage. Furthermore, we obtain only real numbers as solutions, and not negative or complex numbers as their model does, and we demonstrate that the solution to the contract regarding the optimal share given to the VC is non-linear and is a fixed point between 0 and 1.

We simulate the model and show that, contrary to the results of Casamatta (2003) and De Bettignies and Brander (2007), the entrepreneur's effort does not monotonically decrease in the share allocated to the VC. This is because the entrepreneur internalizes, in his effort reaction function, the share allocated to the VC and the elasticity and efficiency of the VC's effort. This is also valid for the VC's best response function. Although the treatment is theoretical, the results have practical implications. In the real world of business, complementarity between the entrepreneur and the VC matters. While the entrepreneur looks not only for the funding of the VC, but also for his experience, networks, and prestige, among other factors, the VC searches for a partner that has the ability to outgrow the project. The model is able to predict that when there is a high degree of complementarity between the effort levels of the two partners, they will tend to share the venture in equal halves. This is an empirical implication that we observe in the data (see for instance Kaplan and Stromberg (2002), Goldfarb et al. (2013), and Cumming (2006)).

We can think of the problem we study as arising from the principal/agent paradigm (see Van Ackere, 1993), and we follow a double-sided moral hazard structure similar to that of Bhattacharyya and Lafontaine (1995). The double-sided moral hazard framework has been used in different transactional contexts, for instance Mann and Wissink (1990) used it to study product warranties, Gupta and Romano (1998) applied it in the context of franchising, and Corbett et al. (2005) used it to study optimal shared-savings contracts in supply chains.

In our model, the VC's investment in the project is also endogenous, following the approach of Casamatta (2003), which is equivalent to assuming that the VC buys a share in the project and pays the price that covers start-up costs, including an upfront payment to the entrepreneur

(Kannianien and Keuschningg, 2003, 2004). We simulate the model under the assumption that project revenue is a Constant Elasticity of Substitution (CES) function, whereby we analyze the effect that complementarity has on effort dynamics, the dynamics of the revenue function and the function of the optimal equity distribution. As a special case, we analyze a scenario in which the efforts are perfect substitutes.

The synergy produced by the complementarity of experiences and know-how between the entrepreneur and the VC explain in big part, the dramatic growth observed in the VC industry in the last three decades. In consequence, we recognize a key real world characteristic in our model, which is that VCs provide an extra-financial value to the venture.

The remainder of the paper is structured as follows: in Section 2, we present and solve the model, in Section 3, we simulate the model, and finally in Section 4, we conclude.

3. The Model

It is assumed that an entrepreneur is endowed with an innovative idea. The project requires three types of inputs: an investment level I and two types of non-observable effort denoted e and a. Effort level e can only be supplied by the entrepreneur, while effort level a may only be supplied by the VC. Both the entrepreneur and the VC are risk-neutral.

Both the entrepreneur and the VC may fund investment I. The opportunity cost of investing in the firm is the risk-free rate r, which is standardized to zero to simplify the algebra. Following Casamatta (2003), we make the VC's investment endogenous. We denote the amount of funding provided by the VC by I_{VC} and the amount provided by the entrepreneur by $I - I_{VC}$. This approach is equivalent to assuming that to purchase an equity share "s" in the project, the VC pays $b_{VC} + I$, which covers the start-up costs and an upfront payment b_{VC} to the entrepreneur (Kannianien and Keuschningg, 2003, 2004).

The project revenues are $R(e, a)$ with success probability p , where $0 < p < 1$. In the event of failure, the project revenues are zero with probability $1 - p$. It is assumed that the first derivatives R_e and R_a are positive, and the second derivatives R_{ee} and R_{aa} are negative, and the cross-derivative $R_{ea} = R_{ae}$ is positive. The efforts are costly. $C(e)$ is the entrepreneur's effort disutility, and $B(a)$ is the VC's effort disutility. It is assumed that both functions are increasing at increasing rates, i.e., C_e, $B_a > 0$, C_{ee}, $B_{aa} > 0$. It is further assumed that $C(0) = B(0) = C_e(0) = B_a(0) = 0$.

The social value of the project is expressed as: $R(e, a) - C(e) - B(a) - I$. Therefore, the first-best solution is expressed as: ${}^{R_e}/_{R_a} = {}^{C_e}/_{B_a}$.

This implies that the funding policy is irrelevant, that is, it does not matter who finances the project. However, in a world in which the entrepreneur's and VC's efforts are not observable, the form of funding and how the project cash flows are distributed affect the way in which efforts are made, creating what the literature calls double-sided moral hazard.

3.1. Double-sided moral hazard case

In our model, both the entrepreneur and the VC make efforts that are not observable by the other player. Thus, this is a double-sided moral hazard problem.

The sequence of events is: in the initial stage, the entrepreneur receives funding from the VC and creates a company. The entrepreneur offers the VC an equity share s in the business. In the

second stage, the partners commit simultaneous, non-observable efforts to develop the business. It is assumed that there is no renegotiation and the entrepreneur holds all of the bargaining power.[1] The level of effort chosen by the entrepreneur comes from his incentive-compatibility constraint:

$$e = argmax\ (1-s)pR(e,a) - C(e) - (I - I_{VC}) \tag{1}$$

That is, the entrepreneur maximizes his expected benefit based on his share of the revenues as stipulated in the contract, $1-s$, his rational expectation of the other player's effort, a, the cost of his effort, $C(e)$, and his financial contribution, $(I - I_{VC})$.
The VC also chooses his level of effort based on his incentive-compatibility constraint:

$$a = argmax\ spR(e,a) - B(a) - I_{VC} \tag{2}$$

Given the assumptions, the problem faced by the entrepreneur is:

$$\max_{s,e,a,I_{VC}} (1-s)pR(e,a) - C(e) - (I - I_{VC}) \tag{3a}$$

subject to

$$e = argmax\ (1-s)pR(e,a) - C(e) - (I - I_{VC}) \tag{3b}$$

$$a = argmax\ spR(e,a) - B(a) - I_{VC} \tag{3c}$$

$$spR(e,a) - B(a) \geq I_{VC} \tag{3d}$$

The entrepreneur's maximization problem involves two additional optimization problems expressed in equations 3b and 3c. To solve these additional problems, these equations are replaced by their first-order conditions, following the approach of Holmström (1979). Thus, the problem can be expressed as:

$$\max_{s,e,a,I_{VC}} (1-s)pR(e,a) - C(e) - (I - I_{VC}) \tag{4a}$$

subject to

$$(1-s)pR_e = C_e \tag{4b}$$

$$spR_a = B_a \tag{4c}$$

$$spR(e,a) - B(a) \geq I_{VC} \tag{4d}$$

The incentive-compatibility equations (4b) and (4c) reflect the double-sided moral hazard problem. It is deduced from these equations that the participation share awarded to the VC cannot be $s = 0$ or $s = 1$. Therefore, the project cash flows must be shared. The participation share s can be expressed as:

$$s = \frac{B_a/R_a}{B_a/R_a + C_e/R_e} \in (0,1) \tag{5}$$

For $s = 0$ to occur, $B_a = 0$ must exist, which means that the VC makes no effort ($a = 0$). For the same reason, if $s = 1$, then $C_e = 0$, in which case the entrepreneur makes no effort

[1] The VC may also possess the bargaining power, and then the optimization problem should be solved from the VC's perspective. The optimization results are identical in this case because of the dual-sided nature of the case at hand.

($e = 0$). Therefore, if the entrepreneur and the VC each supply a positive effort level, the project cash flows must be shared. Equation (5) guarantees that the level of participation will be in the range of 0 to 1, but this equation does not identify the optimal level of equity participation. To find s^*, problem (4a) must be solved for the effort levels, equity shares, and investment.

Under the assumptions of De Bettignies and Brander (2007) concerning the revenue and effort functions, namely $R(e,a) = \alpha e + \beta a$, $C(e) = {}^{e^2}/_2$ and $B(a) = {}^{a^2}/_2$, equation (5) is given by:

$$s = \frac{{}^{a}/_{\beta}}{{}^{a}/_{\beta} + {}^{e}/_{\alpha}} \tag{6}$$

De Bettignies and Brander assume that the efforts are perfect substitutes, meaning that the entrepreneur's effort decreases by s when the VC's effort increases by s. This is because the partners do not internalize either the characteristics of the other partner or the equity share in their effort best-response functions.
Below, we solve the entrepreneur's maximization problem. We show that the VC's participation constraint is binding, and hence there is no unrestricted problem, in contrast to the argument by De Bettignies and Brander (2007). The optimization problem is always restricted, and the equity share assigned to the VC that solves the problem is $s^* \in (0,1)$. Because the restriction is binding, we do not observe the "efficiency wage" outcome of De Bettignies and Brander, that is the entrepreneur does not have to offer a higher equity share in order to increase the productivity of the VC.[2]

Proposition 1

a) The entrepreneur's problem is restricted because $\lambda_3 > 0$, where λ_3 represents the shadow price of the VC's participation constraint.[3]

b) The level of equity participation s^* that solves the entrepreneur's problem is found when the marginal value of the entrepreneur's effort is equal to the marginal value of the VC's effort, i.e., $\lambda_1 R_e = \lambda_2 R_a$, where λ_1 and λ_2 represent the shadow prices of the incentive-compatibility constraints of both the entrepreneur and the VC.

Proof.

The entrepreneur's optimization problem can be expressed via Lagrange multipliers as:

$L = (1-s)pR(e,a) - C(e) - (I - I_{VC}) + \lambda_1((1-s)pR_e - C_e) + \lambda_2(spR_a - B_a) + \lambda_3(spR(e,a) - B(a) - I_{VC})$

1. The First Order Condition for the VC's investment level I_{VC} is:

$$\frac{\partial L}{\partial I_{VC}} = 1 - \lambda_3 = 0 \tag{7}$$

[2] "Efficiency wage" refers to the economic theory postulating that firms may be willing to pay wages above market-clearing conditions because it increases workers' productivity (see Katz, 1986)

[3] The multiplier or shadow price, measures the response of the optimal value of the objective function to changes in the constraint, which the multiplier is attached to. In this case, λ_3 measures the sensitivity of the entrepreneur's objective function to a change in the amount of funding provided by the VC, I_{VC}.

2. The First Order Condition for the entrepreneur's effort e is:

$$\frac{\partial L}{\partial e} = ((1-s)pR_e - C_e) + \lambda_1((1-s)pR_{ee} - C_{ee}) + \lambda_2(spR_{ea}) + \lambda_3(spR_e) = 0$$

where the first term in brackets is zero because of the entrepreneur's incentive-compatibility constraint in equation (4b). Given (7), the above expression would be:

$$\lambda_1((1-s)pR_{ee} - C_{ee}) + \lambda_2(spR_{ea}) = -(spR_e) \tag{8}$$

3. The First Order Condition for the VC's effort a is:

$$\frac{\partial L}{\partial a} = (1-s)pR_a + \lambda_1((1-s)pR_{ae}) + \lambda_2(spR_{aa} - B_{aa}) + \lambda_3(spR_a - B_a) = 0$$

where the last term is zero because of the VC's incentive-compatibility constraint in equation (4c). Thus, the above expression would be:

$$\lambda_1((1-s)pR_{ae}) + \lambda_2(spR_{aa} - B_{aa}) = -(1-s)pR_a \tag{9}$$

4. At a given level of equity participation s awarded to the VC:

$$\frac{\partial L}{\partial s} = -pR(e,a) + \lambda_1(-pR_e) + \lambda_2(pR_a) + \lambda_3(pR(e,a)) = 0$$

Given (7), the above expression would be:

$$\lambda_1 R_e = \lambda_2 R_a \tag{10}$$

Equation (7) shows that $\lambda_3 = 1$ and, therefore, that the VC's equity participation is binding. Equation (10) shows that the entrepreneur's problem is solved when the marginal value of the entrepreneur's effort is identical to the marginal value of the VC's effort. It is deduced from this equation that $\lambda_1, \lambda_2 > 0$. If this does not occur, equations (8) and (9) reduce to $spR_e = 0$ and $(1-s)pR_a = 0$, which cannot be the case because $s, p, R_e, R_a > 0$. □

The above system of equations is reduced to:

$$\lambda_3 = 1 \tag{11}$$

$$\lambda_1((1-s)pR_{ee} - C_{ee}) + \lambda_2(spR_{ea}) = -spR_e \tag{12}$$

$$\lambda_1((1-s)pR_{ae}) + \lambda_2(spR_{aa} - B_{aa}) = -(1-s)pR_a \tag{13}$$

$$\lambda_1 R_e = \lambda_2 R_a \tag{14}$$

The next proposition provides the equity share given to the VC that maximizes the entrepreneur's problem.

Proposition 2

The equity participation level given to the VC that solves the entrepreneur's problem is non-linear and at a fixed point takes the form of $s^* = l(s^*)$, where:

$$l(s^*) = \frac{((1-s^*)pR_{ee} - C_{ee})R_a^2}{((1-s^*)pR_{ee} - C_{ee})R_a^2 + (s^*pR_{aa} - B_{aa})R_e^2}$$

Proof.

Using equations (12) and (13), we solve for λ_1 and λ_2 from the following system of equations:

$$\begin{bmatrix} (1-s)pR_{ee} - C_{ee} & spR_{ea} \\ (1-s)pR_{ae} & spR_{aa} - B_{aa} \end{bmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -spR_e \\ -(1-s)pR_a \end{pmatrix} \tag{15}$$

then:

$$\lambda_1 = \frac{|A_1|}{|A|} \qquad \lambda_2 = \frac{|A_2|}{|A|} \tag{16}$$

where

$$|A| = ((1-s)pR_{ee} - C_{ee})(spR_{aa} - B_{aa}) - s(1-s)p^2R_{ae}^2 \tag{17}$$

$$|A_1| = s(1-s)p^2R_aR_{ea} - sp(spR_{aa} - B_{aa})R_e \tag{18}$$

$$|A_2| = s(1-s)p^2R_{ae}R_e - (1-s)((1-s)pR_{ee} - C_{ee})pR_a \tag{19}$$

plugging λ_1 and λ_2 in (14), we obtain:

$$s(spR_{aa} - B_{aa})R_e^2 = (1-s)((1-s)pR_{ee} - C_{ee})R_a^2 \tag{20}$$

and re-ordering, we solve for the optimal equity share given to the VC, and that solves the entrepreneur problem:

$$s^* = \frac{((1-s^*)pR_{ee} - C_{ee})R_a^2}{((1-s^*)pR_{ee} - C_{ee})R_a^2 + (s^*pR_{aa} - B_{aa})R_e^2} \tag{21}$$

□

Equation (21) generalises the model of De Bettignies and Brander (2007).

Using the assumptions of De Bettignies and Brander, equation (21) is expressed as follows:

$R(e,a) = \alpha e + \beta a$; $C(e) = \frac{e^2}{2}$; $B(a) = \frac{a^2}{2}$; $R_e = \alpha$; $C_e = e$;$B_a = a$; $R_a = \beta$; $C_{ee} = 1$; $B_{aa} = 1$

$$s^* = \frac{R_a^2}{R_a^2 + R_e^2} = \frac{\beta^2}{\beta^2 + \alpha^2} \in (0,1) \tag{22}$$

In the De Bettignies and Brander model, the elasticity of the efforts is what determines how the project cash flows are distributed. As we can see from equation (22), if the elasticity of the VC's effort rises, he receives a greater share of the business, while if the elasticity of the entrepreneur's effort increases, the VC receives a smaller share of the project cash flows.

We now discuss the effects that the degree of complementarity of efforts, the elasticity and the efficiency of the entrepreneur's efforts and the VC's efforts all have on the dynamics of the effort best-response functions and on the optimal equity participation expressed in equation (21).

4. Effort dynamics and the equity shares

We simulate equation (21) by assuming a Constant Elasticity of Substitution (CES) project revenue function in the form of $R(e,a) = A[\alpha e^{\rho} + \beta a^{\rho}]^{1/\rho}$.[4] The parameters α and β correspond to the elasticity of the partners' efforts, A is a productivity parameter and ρ is a substitution parameter. This is a well-known function used extensively in the production part of the microeconomics literature (See Varian, 1992, pag.19-20). Here, the two inputs of production, e and a, can contribute to the revenues of the project. The CES function allows any degree of complementarity; at one extreme we have the case of perfect substitutes (zero complementarity between the inputs), which is the firm can obtain revenues from using either entrepreneurial effort or venture capital effort independently. At the other extreme, the inputs can be perfect complements, which is, they must be used in fixed proportions to produce revenues.[5] In between these two extreme cases, we have different degrees of complementarity between the inputs, that is, they are mixed together in different proportions in order to produce revenues.

Mathematically, it can be proven that if $\rho = 1$, efforts are perfect substitutes; if $\rho = -\infty$, efforts are perfect complements; and if $-\infty < \rho < 1$, there is complementarity of efforts (for the proof, see Saito 2011).

We will also assume that the disutility of the entrepreneur's effort is given by $C(e) = \delta_E e^2/2$, while the disutility of the VC's effort is $B(a) = \delta_{VC} a^2/2$. These are the same functions used by De Bettignies and Brander (2007), Casamatta (2003) and Fairchild (2011), but for the efficiency parameters $\delta_E > 0$, and $\delta_{VC} > 0$, which permit seeing the effects of being more or less efficient in the delivery of efforts.[6,7]

From the incentive-compatibility constraints, we obtain the best-response functions concerning the equity share awarded to the VC in the first stage of the game. As the production technology is CES, and given the assumptions regarding the disutility of efforts, the effort dynamics of the entrepreneur and the VC are expressed by (see Appendix 1 for the proof):

$$e = \frac{(1-s)\alpha p A}{\delta_E}\left[\alpha + \beta\left[\frac{s\beta/\delta_{VC}}{(1-s)\alpha/\delta_E}\right]^{\frac{\rho}{2-\rho}}\right]^{\frac{1-\rho}{\rho}} \tag{23}$$

$$a = \frac{s\beta p A}{\delta_{VC}}\left[\beta + \alpha\left[\frac{(1-s)\alpha/\delta_E}{s\beta/\delta_{VC}}\right]^{\frac{\rho}{2-\rho}}\right]^{\frac{1-\rho}{\rho}} \tag{24}$$

Proposition 3

In the context in which the two effort levels are perfect substitutes: the share s assigned to the VC is inversely related to the effort of the entrepreneur, whereas it is positively related to the effort of the VC.

[4]Notice that other functions presented in the literature are not more complex than the CES. See for instance De Bettignies (2008) and Bolton and Dewatripont (2005).

[5]The classic example of perfect complementarity is the case of left and right shoes; you have to use them in the fixed proportion 1:1.

[6]The lower the efficiency parameter, the more efficient effort is, and thus the lower the cost of providing effort.

[7]Assuming more general disutility functions does not add new insights to the complementarity analysis, but makes the solution of the model more complex because we will need to simulate for three fixed pointts: e^*, a^*, and s^*.

Proof.

The result comes directly from the best-response functions of the entrepreneur and the VC expressed in (23) and (24). If $\rho = 1$, the efforts are represented by $e = (1 - s)\alpha pA/\delta_E$ and $a = s\beta pA/\delta_{VC}$. Then, $\partial e/\partial s = -\alpha pA/\delta_E < 0$, and $\partial a/\partial s = \beta pA/\delta_{VC} > 0$. □

Proposition 3 places us in the world of perfect substitution of effort levels, which is exactly the world of De Bettignies and Brander (2007), where an increase in the equity share awarded to the VC causes the entrepreneur's effort to monotonically decrease and the VC's effort to monotonically increase. This occurs because the entrepreneur and the VC do not incorporate the characteristics of the other partner or the equity participation in their effort functions.

Figure 1 depicts the effort dynamics of each player at different levels of equity participation s using the parameters in Table 1 (see Appendix 2).[8] The aim of the exercise is to observe the effect that the substitution parameter ρ has on the dynamics of the efforts supplied by the entrepreneur and the VC. In case 1, perfect substitution of efforts is assumed, and as established in Proposition 3, it can be seen that when the entrepreneur retains full property of the endeavor, $s = 0$, he exerts maximum effort. His effort level monotonically decreases by s, reaching no effort when $s = 1$. On the contrary, the VC will deploy no effort when $s = 0$, and it increases it monotonically by s, reaching a maximum when all the equity is assigned to him, $s = 1$. It can also be observed, that when they share the endeavor in equals parts, $s = 0, 5$, they exert the same effort levels.

In cases 2-4, different degrees of effort complementarity are assumed. It becomes graphic that when there is complementarity, effort levels become concave on the equity share. The graphs indicate that there must be a level of equity participation awarded to VC that maximizes the entrepreneur's effort. The same holds for the VC's effort, i.e., there must be a level of equity that maximizes his effort. This phenomenon is established in Proposition 4.

Notice that as the degree of effort complementarity increases, for instance case 4: $\rho = -10$, both the entrepreneur and the VC deploy no effort if $s = 0$ or $s = 1$. A large degree of complementarity means that in order for the endeavour to be successful, both partners must put their abilities at work at the same time. If one of them is not part of the project, $s = 0$ or $s = 1$, then the other partner does not exert effort at all because it is worthless.[9]

Proposition 4

If efforts are complementary:

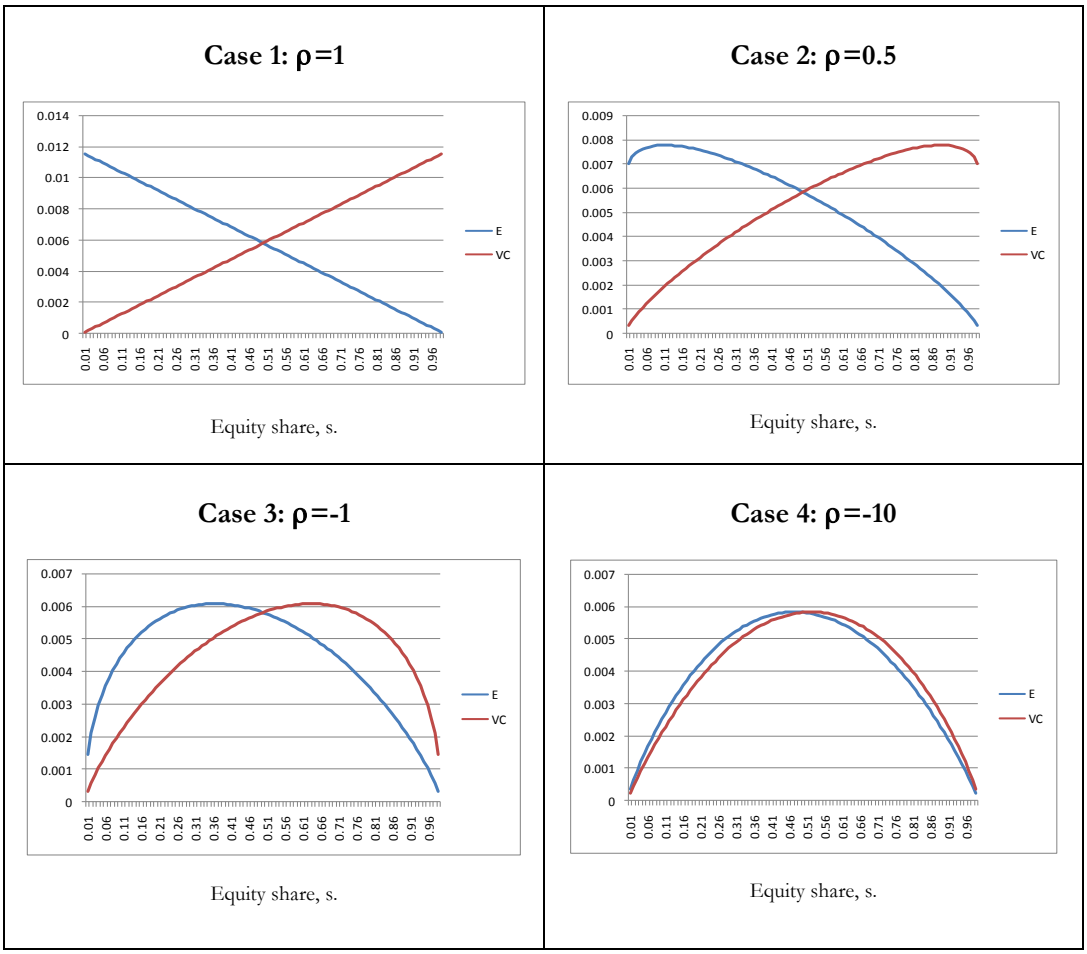
a) There is an equity participation level s_e^* allocated to the VC that maximizes the entrepreneur's effort. This level of equity is non-linear and at a fixed point takes the form of $s_e^* = g(s_e^*)$.

b) There is an equity participation level s_{VC}^* allocated to the VC that maximizes the VC's effort. This level of equity is non-linear and at a fixed point takes the form of $s_{VC}^* = h(s_{VC}^*)$.

[8] We use these parameters for convenience as in Fairchild (2011).

[9] If one of your shoes is missing, there is no point in using only one of them. Indeed, in this case you would walk or run better without them at all.

Figure 1: *Effort dynamics for different degrees of effort complementarity* ρ



Proof.

See Appendix 3.

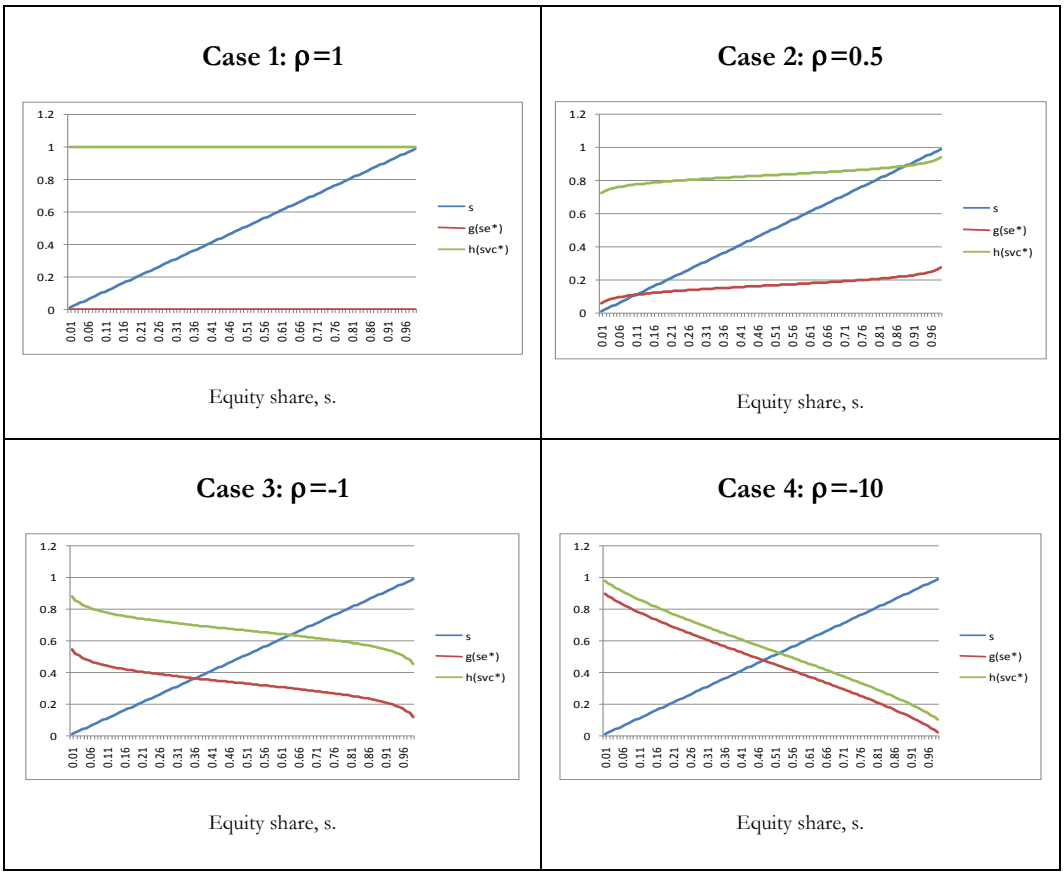
Figure 2 illustrates the results of Proposition 4. The graphs depict the simulation results obtained using the parameters in Table 1 (see Appendix). The figure shows the equilibrium point in the equity participation that maximizes the efforts of both the entrepreneur and the VC. The equilibrium point occurs when the functions $g(s_e^*)$ and $h(s_{VC}^*)$ intersect the 45° line.

As should be expected when $\rho = 1$, the equity share that maximizes the entrepreneur's effort is $s_e^* = 0$, whereas the level that maximizes the VC's effort is $s_{VC}^* = 1$. This result occurs because when $\rho = 1$, the entrepreneur's effort decreases by the participation level. However, the VC's effort increases by the participation level.

When $\rho < 1$, namely when efforts are complements, the share that maximizes the entrepreneur's effort is $s_e^* > 0$ and the share that maximizes the VC's effort is $s_{VC}^* < 1$.[10] The graphs show that $s_e^* < s_{VC}^*$. However, the distance between the two participation levels falls as

[10]If we replace the assumption of risk-neutrality for risk-aversion on any of the agents, this will increase the share they are willing to accept in order to compensate for the risk they are taking. Thus, risk-aversion will not change the implications of the model, but will increase the level of complexity of it.

Figure 2: *Equity share that maximizes effort, for different degrees of complementarity, ρ.*



the complementarity of efforts rises, meaning the levels that maximize efforts tend to be similar. Indeed, as efforts tend to be more complementary, the project cash flows are distributed in similar proportions, at approximately 50% for each partner. Notice that this result is in line with data reported in the literature. Kaplan and Stromberg (2002), studied actual contracts between entrepreneurs and VCs and found that the "VC typically controls roughly 50% of the cash flow rights; founders, 30%; and others, 20%." Goldfarb et al. (2013) obtained electronic data from a law firm that operated in California and the Western United States, and reported on the deals between founders, angel capitalists and VCs. They found that, on average, the final VC ownership was between 49% (in large deals) and 52% (in VC only deals), and Cumming (2006) examined 214 Canadian VC funds and reported that the mean VC share is 51% with a standard deviation of .275.

This evidence reaffirms the idea that entrepreneurs look for a VC that can complement the skills they have, and not only are looking for the VC that offers the highest valuation. Smith (2001) surveyed 415 firms, of which 143 responded, and reported, "Of the 97 firms that received more than one offer to invest, 36 did not accept the highest offer". In the same survey reputational factors along with value-added factors such as formulating business strategy, and monitoring the company's performance were also valued for entrepreneurs. Thus, those VCs that can propose some attributes that entrepreneurs are lacking will probably be able to obtain half of the firm cash flows rights.

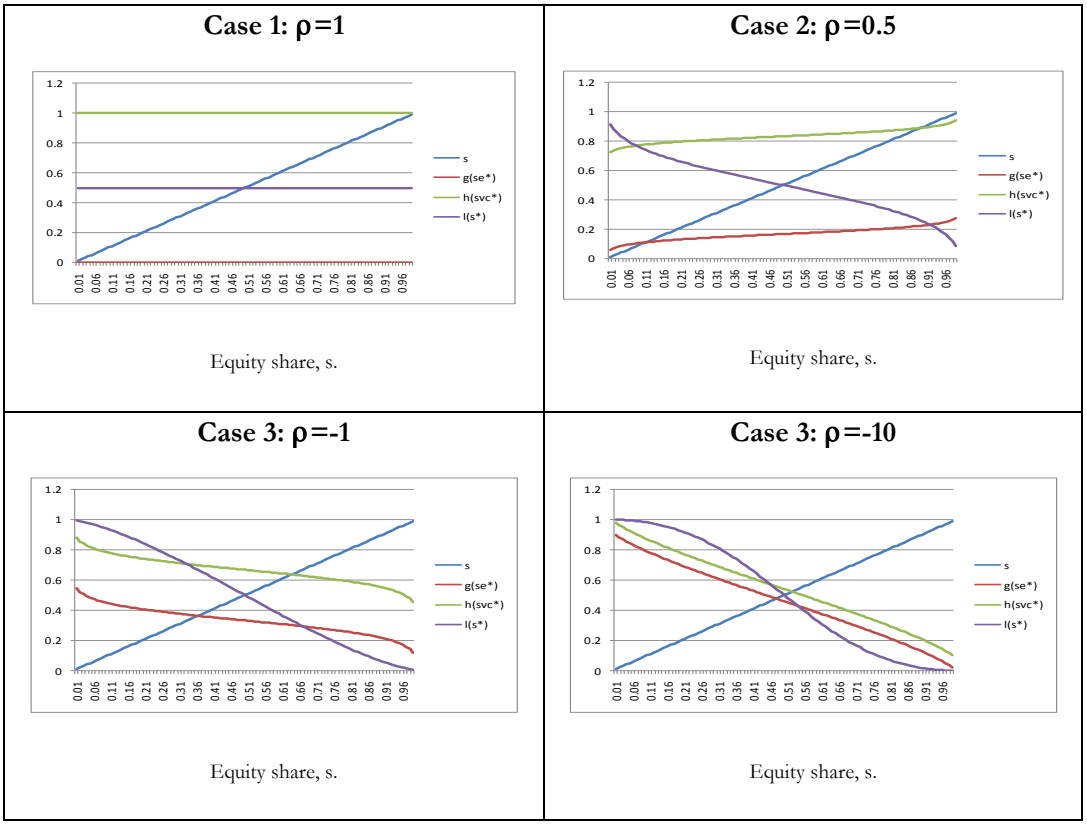
Note that the equilibrium points of the functions $g(s_e^*)$ and $h(s_{VC}^*)$ do not solve the entrepreneur's problem because we have only considered the problem at the level of the partners'

incentive-compatibility constraints. The equilibrium point that solves the entrepreneur's problem is expressed in equation (21). The share assigned to the VC that solves the entrepreneur's problem is at a fixed point $s^* = l(s^*)$, where $s^* \in (s_e^*, s_{VC}^*)$.

The results of Propositions 2 and 4 are illustrated in Figure 3. The graphs depict the results of simulations using the parameters in Table 1 (see Appendix). Figure 3 reflects the trade-off in the equilibrium efforts caused by the level of participation awarded to VC. The figure indicates that at different degrees of complementarity, the equity participation that solves the entrepreneur's problem is found between the share that maximizes the entrepreneur's effort and the share that maximizes the VC's effort. An increase in the complementarity of efforts causes the equilibrium points in Proposition 4 to approach the level that would solve the entrepreneur's problem.[11]

[11]The optimal share awarded to the VC not only depends upon the complementarity of efforts but also upon the elasticity parameters of the partners, and their efficiency. We also carried out simulations for different elasticity and efficiency parameters, and as expected, greater elasticity or more efficient effort is rewarded with a larger equity share. However, regardless of this, the effect on the distribution of cash flows is diluted as complementarity increases. That is, the complementarity effect dominates the effects of the other parameters. Results are available upon request.

Figure 3: *Optimal equity share for different degrees of complementarity, ρ.*



5. Conclusion

This paper focuses on how the complementarity of efforts between an entrepreneur and a VC affect the equity share that the entrepreneur is willing to allocate to the VC. The complementarity of efforts is key to the success of a firm. The advisory services, networking and the experience of VCs are complemented by the technological and innovation skills of entrepreneurs. These complementary skills create a synergy that has a considerable impact on the company's value.

The contributions we make are both technical and practical. With respect to the former, we solve a model in which the solutions belong to the interval [0,1], and thus are empirically plausible, and we also show that De Bettignies and Brander (2007) is a particular case of our setup. Furthermore, we show that when efforts are perfect substitutes, as argued by Casamatta (2003) and De Bettignies and Brander, the entrepreneur's effort monotonically decreases by the share given to the VC, whereas the VC's effort monotonically grows by the share of the project's cash flows that he receives. However, when complementarity occurs, this phenomenon is not replicated because both the entrepreneur and the VC incorporate, in their best-response functions, the characteristics of the other partner and the equity shares, and hence their efforts are non-linear with respect to the equity participation levels. The optimal share awarded to the VC depends upon the elasticity and efficiencies of the players' efforts and on the complementarity of those efforts. Increasing the complementary of efforts increases the proportion of equity that the entrepreneur is willing to give to the VC. Indeed, when the efforts of the entrepreneur and the VC are highly complementary, they will tend to divide the equity of the new firm into equal shares. This is supported by empirical evidence as reported by Kaplan and Stromberg (2002), Goldfarb et al. (2013), and Cumming (2006), all of whom stated that in actual transactions, the VC obtains around 50% of the equity share in the new venture. Thus, this study formalizes what is actually observed in real deals and gives a rationality for the equal split of equity between entrepreneurs and VCs.
Further work using the complementarity approach should include, among others, the consequences on equity share allocation of risk aversion of both partners, how the level of complementarity affects the number of VCs that the entrepreneur contacts, the way in which the entrepreneurs search for complementarity has an impact on crowd-funding as it provides funding but no advising, and how the existence of more or less complementarity between the entrepreneur and the VC may affect the size of the VC portfolio.

6. References

[1] Zoltan. Acs. How is entrepreneurship good for economic growth. *Innovations: Technology, Governance, Globalization*, 1(1):97–107, 2006.

[2] William Baumol. *The Free Market Innovation Machine: Analyzing the Growth Miracle of Capitalism*. Princeton University Press, 2002.

[3] Sugato Bhattacharyya and Francine Lafontaine. Double-sided moral hazard and the nature of share contracts. *The RAND Journal of Economics*, 26(4):761–781, 1995.

[4] Patrick Bolton and Mathias Dewatripont. *Contract theory*. MIT press, 2005.

[5] Catherine Casamatta. Financing and advising: Optimal financial contracts with venture capitalists. *Journal of Finance*, 58(5):2059–2086, 2003.

[6] Charles J. Corbett, Gregory A. DeCroix, and Albert Y. Ha. Optimal shared-savings contracts in supply chains: Linear contracts and double moral hazard. *European Journal of Operational Research*, 163(3):653 – 667, 2005.

[7] Douglas J. Cumming. The determinants of venture capital portfolio size: Empirical evidence. *Journal of Business*, 79:1083–1126, 2006.

[8] Jean-Etienne de Bettignies. Financing the entrepreneurial venture. *Management Science*, 54(1):151–166, 2008.

[9] Jean-Etienne de Bettignies and James Brander. Financing entrepreneurship: Bank finance versus venture capital. *Journal of Business Venturing*, 22(6):808–832, 2007.

[10] Ramy Elitzur and Arieh Gavious. Selection of entrepreneurs in the venture capital industry: An asymptotic analysis. *European Journal of Operational Research*, 215(3):705–712, 2011.

[11] Richard Fairchild. An entrepreneur's choice of venture capitalist or angel-financing: A behavioral game-theoretic approach. *Journal of Business Venturing*, 26(3):359 – 374, 2011.

[12] Arieh Gavious and Ramy Elitzur. A multi-period game theoretic model of venture capitalists and entrepreneurs. *European Journal of Operational Research*, 144:440–453, 2003.

[13] Brent D. Goldfarb, Gerard Hoberg, David Kirsch, and Alexander J. Triantis. Are angels different? an analysis of early venture financing. *Robert H. Smith School Research Paper No. RHS 06-072.*, 2013.

[14] Michael Gorman and William A. Sahlman. What do venture capitalists do? *Journal of Business Venturing*, 4(4):231–248, 1989.

[15] Srabana Gupta and Richard Romano. Monitoring the principal with multiple agents. *RAND Journal of Economics*, 29(2):427–442, 1998.

[16] Thomas Hellmann and Manju Puri. Venture capital and the professionalization of start-up firms: Empirical evidence. *Journal of Finance*, 57(1):169–197, 2002.

[17] Yael V. Hochberg, Alexander Ljungqvist, and Yang Lu. Whom you know matters: Venture capital networks and investment performance. *The Journal of Finance*, 62(1):251–301, 2007.

[18] Bengt Holmstrom. Moral hazard and observability. *Bell Journal of Economics*, 10(1):74–91, 1979.

[19] E. Houben. Venture capital, double-sided adverse selection, and double-sided moral hazard. *Unpublished Working Paper*, 2003.

[20] David H. Hsu. What do entrepreneurs pay for venture capital affiliation? *The Journal of Finance*, 59(4):1805–1844, 2004.

[21] Vesa Kanniainen and Christian Keuschnigg. The optimal portfolio of start-up firms in venture capital finance. *Journal of Corporate Finance*, 9(5):521–534, 2003.

[22] Vesa Kanniainen and Christian Keuschnigg. Start-up investment with scarce venture capital support. *Journal of Banking & Finance*, 28(8):1935–1959, 2004.

[23] Steven N. Kaplan and Per Strömberg. Financial contracting theory meets the real world: An empirical analysis of venture capital contracts. *The Review of Economic Studies*, 70(2):281–315, 2003.

[24] Lawrence F. Katz. Efficiency wage theories: A partial evaluation. *NBER Macroeconomics Annual*, (1):235–290, 1986.

[25] Elmar Lukas, Sascha Mölls, and Andreas Welling. Venture capital, staged financing and optimal funding policies under uncertainty. *European Journal of Operational Research*, 250(1):305–313, 2016.

[26] D. N. P. Murthy and . Djamaludin. New product warranty: A literature review. *International Journal of Production Economics*, 79(3):231–260, 2002.

[27] NVCA. *National Venture Capital Association Yearbook*. Thomson Reuters, 2015.

[28] Joseph Schumpeter. *The Theory of Economic Development: An Inquiry into Profits, Credit, Interest and the Business Cycle*. 2nd ed. Harvard University Press, 1934.

[29] Gordon Smith. How early stage entrepreneurs evaluate venture capitalists. *The Journal of Private Equity*, 4(2):33–45, 2001.

[30] Morten Sorensen. How smart is smart money? a two-sided matching model of venture capital. *The Journal of Finance*, 62(6):2725–2762, 2007.

[31] Saito Tetsuya. How do we get cobb-douglas and leontief functions from ces function: A lecture note on discrete and continuum differentiated object models. *Journal of Industrial Organization Education*, 6(1):1–13, 2012.

[32] Ann Van Ackere. The principal/agent paradigm: Its relevance to various functional fields. *European Journal of Operational Research*, 70(1):83–103, 1993.

[33] Hal R. Varian. *Microeconomic Analysis*. Norton, 3rd. edition, 1992.

[34] Gornall Will and Ilya A. Strebulaev. The economic impact of venture capital: Evidence from public companies. *Stanford University Graduate School of Business Research Paper*, 15-55, 2015.

7. Appendices

7.1. ***Appendix 1: Effort Best-Response Functions***

If $R(e,a) = A[\alpha e^{\rho} + \beta a^{\rho}]^{\frac{1}{\rho}}$, and $C(e) = \delta_E e^2/2$, and $B(a) = \delta_{VC} a^2/2$, then:
$R_e = A[\alpha e^{\rho} + \beta a^{\rho}]^{\frac{1}{\rho}-1} \alpha e^{\rho-1}$
$R_a = A[\alpha e^{\rho} + \beta a^{\rho}]^{\frac{1}{\rho}-1} \beta a^{\rho-1}$
$C_e = \delta_E e$
$B_a = \delta_{VC} a$

If the partners' incentive-compatibility constraints (equations 4b and 4c) are replaced by the expressions above, then:

$$(1-s)pA[\alpha e^{\rho} + \beta a^{\rho}]^{\frac{1}{\rho}-1} \alpha e^{\rho-1} = \delta_E e \tag{25}$$

$$spA[\alpha e^{\rho} + \beta a^{\rho}]^{\frac{1}{\rho}-1} \beta a^{\rho-1} = \delta_{VC} a \tag{26}$$

If we divide equation 25 by equation 26 and reorder, we obtain:

$$e = a\left[\frac{(1-s)\alpha}{s\beta}\frac{\delta_{VC}}{\delta_E}\right]^{\frac{1}{2-\rho}} \tag{27}$$

Then, plugging 27 into 26 to obtain the VC best-response function:

$$a = \frac{s\beta pA}{\delta_{VC}}\left[\beta + \alpha\left[\frac{(1-s)\alpha/\delta_E}{s\beta/\delta_{VC}}\right]^{\frac{\rho}{2-\rho}}\right]^{\frac{1-\rho}{\rho}} \tag{28}$$

and plugging 28 into 27, we solve for the entrepreneur best-response function:

$$e = \frac{(1-s)\alpha pA}{\delta_E}\left[\alpha + \beta\left[\frac{s\beta/\delta_{VC}}{(1-s)\alpha/\delta_E}\right]^{\frac{\rho}{2-\rho}}\right]^{\frac{1-\rho}{\rho}} \tag{29}$$

7.2. ***Appendix 2***

Table 1: Simulation Parameters

Parameter	Case 1	Case 2	Case 3	Case 4
α	0.5	0.5	0.5	0.5
β	0.5	0.5	0.5	0.5
δ_E	30	30	30	30
δ_{VC}	30	30	30	30
A	1	1	1	1
p	0.7	0.7	0.7	0.7
ρ	1	0.5	-1	-10

7.3. ***Appendix 3***

Proof. Proof of Proposition 4, part a:

The derivative of equation 23 with respect to the share allocated to the VC is:

$$\frac{\partial e}{\partial s} = -\frac{\alpha p A}{\delta_E}\left[\alpha + \beta\left(\frac{s}{1-s}\frac{\beta}{\alpha}\frac{\delta_E}{\delta_{VC}}\right)^{\frac{\rho}{2-\rho}}\right]^{\frac{1-\rho}{\rho}} + \frac{(1-s)\alpha p A}{\delta_E}\frac{(1-\rho)}{\rho}\left[\alpha + \beta\left(\frac{s}{1-s}\frac{\beta}{\alpha}\frac{\delta_E}{\delta_{VC}}\right)^{\frac{\rho}{2-\rho}}\right]^{\frac{1-2\rho}{\rho}} \times \beta\frac{\rho}{2-\rho}\left(\frac{s}{1-s}\frac{\beta}{\alpha}\frac{\delta_E}{\delta_{VC}}\right)^{\frac{2\rho-2}{2-\rho}}\left[\frac{\beta}{\alpha}\frac{\delta_E}{\delta_{VC}}\left(\frac{1}{(1-s)^2}\right)\right]$$

If we set the expression above to zero and reorder, then:

$$s_E^* = g(s_E^*) = \frac{\frac{1-\rho}{2-\rho}\beta\left(\frac{s_E^*}{1-s_E^*}\frac{\beta}{\alpha}\frac{\delta_E}{\delta_{VC}}\right)^{\frac{\rho}{2-\rho}}}{\left[\alpha + \beta\left(\frac{s_E^*}{1-s_E^*}\frac{\beta}{\alpha}\frac{\delta_E}{\delta_{VC}}\right)^{\frac{\rho}{2-\rho}}\right]}$$

Proof of Proposition 4, part b:

The derivative of equation 24 with respect to the share allocated to the VC is:

$$\frac{\partial a}{\partial s} = \frac{\beta p A}{\delta_{VC}}\left[\beta + \alpha\left(\frac{(1-s)}{s}\frac{\alpha}{\beta}\frac{\delta_{VC}}{\delta_E}\right)^{\frac{\rho}{2-\rho}}\right]^{\frac{1-\rho}{\rho}} - \frac{s\beta p A}{\delta_{VC}}\frac{(1-\rho)}{\rho}\left[\beta + \alpha\left(\frac{(1-s)}{s}\frac{\alpha}{\beta}\frac{\delta_{VC}}{\delta_E}\right)^{\frac{\rho}{2-\rho}}\right]^{\frac{1-2\rho}{\rho}} \times \alpha\frac{\rho}{2-\rho}\left(\frac{(1-s)}{s}\frac{\alpha}{\beta}\frac{\delta_{VC}}{\delta_E}\right)^{\frac{2\rho-2}{2-\rho}}\left[\frac{\alpha}{\beta}\frac{\delta_{VC}}{\delta_E}\left(\frac{1}{s^2}\right)\right]$$

If we set the expression above to zero and reorder, then:

$$s_{VC}^* = h(s_{VC}^*) = 1 - \frac{\frac{1-\rho}{2-\rho}\alpha\left(\frac{(1-s_{VC}^*)}{s_{VC}^*}\frac{\alpha}{\beta}\frac{\delta_{VC}}{\delta_E}\right)^{\frac{\rho}{2-\rho}}}{\left[\beta + \alpha\left(\frac{(1-s_{VC}^*)}{s_{VC}^*}\frac{\alpha}{\beta}\frac{\delta_{VC}}{\delta_E}\right)^{\frac{\rho}{2-\rho}}\right]}$$

which shows that the solution is a fixed point and that, it is non-linear. □